

EUROPEAN CENTRAL BANK  
WORKING PAPER SERIES



**WORKING PAPER NO. 292**

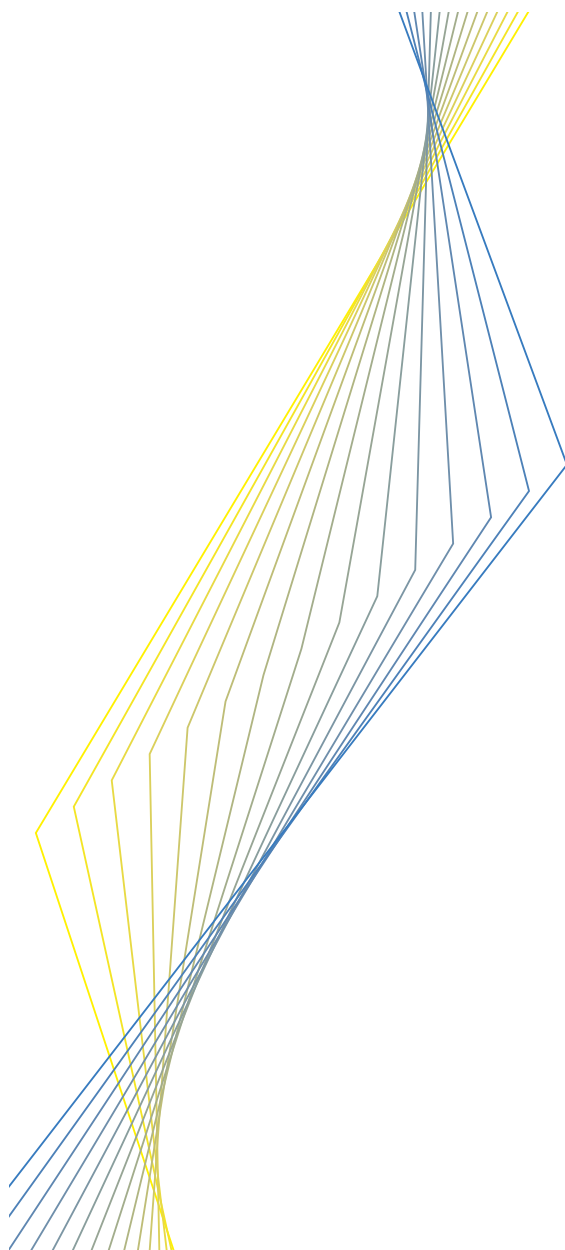
**BANK MERGERS, COMPETITION  
AND LIQUIDITY**

**BY ELENA CARLETTI,  
PHILIPP HARTMANN AND  
GIANCARLO SPAGNOLO**

**November 2003**

EUROPEAN CENTRAL BANK

WORKING PAPER SERIES



**WORKING PAPER NO. 292**

**BANK MERGERS, COMPETITION  
AND LIQUIDITY<sup>1</sup>**

**BY ELENA CARLETTI<sup>2</sup>,  
PHILIPP HARTMANN<sup>3</sup> AND  
GIANCARLO SPAGNOLO<sup>4</sup>**

**November 2003**

<sup>1</sup> We would like to thank Franklin Allen, Giuseppe Bertola, Ulrich Bindseil, Urs Birchler, Jürg Blum, Vittoria Cerasi, Hans Degryse, Fiorella De Fiore, Charles Goodhart, Martin Hellwig, Cornelia Holthausen, Haizhou Huang, Roman Inderst, Andreas Irmen, Simone Manganelli, Bruno Parigi, Rafael Repullo, Jean-Charles Rochet, Richard Rosen, Martin Ruckes, Rune Stenbacka and Jürgen Weigand for comments and suggestions. Thanks also to the participants at the Fed Chicago Bank Structure Conference 2003, the Launching Workshop of the ECB-CFS network in Frankfurt, the Third Joint Central Bank Conference in Basel, the CEPR/Fundacion BBVA conference on universal banking in Madrid, EFA 2002 in Berlin, EEA and ESEM 2002 in Venice, GFA 2002 in Cologne, ESSFM 2002 in Gerzensee; and to seminar participants at the Bank of England, ECB, Ente Enaudi, EUI, Fed Board, New York Fed, George Washington University, IMF, Swiss National Bank, University Carlos III, and University of Zurich. We appreciated the excellent research assistance by Andres Manzanares and Sandrine Corvoisier. Any views expressed are only the authors' own and do not necessarily coincide with the views of the ECB or the Eurosystem. This paper can be downloaded without charge from <http://www.ecb.int> or from the Social Science Research Network electronic library at [http://ssrn.com/abstract\\_id=487471](http://ssrn.com/abstract_id=487471).

<sup>2</sup> University of Mannheim, Department of Economics, 68131 Mannheim, Germany, email: [carletti@rumms.uni-mannheim.de](mailto:carletti@rumms.uni-mannheim.de).

<sup>3</sup> European Central Bank, DG Research, Kaiserstrasse 27, 60311 Frankfurt, Germany, and CEPR, email: [philipp.hartmann@ecb.int](mailto:philipp.hartmann@ecb.int).

<sup>4</sup> University of Mannheim, Department of Economics, 68131 Mannheim, Germany, and CEPR, email: [gianca@uni-mannheim.de](mailto:gianca@uni-mannheim.de).

© European Central Bank, 2003

<b>Address</b>	<b>Kaiserstrasse 29 D-60311 Frankfurt am Main Germany</b>
<b>Postal address</b>	<b>Postfach 16 03 19 D-60066 Frankfurt am Main Germany</b>
<b>Telephone</b>	<b>+49 69 1344 0</b>
<b>Internet</b>	<b><a href="http://www.ecb.int">http://www.ecb.int</a></b>
<b>Fax</b>	<b>+49 69 1344 6000</b>
<b>Telex</b>	<b>411 144 ecb d</b>

*All rights reserved.*

*Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.*

*The views expressed in this paper do not necessarily reflect those of the European Central Bank.*

*The statement of purpose for the ECB Working Paper Series is available from the ECB website, <http://www.ecb.int>.*

ISSN 1561-0810 (print)

ISSN 1725-2806 (online)

## Contents

Abstract	4
Non-technical summary	5
1 Introduction	8
2 The model	12
3 The status quo	15
4 The effects of a merger on banks' behavior	17
4.1 Internal money market and choice of reserves	17
4.2 Cost structures, choice of loan rates and balance sheets	19
4.3 Banks' individual liquidity risk	21
5 The effects of a merger on aggregate liquidity	21
5.1 Asymmetry channel without internal money market	22
5.2 Interaction with the reserve channel	23
6 Competition and aggregate liquidity	25
7 Discussion	26
References	28
Appendix	31
Tables and figures	41
European Central Bank working paper series	45

## Abstract

We model the impact of bank mergers on loan competition, banks' reserve holdings and aggregate liquidity. Banks compete in a differentiated loan market, hold reserves against liquidity shocks, and refinance in the interbank market. A merger creates an internal money market that induces financial cost advantages and may increase reserve holdings. We assess changes in liquidity risk and expected liquidity needs for each bank and for the banking system. Large mergers tend to increase expected aggregate liquidity needs, and thus the liquidity provision by the central bank. Comparative statics suggest that a more competitive environment moderates this effect.

*JEL Classification:* D43, G21, G28, L13

*Keywords:* Credit market competition, bank reserves, internal money market, banking system liquidity

## **Non-technical summary**

The last decade has witnessed an intense process of consolidation in the financial sectors of many industrial countries. This “merger movement” was particularly concentrated among banking firms and occurred mostly within national borders. As a consequence, many countries reached a situation of high banking sector concentration or faced a further deterioration of an already concentrated sector. Often a small number of large banks constitutes more than two thirds of the national banking sector (e.g. measured by deposits). The present paper is the first theoretical exploration of the potential joint consequences of this extensive consolidation process for the competitiveness of bank intermediation, for reserve management and for banking system liquidity. The results are suggestive for competition policies, monetary implementation and prudential supervision.

Market power in loan markets may have adverse effects on borrower welfare, real investment and growth if not counterbalanced by substantial efficiency gains. Available evidence indicates that mergers often lead to upward pressure on loan rates, suggesting that efficiency gains are relatively small. Individual banks’ reserve holdings reflect their fundamental role as liquidity providers, as they determine their ability to meet depositors’ unexpected withdrawals and consumption needs. From a micro-prudential perspective, thus, consolidation may change individual banks’ resiliency against liquidity shocks through changes in their reserve holdings.

Banking system liquidity is important in several respects. First, large liquidity fluctuations may conflict with the objectives of central banks in money market operations. In particular, frequent large liquidity injections can be inconsistent with a lean, simple and transparent implementation of monetary policy; and they may strain banks’ collateral pools, thus complicating risk management. Second, from a macro-prudential perspective consolidation may increase banking system liquidity fluctuations. Hence, in the absence of timely and accurate central bank operations, large liquidity shortages may sometimes endanger the stability of the banking system. In line with these arguments, the 2001 G-10 “Report on Consolidation in the Financial Sector” expresses the concern that “...by internalising what had previously been interbank transactions, consolidation could reduce the liquidity of the market for central bank reserves, making it less efficient in reallocating balances across institutions and increasing market volatility”.

To address these issues we develop a model which allows for the joint analysis of the impact of bank mergers on credit market competition, individual and aggregate liquidity

management. Banks raise retail deposits to invest in long-term loans to entrepreneurs and in liquid short-term assets (reserves). On the loan market banks compete in prices and retain some market power through differentiation. They hold reserves as a cushion against stochastic liquidity shocks (depositors' withdrawals) distributed independently across banks. If liquidity demand exceeds reserves, a bank can fund the difference by borrowing in the interbank (money) market, which redistributes reserves from banks with excess liquidity to banks with shortages. When the aggregate demand for liquidity exceeds the total stock of available reserves, the central bank intervenes to provide the missing liquidity.

The occurrence of a merger modifies banks' behaviour concerning both liquidity management and loan market competition. As regards the former, an important feature of our analysis is that a merger creates an internal money market where liquidity can be freely reshuffled. Surprisingly, we find that this may sometimes lead the merged banks to increase reserve holdings. On the one hand, the possibility to reshuffle reserves internally increases their marginal value, which implies such upward pressure. On the other hand, the typical diversification effect related to the pooling of independent liquidity shocks induces the merged banks to reduce reserves. We show that the internalisation effect dominates when the relative cost of refinancing on the interbank market is low, since then banks hold few reserves and face a high probability of needing additional reserves. When the relative cost of refinancing is high, the diversification effect dominates and banks reduce reserve holdings. In both circumstances, however, the merged banks always improve their liquidity situation.

The effect of a merger on the loan market depends on the relative strength of the increase in market power and potential cost efficiency gains. A merger allows the banks involved to internalise the effect of their pricing on the demand of their companion bank and to set higher loan rates. At the same time, potential efficiency gains make them more aggressive in setting loan rates. As known from the industrial organisation literature, the overall effect on loan rates depends on how strong these cost reductions are. The novelty in our model is that, by lowering interbank refinancing costs, the internal money market generates endogenous financial cost efficiencies, which reduce the anti-competitive effects of bank mergers.

Loan market shares across banks move in line with loan rates. The merged banks gain market shares at the expense of competitors when loan rates fall, and lose market shares otherwise. Thus, consolidation changes banks' balance sheets, creating (or reducing) heterogeneity through changes in equilibrium loan market shares. This has an important effect on banking system liquidity, since changes in the size distribution of banks' balance sheets affect

aggregate liquidity demand and, hence, expected aggregate liquidity needs (the expected amount of publicly provided liquidity the system needs).

We identify two channels through which mergers affect banking system liquidity. The reserve channel is directly related to individual banks' changes in reserve holdings, as described above. When the relative cost of refinancing on the interbank market is low, a merger leads banks to increase reserves, thus pushing up aggregate liquidity supply and reducing the expected liquidity needs of the system. The opposite happens when the relative cost of refinancing is high. The asymmetry channel is linked to changes in the heterogeneity of banks' balance sheets generated by mergers occurring in imperfectly competitive environments. We show that greater heterogeneity increases the variance of the aggregate liquidity demand, which leads to higher expected aggregate liquidity needs.

Depending on the size of the relative cost of refinancing, the reserve and asymmetry channels can work in the same or opposite directions. When interbank refinancing is relatively expensive, the two channels lead to a deterioration of aggregate liquidity in the banking system. When interbank refinancing is relatively inexpensive, the two channels push instead in opposite directions and the net effect on aggregate liquidity depends on their relative strength. We conclude that if we face a merger wave that leads to a "polarisation" of the banking system with large and small institutions, this wave is likely to generate an adverse outcome in terms of aggregate liquidity need, particularly where interbank refinancing is more costly. In contrast, a merger movement that leaves behind relatively little heterogeneity in banks' balance sheets may leave interbank market liquidity unaffected or even improve it. This result is noteworthy given that the banking sector consolidation of the 1990s led to greater asymmetry between the largest and smaller banks in most industrial countries.

To further explore the role of competition in the aggregate liquidity effects of bank mergers, we undertake a comparative statics exercise, varying the competition parameters of the model. It turns out that in the most plausible parameter configurations, a more competitive environment is favourable for banking system liquidity. More banks or a greater substitutability of loans decrease the asymmetry in banks' balance sheets caused by a merger, thus reducing expected aggregate liquidity needs.



## 1 Introduction

The last decade has witnessed an intense process of consolidation in the financial sector of many industrial countries. This ‘merger movement’, documented in a number of papers and official reports, was particularly concentrated among banking firms and occurred mostly within national borders.<sup>1</sup> As shown in Figure 1, in Canada, Italy and Japan more than half the average number of banks combined forces over the 1990s.

[FIGURE 1 ABOUT HERE]

As a consequence, many countries (e.g., Belgium, Canada, France, the Netherlands, and Sweden) reached a situation of high banking sector concentration or faced a further deterioration of an already concentrated sector. As it can be seen from Table 1, a small number of large banks often constitutes more than 70 per cent of the national banking sector.

[TABLE 1 ABOUT HERE]

This paper is the first theoretical exploration of the potential joint consequences of this extensive consolidation process for the competitiveness of bank intermediation, reserve management and banking system liquidity.

These three issues are important. Market power in loan markets may have adverse effects on borrower welfare, real investment and growth if not counterbalanced by substantial efficiency gains.<sup>2</sup> Available evidence indicates that mergers often lead to upward pressure on loan rates, suggesting that efficiency gains are relatively small.<sup>3</sup>

Individual banks’ reserve holdings reflect their fundamental role as liquidity providers, as they determine their ability to meet depositors’ unexpected withdrawals and consumption needs. From a micro-prudential perspective, thus, consolidation may change individual banks’ resiliency against liquidity shocks through changes in their reserve holdings.

Banking system liquidity is important in several respects. First, large liquidity fluctuations may conflict with the objectives of central banks in money market operations. In particular, frequent large liquidity injections can be inconsistent with a lean, simple and transparent implementation of monetary policy; and they may strain banks’ collateral pools, thus complicating risk management. Second, from a macro-prudential perspective consolidation may increase banking system liquidity fluctuations. Hence, in the absence of timely and accurate central bank operations, large liquidity shortages may sometimes endanger the stability of the banking system. In line with these arguments, the G-10 ‘Report on Consolidation in the Financial Sector’ expresses the concern that ‘...by internalizing what

---

<sup>1</sup>See e.g., Boyd and Graham (1996), Berger et al. (1999), Hanweck and Shull (1999), Dermine (2000); ECB (2000), OECD (2000) and Group of Ten (2001).

<sup>2</sup>For example, Spagnolo (2000) shows that poor credit market competition may hinder competition *in the whole economy*, and Cetorelli (2002) provides empirical evidence of this effect.

<sup>3</sup>See, e.g., the surveys by Rhoades (1998) and Carletti et al. (2002).

had previously been interbank transactions, consolidation could reduce the liquidity of the market for central bank reserves, making it less efficient in reallocating balances across institutions and increasing market volatility' (Group of Ten, 2001, p. 20).

To address these issues we develop a model which allows for the joint analysis of the impact of bank mergers on credit market competition, individual and aggregate liquidity management. Banks raise retail deposits to invest in long-term loans to entrepreneurs and in liquid short-term assets (reserves). On the loan market banks compete in prices and retain some market power through differentiation. They hold reserves as a cushion against stochastic liquidity shocks (depositors' withdrawals) distributed independently across banks. If liquidity demand exceeds reserves, a bank can fund the difference by borrowing in the interbank (or money) market, which redistributes reserves from banks with excess liquidity to banks with shortages. When the aggregate demand for liquidity exceeds the total stock of available reserves, the central bank intervenes to provide the missing liquidity.

Banks choose reserves balancing the marginal benefit of lower interbank refinancing needs with the marginal cost of having to raise more deposits. At the optimum, reserve holdings grow when the cost of refinancing increases relative to the cost of raising deposit. Equilibrium loan rates are set at the level that equates the marginal revenue of granting loans with the marginal cost of providing them, refinancing in the interbank market and raising deposits.

The occurrence of a merger modifies banks' behavior concerning both liquidity management and loan market competition. As regards the former, an important feature of our analysis is that a merger creates an *internal money market* where liquidity can be freely reshuffled. Surprisingly, we find that this may lead the merged banks to increase reserve holdings. On the one hand, the typical *diversification effect* related to the pooling of their independent liquidity shocks induces the merged banks to reduce reserves. On the other hand, the possibility to reshuffle reserves internally increases their marginal value, thus leading the merged banks to increase reserves. We show that this *internalization effect* dominates when the relative cost of refinancing on the interbank market is low, since then banks hold few reserves and face a high probability of needing additional reserves. When the relative cost of refinancing is high, the diversification effect dominates and banks reduce reserve holdings. In both circumstances, however, the merged banks always improve their liquidity situation, in terms of both liquidity risk (the probability of facing a liquidity shortage) and expected liquidity needed.

The effect of a merger on the loan market depends on the relative strength of the increase in market power and potential cost efficiency gains. A merger allows the banks involved to internalize the effect of their pricing on the demand of their companion bank and to set, *ceteris paribus*, higher loan rates. At the same time, potential efficiency gains make them more aggressive in setting loan rates. As known from the industrial organization literature, the overall effect on loan rates depends on how strong these cost reductions are.

The novelty in our model is that, by lowering interbank refinancing costs, the internal money market generates endogenous *financial cost efficiencies*, which reduce, *ceteris paribus*, the anti-competitive effects of mergers between banks.

Loan market shares across banks move in line with loan rates. The merged banks gain market shares at the expense of competitors when loan rates fall, and lose market shares otherwise. Thus, consolidation changes banks' balance sheets, creating (or reducing) heterogeneity through changes in equilibrium loan market shares. This has an important effect on banking system liquidity, since changes in the size distribution of banks' balance sheets affect aggregate liquidity demand and thus expected aggregate liquidity needs (the expected amount of publicly provided liquidity the system needs).

We identify two channels through which mergers affect banking system liquidity. The *reserve channel* is directly related to individual banks' changes in reserve holdings, as described earlier. When the relative cost of refinancing on the interbank market is low, a merger leads banks to increase reserves, thus pushing up aggregate liquidity supply and reducing the expected liquidity needs of the system. The opposite happens when the relative cost of refinancing is high. The *asymmetry channel* is linked to changes in the heterogeneity of banks' balance sheets generated by mergers occurring in imperfectly competitive environments. We show that greater heterogeneity increases the variance of the aggregate liquidity demand, thus leading, *ceteris paribus*, to higher expected aggregate liquidity needs.

Depending on the size of the relative cost of refinancing, the reserve and asymmetry channels can then work in the same or opposite directions. When interbank refinancing is relatively expensive, the two channels lead to a deterioration of aggregate liquidity in the banking system. Both banks' lower reserves and greater balance sheet heterogeneity increase expected aggregate liquidity needs. When interbank refinancing is relatively inexpensive, the two channels push instead in opposite directions and the net effect on aggregate liquidity depends on their relative strength. We conclude that if we face a merger wave that leads to a 'polarization' of the banking system with large and small institutions, this wave is likely to generate an adverse outcome in terms of aggregate liquidity need, particularly where interbank refinancing is more costly. In contrast, a merger movement that leaves behind relatively little heterogeneity in banks' balance sheets may leave interbank market liquidity unaffected or even improve it. This result is particularly noteworthy in the light of Table 1, which suggests that the banking sector consolidation of the 1990s led to greater asymmetry between the largest and smaller banks in most industrial countries.

To further explore the role of competition in the aggregate liquidity effects of bank mergers, we undertake a comparative statics exercise varying the competition parameters of the model. It turns out that in the most plausible parameter configurations, a more competitive environment is favorable for banking system liquidity. More banks or a greater substitutability of loans decrease the asymmetry in banks' balance sheets caused by a merger, thus reducing, *ceteris paribus*, expected aggregate liquidity needs.

The paper builds on the industrial organization literature on the implications of exogenous mergers under imperfect competition, in particular on Deneckere and Davidson (1985) and Perry and Porter (1985), and combines it with the analysis of financial intermediation and market liquidity. Starting with Diamond and Dybvig (1983), there is an important field of research studying the role of banks as liquidity providers. Recent examples are Kashyap, Rajan and Stein (2002), who describe the links between banks' liquidity provision to depositors and their liquidity provision to borrowers through credit lines; and Diamond (1997), who discusses the relationship between the activities of Diamond-and-Dybvig-type banks and liquidity of financial markets. Concerning liquidity provision by public authorities, Holmstrom and Tirole (1998) analyze the role of government debt management in meeting the liquidity needs of the productive sector. This literature, however, has not yet considered the implications of imperfect competition and financial consolidation for private and public provision of liquidity.

Several authors have studied the rationale for an interbank market and its effect on reserve holdings. For example, Bhattacharya and Gale (1987) show that banks can optimally cope with liquidity shocks by borrowing and lending reserves; but they also argue that moral hazard and adverse selection lead to under-investment in reserves. Bhattacharya and Fulghieri (1994) clarify that, if the timing of returns on reserves is uncertain, reserve holdings can instead be excessive. These authors argue that the central bank has a role in healing these imperfections. Allen and Gale (2000) and Freixas et al. (2000) analyze how small unexpected liquidity shocks can lead to liquidity shortages in the banking system and thus, in the absence of a central bank, to contagious crises. We discuss how the likelihood and the extent of such shortages vary with changes in market structure when a central bank stands ready to offset private market liquidity fluctuations.

The paper is also related to the literature on firms' internal capital markets. Gertner et al. (1994) and Stein (1997) discuss the potentially efficiency-enhancing role of internal capital markets, while Scharfstein and Stein (2000) and Rajan et al. (2000) warn that these may become inefficient if internal incentive problems and power struggles lead to excessive cross-divisional subsidies. The empirical results of Graham et al. (2002) suggest, however, that 'value destruction' in firms is not related to consolidation. Regarding banks, Houston et al. (1997) provide evidence that loan growth at subsidiaries of US bank holding companies (BHCs) is more sensitive to the holding company's cash flow than to the subsidiaries' own cash flow. Campello (2002) shows that the funding of loans by small affiliates of US BHCs is less sensitive to affiliate-level cash flows than independent banks of comparable size. Focusing on short-term assets, we show how the creation of an internal money market can cushion external liquidity shocks and affect banks' reserve choices and banking system liquidity. We also show that the financial cost advantages associated with the internal money market lead the merged banks, *ceteris paribus*, to be more aggressive on the loan market.

Aggregate liquidity risk is related to financial stability. Although we are not covering solvency problems in our model, in practice severe liquidity problems may cause default if there is no adequate intervention. The ‘charter value’ literature studies the relationship between competition and bank stability, arguing that some monopoly rents are desirable to reduce incentives for excessive risk-taking (see, e.g., Keeley, 1990; Hellman et al., 2000, and Matutes and Vives, 2000). Perotti and Suarez (2002) argue that a succession of bank takeovers and an active entry policy may ensure stability, while keeping competition intact. More recent empirical work indicates that less competitive banking systems are not necessarily more stable.<sup>4</sup> Our model links monopoly rents in loan competition to individual and aggregate liquidity fluctuations, suggesting that bank competition may reduce liquidity shortages.

The remainder of the paper is structured as follows. Section 2 sets up the model. Section 3 derives the equilibrium before a merger (‘status quo’). The subsequent section characterizes the effects of the merger on individual banks’ behavior, and Section 5 looks at its implications for aggregate liquidity. Comparative statics analysis is conducted in Section 7. The final section discusses the robustness of the results.

## 2 The Model

Consider a three date ( $T = 0, 1, 2$ ) economy with three classes of risk neutral agents:  $N$  banks ( $N > 3$ ), numerous entrepreneurs, and numerous individuals. At date 0 banks raise funds from individuals in the form of retail deposits, and invest the proceeds in loans to entrepreneurs and liquid short-term assets denoted as reserves. Thus, the balance sheet for each bank  $i$  is

$$L_i + R_i = D_i, \tag{1}$$

where  $L_i$  denotes loans,  $R_i$  reserves, and  $D_i$  deposits.

### *Competition in the loan market*

Banks offer differentiated loans and compete in prices. The differentiation of loans may emerge from long-term lending relationships (see, e.g., Sharpe, 1990; Rajan, 1992), specialization in certain types of lending (e.g., to small/large firms or to different sectors) or in certain geographical areas. Following Shubik and Levitan (1980), we assume that each bank  $i$  faces a linear demand for loans given by

$$L_i = l - \gamma \left( r_i^L - \frac{1}{N} \sum_{j=1}^N r_j^L \right), \tag{2}$$

---

<sup>4</sup>Carletti and Hartmann (2003) provide a more comprehensive review of the literature on competition and stability in banking.

where  $r_i^L$  and  $r_j^L$  are the loan rates charged by banks  $i$  and  $j$  (with  $j = 1, \dots, i, \dots, N$ ), and the parameter  $\gamma \geq 0$  represents the degree of substitutability of loans. The larger  $\gamma$  the more substitutable are the loans. Note that expression (2) implies a constant aggregate demand for loans  $\sum_{i=1}^N L_i = Nl$ , as in Salop (1979).

Processing loans involves a per-unit provision cost  $c$ , which can be thought of as a set up cost or a monitoring cost. Loans mature at date 2 and yield nothing if liquidated before maturity.

### *Deposits, individual liquidity shocks and reserve holdings*

Banks raise deposits in  $N$  distinct ‘regions’. A region can be interpreted as a geographical area, a specific segment of the population, or an industry sector in which a bank specializes for its deposit business. There is a large number of potential depositors in every region, each endowed with one unit of funds at date 0. Depositors are offered demandable contracts, which pay just the initial investment in case of withdrawal at date 1 and a (net) rate  $r^D$  at date 2. The deposit rate  $r^D$  can be thought of as the reservation value of depositors (the return of another investment opportunity), or, alternatively, as the equilibrium rate in a competition game between banks and other deposit-taking financial institutions.

As in Diamond and Dybvig (1983), a fraction  $\delta_i$  of depositors at each bank develops a preference for early consumption, and withdraws at date 1. The remaining  $1 - \delta_i$  depositors value consumption only at date 2, and leave their funds at the bank a period longer.<sup>5</sup> The fraction  $\delta_i$  is assumed to be stochastic; specifically,  $\delta_i$  is uniformly distributed between 0 and 1, and is i.i.d. across banks.<sup>6</sup> This introduces uncertainty at the level of each individual bank and in the aggregate. All uncertainty is resolved at date 1, when liquidity shocks materialize.

Each bank keeps reserves  $R_i$  to face its date 1 *demand for liquidity*  $x_i = \delta_i D_i$ . Reserves represent a storage technology that transfers the value of investment from one period to the next. We may think of cash, reserve holdings at the central bank, or even short-term government securities, and other safe and low yielding assets. (The interest rate on reserves needs not be zero.)

The stochastic nature of  $\delta_i$  implies that the realized demand for liquidity  $x_i$  may exceed or fall short of  $R_i$ . Denoting as  $f(x_i)$  the density function of  $x_i$ , from an ex ante perspective each bank faces a *liquidity risk* – the probability to experience a liquidity shortage at date

---

<sup>5</sup>The fraction  $\delta_i$  can also be interpreted as a regional macro shock. For example, weather conditions may change the general consumption needs in a region, so that each depositor withdraws a fraction  $\delta_i$  of his initial investment.

<sup>6</sup>The simplifying assumption that liquidity shocks are independent across banks is by no means necessary. As long as the shocks are not perfectly correlated, all our results below remain valid, also for the case of dependence. In the extreme case of perfectly correlated liquidity shocks the problem is not interesting, as the rationale for a money market disappears. We like to thank an anonymous working paper referee for having pointed out that this should be clarified.

1 – given by

$$\phi_i = \text{prob}(x_i > R_i) = \int_{R_i}^{D_i} f(x_i) dx_i, \quad (3)$$

and has *expected liquidity needs* – the expected size of liquidity shortage that needs to be refinanced at date 1 – equal to

$$\omega_i = \int_{R_i}^{D_i} (x_i - R_i) f(x_i) dx_i. \quad (4)$$

#### *Interbank refinancing and aggregate liquidity*

As liquidity shocks are independent across banks, there is room for reshuffling liquidity from banks with reserve excesses ( $x_i < R_i$ ) to banks with reserve shortages ( $x_i > R_i$ ) on an interbank (or money) market.<sup>7</sup> The presence of aggregate uncertainty implies, however, that there may be an aggregate shortage or an aggregate excess of liquidity. An aggregate shortage of private liquidity occurs whenever the aggregate demand for liquidity is higher than the aggregate supply of liquidity represented by the sum of individual banks' reserves, i.e., whenever

$$\sum_{i=1}^N x_i > \sum_{i=1}^N R_i. \quad (5)$$

Denoting as  $X_i = \sum_{i=1}^N x_i$  the aggregate demand for liquidity with density function  $f(X_i)$ , we express the frequency with which aggregate shortages occur through the *aggregate (or systemic) liquidity risk*

$$\Phi = \text{prob} \left( X_i > \sum_{i=1}^N R_i \right) = \int_{\sum R_i}^{\sum D_i} f(X_i) dX_i, \quad (6)$$

and the expected size through the *expected aggregate (or systemic) liquidity needs*

$$\Omega = \int_{\sum R_i}^{\sum D_i} \left( X_i - \sum_{i=1}^N R_i \right) f(X_i) dX_i. \quad (7)$$

In order to concentrate on public liquidity management, we assume that the central bank supplies (or demands) the liquidity necessary to clear the interbank market and avoid a crisis, and that the loan market is sufficiently profitable for banks to borrow in the interbank market against loan market profits. This ensures that the interbank market is stable and clears at the rate justified by the current stance of monetary policy. Banks can then borrow at date 1 at a fixed rate  $r^{IB}$ , either from other banks or from the central bank, and lend

---

<sup>7</sup>We can think of it in terms of wholesale overnight deposits, such as the interbank overnight deposit market in the euro area and the Fed funds market in the United States.

at a fixed rate  $r^{IL}$ , where  $r^{IB} > r^{IL}$ .<sup>8</sup> The aggregate liquidity risk (6) and the expected aggregate liquidity needs (7) can then be interpreted as measures of the degree to which the banking system depends on public supply of liquidity.

The timing of the model is summarized in Figure 2. At date 0 banks compete in prices in the loan market, choose reserve holdings, and raise deposits. After liquidity shocks materialize at date 1, banks borrow or lend in the interbank market, which is completed by the central bank if necessary. At date 2 loans mature, and remaining claims from deposits and interbank market are settled.

Figure 2: Timing of the model

T=0	T=1	T=2
price competition in the loan market, choice of $R_i$ , $D_i = L_i + R_i$ are raised	shocks $\delta_i$ materialize, banks operate in the interbank market, and central bank intervenes	loans mature, claims are settled, and profits materialize

### 3 The Status Quo

In this section we characterize the equilibrium when all banks are identical. We start with noting two features of the model. First, bank runs never occur in this model. The illiquidity of loans together with  $r^D > 0$  guarantees that depositors withdraw prematurely only if hit by liquidity shocks. Second, because banks can always repay depositors and creditors in the interbank market, we can directly focus on the date 0 maximization problem.

With these considerations in mind, at date 0 each bank  $i$  chooses the loan rate  $r_i^L$  and the reserves  $R_i$  so as to maximize the following expected profit (for simplicity, the intertemporal discount factor is normalized to one):

$$\Pi_i = (r_i^L - c)L_i + \int_0^{R_i} r^{IL}(R_i - x_i)f(x_i)dx_i - \int_{R_i}^{D_i} r^{IB}(x_i - R_i)f(x_i)dx_i - r^D D_i(1 - E(\delta_i)). \quad (8)$$

The first term in (8) represents the profit from the loan market, the second term is the expected profit from lending at date 1 when the bank is in excess of reserves, the third

---

<sup>8</sup>For interbank lending the difference between  $r^{IB}$  and  $r^{IL}$  reflects transaction costs, such as bid-ask spreads charged by market makers or brokers (see e.g., Hartmann et al., 2001). For transactions with the central bank we can interpret  $r^{IB}$  and  $r^{IL}$  as the policy rates set for standing facilities. For example, the US Federal Reserve's (Fed) new primary credit facility and the European Central Bank's (ECB) marginal lending facility allow sound banks to receive overnight credit. The ECB offers also an overnight deposit facility (paying a much lower rate) for banks with excess liquidity, whereas banks' holdings of overnight balances with the Fed do not earn any interest. For simplicity, we assume that market and central bank borrowing and lending rates are identical, but the working of our model and the qualitative results do not hinge on this assumption.



term is the expected cost of refinancing at date 1 when the bank needs refinancing, and the fourth term is the expected repayment to depositors leaving their funds until date 2. Taken together, the last two terms represent bank  $i$ 's *financing costs*.

For expositional convenience, and without loss of generality, we set  $r^{IL} = 0$ , and denote  $r^{IB}$  simply as  $r^I$ . (No qualitative result depends on this simplification, which also captures the idea that banks do not keep reserves to make profits, but only to protect themselves against liquidity shocks.)

The following proposition characterizes the symmetric equilibrium in the status quo. All proofs are in the appendix.

**Proposition 1** *The symmetric status quo equilibrium is characterized as follows:*

1. Each bank sets a loan rate  $r_{sq}^L = \frac{l}{\gamma(\frac{N-1}{N})} + c_{sq}$ , where  $c_{sq} = c + \sqrt{r^I r^D}$ ;
2. It has a loan market share  $L_{sq} = l$ ;
3. If  $r^I > r^D$ , it keeps reserves  $R_{sq} = \left( \sqrt{\frac{r^I}{r^D}} - 1 \right) L_{sq}$ , and raises deposits  $D_{sq} = L_{sq} \sqrt{\frac{r^I}{r^D}}$ .

The equilibrium loan rate  $r_{sq}^L$  diverges from the total marginal cost  $c_{sq}$  via the mark up  $\frac{l}{\gamma(\frac{N-1}{N})}$ . This decreases with both the number of banks  $N$  and the loan substitutability parameter  $\gamma$ , while it increases with the level of loan demand  $l$ . The total marginal cost includes the loan provision cost  $c$  and the marginal financing costs  $\sqrt{r^I r^D}$ , i.e., the sum of the expected cost of refinancing and of raising deposits.

Equilibrium reserve holdings balance the marginal benefit of reducing the expected cost of refinancing with the marginal cost of increasing deposits, and they are positive as long as  $r^I > r^D$ . We restrict our attention to this plausible case, the other case being also theoretically uninteresting. Both reserves and deposits increase with the interbank rate  $r^I$  and with the demand for loans  $L_{sq}$ , while they decrease with the deposit rate  $r^D$ . The ratio  $\frac{r^I}{r^D}$  is *the relative cost of refinancing*, which will play an important role as we go along. It is a measure of how costly refinancing at date 1 is relative to raising deposits and holding reserves at date 0.

Two further implications of Proposition 1 are important for comparing this equilibrium with the post-merger equilibrium in the next section. First, using the balance sheet equality (1), we can express equilibrium reserve holdings in terms of an *optimal reserve-deposit ratio* as

$$k_{sq} = \frac{R_{sq}}{D_{sq}} = \left( 1 - \sqrt{\frac{r^D}{r^I}} \right). \quad (9)$$

Note that, whereas the equilibrium reserve holdings in Proposition 1 depend on the loan market outcome, the reserve-deposit ratio in (9) does not. To exploit this simplification, in

what follows we will mostly focus on this ratio. Second, Proposition 1 implies the following corollary.

**Corollary 1** *In the status quo equilibrium, each bank has liquidity risk  $\phi_{sq} = \sqrt{\frac{r^D}{r^I}}$  and expected liquidity needs  $\omega_{sq} = \frac{r^D}{2r^I} D_{sq} = \frac{L_{sq}}{2} \sqrt{\frac{r^D}{r^I}}$ .*

The equilibrium liquidity risk  $\phi_{sq}$  is increasing in the deposit rate  $r^D$  and decreasing in the interbank rate  $r^I$ . An increase in  $r^D$  induces banks to reduce reserves and thus deposits. Lower reserves mean lower protection against early liquidity demand, while lower deposits reduce the size of such demand. As liquidity shocks hit only a fraction  $\delta_i$  of deposits, the negative effect of lower reserves dominates, so that individual liquidity risk  $\phi_{sq}$  increases. A similar mechanism explains the negative dependence of  $\phi_{sq}$  on  $r^I$ , as well as the relationships between the expected liquidity needs  $\omega_{sq}$ , the rates  $r^D$  and  $r^I$ , and the equilibrium demand for loans  $L_{sq}$ .

## 4 The Effects of a Merger on Banks' Behavior

In this section we analyze what happens at the individual bank level when a merger takes place. The behavior of the merged banks changes in several ways. First, they can exchange reserves internally, which changes their way to insure against liquidity risk. Second, this 'internal money market' gives them a financing cost advantage, whose size is endogenously determined. Third, the merged banks may enjoy cost efficiencies in terms of lower loan provision costs. Fourth, they gain market power in setting loan rates. All these factors affect banks' equilibrium balance sheets and, in turn, the demand and supply of liquidity.

We begin with how the merger modifies banks' reserve holdings, and then we turn to its effects on loan market competition. As noted earlier, one can look at these issues separately by focussing on the optimal reserve-deposit ratios, rather than on the absolute levels of reserves.

### 4.1 Internal Money Market and Choice of Reserves

We note first that the merger does not affect the optimal reserve-deposit ratio of the  $N - 2$  competitors. As they have the same cost structure as in the status quo, they still choose their reserve-deposit ratios according to (9), i.e.,  $k_c = k_{sq}$ .

By contrast, the merged banks, say bank 1 and bank 2, choose a different reserve-deposit ratio. As their liquidity shocks are independently distributed, they can pool their reserves to meet the total demand for liquidity. Thus, as long as the two banks continue to raise deposits in two separate regions, the merger leaves room for an *internal money market* in which they can reshuffle reserves according to their respective needs. For simplicity, we assume a 'perfect' internal money market, so that exchanging reserves internally involves

no cost. (All qualitative results go through as long as the cost of using the internal money market is lower than the interbank rate.)

Let  $x_m = \delta_1 D_1 + \delta_2 D_2$  be the total demand for liquidity of the merged banks at date 1,  $R_m = R_1 + R_2$  be their total reserves and  $D_m = D_1 + D_2$  be their total deposits. The combined profits of the merged banks are then given by

$$\begin{aligned} \Pi_m = & (r_1^L - \beta c)L_1 + (r_2^L - \beta c)L_2 - \int_{R_m}^{D_m} r^I(x_m - R_m)f(x_m)dx_m \\ & - r^D [D_1(1 - E(\delta_1)) + D_2(1 - E(\delta_2))]. \end{aligned} \quad (10)$$

The first two terms in (10) represent the combined profits from the loan market, with  $\beta \leq 1$  reflecting potential efficiency gains in the form of reduced loan provision costs, the third term is the total expected cost of refinancing, and the last one is the total expected repayment to depositors. The operation of the internal money market can be seen in the third term of (10), where demands for liquidity and reserves are pooled together.

A preliminary step before deriving their optimal reserve-deposit ratio is to understand the ‘deposit market policy’ of the merged banks. Whether they raise equal or different amounts in both regions affects the distribution of the demand for liquidity  $x_m$ , and thus the size of the expected cost of refinancing. We have the following lemma.

**Lemma 1** *The merged banks raise an equal amount of deposits in each region, i.e.,  $D_1 = D_2 = \frac{D_m}{2}$ .*

Lemma 1 shows that the merged banks not only raise deposits in both regions, but they even do it symmetrically. Choosing equal amounts of deposits in both regions minimizes the variance of  $x_m$  and maximizes the benefits of diversification, thus reducing the expected refinancing cost. (We will come back to this point in Section 6 when studying the effect of the merger on aggregate liquidity demand.)

Given  $D_1 = D_2$ , the merged banks choose reserves  $R_m$  so as to maximize their combined profits in (10). Let  $k_m = \frac{R_m}{D_m}$  be the reserve-deposit ratio for the merged banks and recall that  $k_{sq}$  is the one for banks in the status quo defined in (9). The following proposition compares these two ratios.

**Proposition 2** *The merged banks choose a higher reserve-deposit ratio than in the status quo ( $k_m > k_{sq}$ ) if the relative cost of refinancing,  $\frac{r^I}{r^D}$ , is low, and a lower one otherwise.*

Contrary to conventional wisdom – suggesting that the *diversification effect* of the internal money market should lead the merged banks to reduce reserves –, Proposition 2 shows that, as long as refinancing is not too costly, the merged banks increase their optimal reserve-deposit ratio. The reason is that the typical diversification effect is offset by an *internalization effect*. When choosing reserves, the merged banks take into account (‘internalize’) an

externality, namely that each unit of reserves can now be used to cover a liquidity demand at either of them.

The merger modifies the demand for liquidity  $x_m$  of the merged banks relative to the demand for liquidity  $x_i$  of each individual bank in the status quo, and the relative cost of refinancing affects banks' reserve choices. As a sum of two independent liquidity shocks,  $x_m$  is more concentrated around the mean than  $x_i$ . Thus, the distribution of  $x_m$  gives a lower probability to events with very low and very high liquidity demand than that of  $x_i$ . If the ratio  $\frac{r^I}{r^D}$  is low, both the merged banks and each individual bank choose relatively small reserve-deposit ratios because refinancing is inexpensive. For any given small level of this ratio, however, the merged banks would be able to cover their demand for liquidity less frequently than the individual bank because of the thinner left tail of the distribution of  $x_m$ . The merged banks have therefore a higher marginal valuation of further reserve units and increase their reserve-deposit ratio  $k_m$  above  $k_{sq}$  (the internalization effect dominates the diversification effect).

The reverse happens if the relative cost of refinancing is high. In this case, all banks tend to have high reserve-deposit ratios. For any given large level of this ratio, the merged banks would experience liquidity shortages less often than an individual bank, because the right tail of the distribution of  $x_m$  is thinner than that of the distribution of  $x_i$ . This makes the merged banks have a lower marginal valuation of further reserve units, and it induces them to decrease their reserve-deposit ratio (the diversification effect dominates).

## 4.2 Cost Structures, Choice of Loan Rates and Balance Sheets

We now examine how the merger modifies the equilibrium in the loan market and banks' balance sheets. Consider first banks' cost structures. As noted earlier, competitors have the same cost structure as in the status quo. Each of them pays a per-unit loan provision cost  $c$  and per-unit financing costs  $\sqrt{r^I r^D}$  (from Proposition 1).

By contrast, the cost structure of the merged banks changes in two ways. First, their loan provision costs reach  $\beta c$ , where the parameter  $\beta \leq 1$  represents the potential efficiency gains that the merger induces for the processing of loans. Second, the emergence of the internal money market affects the merged banks' expected costs of refinancing. We have the following result.

**Lemma 2** *The merged banks have lower financing costs than competitors.*

This advantage for the merged banks is endogenous to the model in that its size is determined by their optimal reserve choices.

The following proposition describes the post-merger equilibrium with symmetric behavior within the 'coalition' (merger) and among competitors.

**Proposition 3** *The post-merger equilibrium with  $r_1^L = r_2^L = r_m^L$  and  $r_i^L = r_c^L$  for  $i = 3, \dots, N$  is characterized as follows:*

1. *Each merged bank sets a loan rate  $r_m^L = \left(\frac{2N-1}{N-2}\right) \frac{l}{2\gamma} + \frac{(N-1)}{2N}c_c + \frac{(N+1)}{2N}c_m$ , and each competitor sets  $r_c^L = \left(\frac{N-1}{N-2}\right) \frac{l}{\gamma} + \frac{(N-1)}{N}c_c + \frac{1}{N}c_m$ ;*
2. *The merged banks have a total loan market share  $L_m = \left(\frac{2N-1}{N}\right) l + \gamma \frac{(N-1)(N-2)}{N^2}(c_c - c_m)$ , and each competitor has  $L_c = \frac{(N-1)^2}{N(N-2)}l - \gamma \frac{(N-1)}{N^2}(c_c - c_m)$ ;*
3. *The merged banks raise total deposits  $D_m = \frac{1}{1-k_m}L_m$ , and each competitor raises  $D_c = \frac{1}{1-k_c}L_c$ ;*

*where  $c_m, c_c$  are the total marginal costs of the merged banks and of the competitors, and  $k_m$  and  $k_c$  are their respective optimal reserve-deposit ratios.<sup>9</sup>*

Since banks compete in strategic complements, in equilibrium the loan rates of competitors move in the same direction as the loan rates of the merged banks. Both  $r_m^L$  and  $r_c^L$  are a weighted average of the mark ups that banks can charge and of the total marginal costs  $c_m$  and  $c_c$ . All marks ups are higher than those in the status quo equilibrium (see  $r_{sq}^L$  in Proposition 1), but as the merged banks gain market power, they charge a higher mark up than competitors. By contrast, their total marginal cost  $c_m$  is lower than those of the competitors, as the merged banks benefit from lower financing costs (see Lemma 2) and from potential efficiency gains in the provision of loans. Thus, the effect of the merger on equilibrium loan rates depends on the relative importance of the increased market power of the merged banks as compared to their lower total marginal cost. Post-merger equilibrium loan rates increase when the merger induces a small cost advantage relative to the increase in market power, whereas they decrease otherwise.

Loan market shares across banks change in line with loan rates. As the merged banks change their loan rates by more than competitors, their total loan market share shrinks when loan rates increase and it expands otherwise, i.e.,  $L_m < 2L_{sq} < 2L_c$  when  $r_m^L > r_c^L$ , and  $L_m > 2L_{sq} > 2L_c$  otherwise.

The modification of loan market shares together with the change in the optimal reserve-deposit ratio described in Proposition 2 determines the effects on the sizes of banks' balance sheets (as measured by the amount of deposits). Most importantly, a merger breaks the symmetry in banks' balance sheets. Whereas in the status quo all banks have the same deposits  $D_{sq}$ , the merged banks have now in general different deposit sizes than competitors, i.e.,  $\frac{D_m}{D_c} \neq 2$ .

---

<sup>9</sup>The expressions for  $c_m, c_c$  are in the proof of this proposition; those for  $k_m$  and  $k_c$  are, respectively, in the proof of Proposition 2 and in equation (9).

### 4.3 Banks' Individual Liquidity Risk

An important implication of Propositions 2 and 3 is how the merger modifies banks' liquidity risks and expected liquidity needs. The results for competitor banks are quite straightforward. As they follow the same optimal reserve rule as in the status quo, they face the same liquidity risk  $\phi_c = \phi_{sq} = \sqrt{\frac{r^D}{r^I}}$  (see Corollary 1). Their expected liquidity needs, however, change with their balance sheet, as  $\omega_c = \frac{r^D}{2r^I}D_c$ . The merged banks experience more far reaching changes in liquidity risks and needs.

**Corollary 2** *The merged banks have lower liquidity risk than a single bank in the status quo.*

This result derives directly from Proposition 2. When the relative cost of refinancing is low, the merged banks increase their reserve-deposit ratio and their liquidity risk goes down. In the other case, although they choose a lower reserve-deposit ratio than in the status quo, they still keep it sufficiently high to decrease the liquidity risk. This effect is so strong that the liquidity risk of the merged banks is not only lower than the risks of two banks in the status quo, but it is even lower than that of a single bank.

**Corollary 3** *The merged banks have lower expected liquidity needs than in the status quo if  $\frac{D_m}{D_{sq}} < h$ , where  $2 < h \leq 4$ , and higher ones otherwise.*

The merger changes the merged banks' expected needs for three reasons. First, it creates the internal money market, which reduces ceteris paribus expected liquidity needs. Second, the merger modifies the merged banks' optimal reserve-deposit ratio, which reduces ceteris paribus expected liquidity needs when the relative cost of refinancing is low. Third, the merger changes the merged banks' deposits, and hence the size of their demand for liquidity. Corollary 3 shows that the first effect dominates unless cost advantages (efficiency gains and reduced financing costs) and competition in the loan market (degree of loan differentiation  $\gamma$  and number of banks  $N$ ) are so strong that the merged banks increase their balance sheets substantially relative to two banks in the status quo.

## 5 The Effects of a Merger on Aggregate Liquidity

Now that we have seen how a merger affects the behavior of individual banks, we can turn to its implications for the banking system as a whole. To see this, we analyze how changes in banks' reserve holdings and in loan market competition modify the aggregate supply and demand of liquidity.

We identify two channels. The first one we call *reserve channel*, as it works through changes in reserve holdings. When looking at the system as a whole, the distinction between the internal money market of the merged banks and the interbank market is blurred, and the

total supply of liquidity is composed of the sum of all banks' reserve holdings. Nevertheless, the existence of the internal money market affects the total supply of liquidity through the change in the reserve holdings of the merged banks. The second channel is an *asymmetry channel*, which affects the distribution of the aggregate liquidity demand. This channel originates in the heterogeneity of balance sheets across banks, which – as shown above – depends on both the different amount of reserves and the different loan market shares that banks have after the merger.

We start with analyzing each of the two channels in isolation; then we examine how they interact in determining aggregate liquidity risk and expected aggregate liquidity needs.

### 5.1 Asymmetry Channel without Internal Money Market

To isolate the working of the asymmetry channel, we assume for a moment that the merged banks cannot make use of the internal money market. In this case, the merged banks do not have any financing cost advantages, and they choose the same optimal reserve rule as their competitors. As a consequence, the asymmetry in banks' balance sheets originates only from the different distribution of market shares due to loan competition.

Because all banks continue to choose reserves according to (9) and the aggregate demand for loans is inelastic, the merger does not affect the total amounts of reserves and deposits, thus leaving the aggregate supply of liquidity unchanged. The heterogeneity of banks' balance sheets, however, modifies the aggregate liquidity demand, which changes from  $X_{sq} = \sum_{i=1}^N \delta_i D_{sq}$  in the status quo to  $X_m = \delta_1 \frac{D_m}{2} + \delta_2 \frac{D_m}{2} + \sum_{i=3}^N \delta_i D_c$  after the merger. Both  $X_{sq}$  and  $X_m$  are weighted sums of  $N$  uniform random variables, but in the first case weights are equal and in the second case they differ (according to deposit sizes). This brings us to the main result about the asymmetry channel.

**Proposition 4** *Suppose the merged banks do not exchange reserves internally. Then:*

1. *The merger decreases aggregate liquidity risk if the relative cost of refinancing is sufficiently low, and increases it otherwise;*
2. *The merger always increases expected aggregate liquidity needs.*

The intuition behind Proposition 4 is as follows. As already mentioned for Lemma 1, moving from a uniformly weighted sum of random variables (in the status quo) to a heterogeneously weighted sum of random variables (after merger) increases the variance of the total sum. Thus, as Figure 3 illustrates, the distribution of  $X_{sq}$  gives lower probability to extreme events – very low and very high realizations of the aggregate liquidity demand – than that of  $X_m$ .

This change in the distribution of  $X_m$  reduces the aggregate liquidity risk if the relative cost of refinancing is low because it increases the probability that the aggregate liquidity

demand is below the total supply. This is illustrated in Figure 3, where total reserves – indicated by the vertical line  $\sum_{i=1}^N R_i$  – are low and the area  $1 - \Phi_m$  is larger than the diagonally striped area  $1 - \Phi_{sq}$ . The opposite happens when the relative cost of refinancing is high.

[FIGURE 3 ABOUT HERE]

Proposition 4 also states that the merger always increases the expected amount of public liquidity needed. The reason is that the expected aggregate liquidity needs depend not only on the frequency with which aggregate liquidity demand exceeds aggregate supply, but also on the magnitude of each excess. As noted earlier, the merger increases the variance of the distribution of  $X_m$  and thus the probability of events with very low and very high demands. If banks do not hold reserves, these increases offset each other and the expected aggregate liquidity needs are the same before and after the merger. By contrast, when banks hold positive reserves, they can cover the events with low aggregate liquidity demand. Hence, the higher probability of extreme events with high aggregate liquidity demand is not outweighed any more by the higher frequency of low demand events, and the expected aggregate liquidity needs grow.

## 5.2 Interaction with the Reserve Channel

In this section we reintroduce the possibility for the merged banks to use the internal money market. We first analyze how this affects aggregate liquidity through the reserve channel. Denote as

$$K_m = \frac{R_m + \sum_{i=3}^N R_c}{D_m + (N-2)D_c} = \frac{k_m D_m + \sum_{i=3}^N k_c D_c}{D_m + (N-2)D_c} \quad (11)$$

the aggregate reserve-deposit ratio after the merger. Since competitors choose the same ratio as in the status quo ( $k_c = k_{sq}$ ), the change in  $K_m$  is solely determined by the change in the merged banks' reserve-deposit ratio. Hence, it follows from Proposition 2 that  $K_m$  increases when the relative cost of refinancing is low (because then  $k_m > k_{sq}$ ), whereas it decreases otherwise. The following lemma describes how the change in the aggregate reserve-deposit ratio alone affects aggregate liquidity.

**Lemma 3** *Suppose the merger does not cause any asymmetry in banks' balance sheets ( $D_m = 2D_c$ ). Then, it decreases aggregate liquidity risk and expected aggregate needs if the relative cost of refinancing is low, and it increases them otherwise.*

When the merger does not generate asymmetry across banks' balance sheets, it affects aggregate liquidity only through the reserve channel. The aggregate liquidity supply changes, whereas the aggregate liquidity demand remains the same. Thus, the merger reduces both aggregate liquidity risk and expected aggregate liquidity needs when the aggregate liquidity supply increases through a higher reserve-deposit ratio of the merged banks. The opposite happens when the aggregate liquidity supply falls.



When the merger generates the internal money market and asymmetry across banks, both the asymmetry and the reserve channel are at work. Depending on the size of the relative cost of refinancing, the two channels can reinforce or offset each other. Therefore, we consider the cases of high and low relative cost of refinancing separately.

**Proposition 5** *If the relative cost of refinancing is high, the merger increases both aggregate liquidity risk and expected aggregate liquidity needs.*

When the relative cost of refinancing is high, the asymmetry channel and the reserve channel work in the same direction. The asymmetry channel increases the variance of the aggregate liquidity demand, and the reserve channel reduces the aggregate liquidity supply through the lower reserve holdings of the merged banks. Both these effects make the system more vulnerable to liquidity shortages and more dependent on public liquidity provision.

**Proposition 6** *If the relative cost of refinancing is low, then:*

- 1. There exists a critical level of the relative cost of refinancing such that the merger reduces aggregate liquidity risk if the cost of refinancing is below such critical level, and increases it otherwise.*
- 2. For any small level of asymmetry induced by the merger, there exists a set of values of the relative cost of refinancing for which the merger reduces expected aggregate liquidity needs.*

When the cost of refinancing is low, the reserve and the asymmetry channels drive aggregate liquidity in opposite directions, and the net effect depends on their relative strength. As shown in Lemma 3, the reserve channel reduces both aggregate liquidity risk and expected liquidity needs. As stated in Proposition 4, however, the asymmetry channel always increases expected aggregate liquidity needs, whereas it reduces aggregate liquidity risk only if the relative cost of refinancing is sufficiently low.

Thus, when the two channels interact, the merger reduces aggregate liquidity risk for a larger range of parameter values than in Proposition 4, where only the asymmetry channel is active. Similarly, it increases aggregate liquidity risk in a larger range of parameter values than in Lemma 3, where only the reserve channel is present.

As for the expected aggregate liquidity needs, the reserve channel dominates when the asymmetry induced by the merger is sufficiently small. Thus, there is a range of values of the relative cost of refinancing for which the merger reduces expected aggregate liquidity needs. The larger the asymmetry in banks' balance sheets, the larger is this range of parameters in which the merger increases expected aggregate liquidity needs.

## 6 Competition and Aggregate Liquidity

We now discuss in greater detail how mergers, loan market competition and reserve choices interact in determining both loan rates and aggregate liquidity (for simplicity, here interpreted only as expected aggregate liquidity needs), and we draw some policy implications.

At the individual bank level, the loan market equilibrium affects banks' reserve holdings (in absolute terms) by determining the amount of deposits required to finance loans, and hence the size of liquidity demands at any given level of reserves. Equilibrium reserve holdings determine banks' financing costs – the sum of the expected cost of refinancing and of the expected repayment to depositors –, and thereby influence the loan market equilibrium. At the aggregate level, loan market competition affects the degree of asymmetry in banks' balance sheets through the distribution of equilibrium loan market shares.

Table 2 summarizes the possible effects of the merger on both loan rates  $r^L$  and expected aggregate liquidity needs  $\Omega$ , as described in Propositions 3, 5 and 6. The rows of the table indicate whether a merger is characterized by low or high efficiency gains in terms of reduced loan provision costs ( $\frac{c_m}{c_c}$  high or low); the two columns show the cases of high and low relative cost of refinancing  $\frac{r^I}{r^D}$ .

[TABLE 2 ABOUT HERE]

When the relative cost of refinancing is low, the merger increases the aggregate reserve-deposit ratio, and the final effect on expected aggregate liquidity needs depends on whether the positive reserve channel dominates the asymmetry channel. When the relative cost of refinancing is high, the merger increases unambiguously expected aggregate liquidity needs because it reduces the aggregate reserve-deposit ratio. When efficiency gains are small, the increase in market power dominates, and the merger increases loan rates. The opposite happens when efficiency gains are large.

Cell I describes a case in which competition and liquidity concerns may be in conflict; loan rates increase, whereas expected aggregate liquidity needs may fall. In this case, the merger would be undesirable from a competition policy perspective, but it would be desirable from the perspective of a central bank that does not want to frequently inject large amounts of liquidity for the reasons discussed earlier. A similar conflict between competition and liquidity concerns may emerge in cell II, where loan rates fall but expected aggregate liquidity needs may rise. By contrast, in cell III the two concerns are aligned; the merger increases both loan rates and expected aggregate liquidity needs. Finally, in cell IV competition and liquidity concerns are always in conflict.

To see under which loan market conditions it is more likely that a merger causes a conflict between competition and liquidity considerations, we now perform some comparative statics. We restrict our attention to the scenario in cell I, since, as already discussed in Section 1, bank mergers seem to produce limited efficiency gains, if at all positive. The following lemma describes how changes in loan market conditions affect equilibrium loan rates and banks' balance sheets.

**Lemma 4** *Suppose mergers increase loan rates and reduce merged banks' balance sheets ( $D_m < 2D_c$ ). Then, an increase in efficiency gains, in the number of banks or in loan substitutability increases the merged banks' balance sheets relative to the ones of the competitors.*

The larger the efficiencies generated by the merger – the lower  $\beta$  –, the lower the equilibrium loan rates, and the larger the loan market shares of the merged banks relative to competitors. This implies larger deposits for the merged banks, due to both higher loan market shares and higher reserve-deposit ratios. Similarly, an increase in competition – either through an increase in the number of banks  $N$  or through a higher loan substitutability  $\gamma$  – reduces all equilibrium loan rates, but relatively more those charged by the merged banks, thereby increasing their relative size.

The following proposition discusses how an increase in merged banks' balance sheets affects expected aggregate liquidity needs.

**Proposition 7** *Suppose mergers reduce merged banks' balance sheets ( $D_m < 2D_c$ ). Then, an increase in efficiency gains, in the number of banks or in loan substitutability reduces expected aggregate liquidity needs if the relative cost of refinancing is low.*

In the parameter region where  $D_m < 2D_c$ , the increase in the merged banks' balance sheets caused by stronger efficiency gains reduces the asymmetry across banks and tends to reduce expected aggregate liquidity needs. If the relative cost of refinancing is low, this effect is reinforced by a parallel increase in the aggregate reserve-deposit ratio. Analogously, by increasing merged banks' relative size, a higher substitutability of bank loans weakens the asymmetry channel, and increases the aggregate reserve-deposit ratio. This reduces expected aggregate liquidity needs. The same happens when the number of banks increases.

Proposition 7 has important policy implications. Under the rather plausible parameter ranges considered, more competitive loan markets (lower  $\beta$ , higher  $N$  and  $\gamma$ ) are beneficial for interbank liquidity. As long as the relative cost of refinancing is low, mergers withdraw less liquidity from the interbank market when they lead to efficiency gains and take place in a more competitive environment. By implication, a successful competition policy in banking will also limit the expected amounts of liquidity a central bank has to inject in the banking system. In this sense competition and liquidity considerations may go 'hand in hand'.

## 7 Discussion

In the model we introduce a merger in a situation where all banks are identical ex ante. This means that the merger leads to some degree of heterogeneity in banks' sizes. In doing this,

we have large mergers in mind. Even though this appears consistent with the bank merger movement of the 1990s as shown in Table 1, not every merger leads to a more asymmetric banking system. For example, in a situation where the system is composed of a group of small banks and another group of large banks, mergers among the small banks would have the opposite effect. This configuration reverses the functioning of the asymmetry channel described in Section 5. A merger that makes the banking system more symmetric is, *ceteris paribus*, more likely to moderate aggregate liquidity fluctuations. Even in this situation, however, financial consolidation can still cause greater liquidity risk and larger expected aggregate liquidity needs, when it induces a reduction of banks' reserve holdings.

We show in Section 4 that, when the relative cost of refinancing is low, the presence of an internal money market leads to an increase in the reserve-deposit ratio of the merged banks, and thus to a larger total supply of liquidity in the system. This seems to be the empirically more plausible range of interbank and deposit rates. It is important to note though that the precise levels of the relative cost of refinancing – which we indicate as low or high – are rather of an indicative nature, because relaxing some assumptions can change those levels. First, the exact size of the range depends on the distribution of liquidity shocks. We have assumed  $\delta_i$  to be uniformly distributed on the support  $[0, 1]$ . Limiting the support to a fraction of the unit interval would reduce the range of the relative cost of refinancing for which reserves increase with the merger. Assuming another symmetric density function would also change the relevant range, although it would not change the qualitative results. Second, in the model we neglect price effects in the choice of reserves by assuming that all banks pay the same rate  $r^f$  to obtain liquidity. It may be argued that in reality this needs not always be the case. Large banks (in our case merged banks) might pay a slightly lower rate, for example because in some smaller countries they may have market power in the interbank market or because they may be perceived as safer thanks to expectations about 'too big to fail' policies. If present, these forces would act against the internalization effect of the internal money market, further limiting the range of parameters for which merged banks increase reserves.

## References

- [1] Allen F. and D. Gale, 2000, "Financial Contagion", *Journal of Political Economy*, 108(1), 1-33
- [2] Bhattacharya S. and D. Gale, 1987, "Preference Shocks, Liquidity and Central Bank Policy", in W. Barnett and K. Singleton (eds) *New Approaches to Monetary Economics*, Cambridge University Press, 69-88
- [3] Bhattacharya S. and P. Fulghieri, 1994, "Uncertain Liquidity and Interbank Contracting", *Economics Letters*, 44, 287-294
- [4] Berger A.N., R.S. Demsetz and P.E. Strahan, 1999, "The Consolidation of the Financial Services Industry: Causes, Consequences, and Implications for the Future", *Journal of Banking and Finance*, 23(2-4), 135-94
- [5] Boyd J.H. and S.L. Graham, 1996, "Consolidation in US Banking: Implications for Efficiency and Risk", Federal Reserve Bank of Minneapolis Working Paper, no. 572, December
- [6] Bradley D.M. and R.C. Gupta, 2002, "On the Distribution of the Sum of  $n$  Non-Identically Distributed Uniform Random Variables", *Annals of the Institute of Statistical Mathematics*, 54(3), 689-700
- [7] Campello, M., 2002, "Internal Capital Markets in Financial Conglomerates: Evidence from Small Bank Responses to Monetary Policy", *Journal of Finance*, 57(6), 2773-2805
- [8] Carletti E. and P. Hartmann, 2003, "Competition and Stability: What's Special about Banking?", in P. Mizen (ed.) *Monetary History, Exchange Rates and Financial Markets: Essays in Honour of Charles Goodhart*, vol. 2, Edward Elgar, Cheltenham, 202-229
- [9] Carletti E., P. Hartmann and G. Spagnolo, 2002, "Implications of the Bank Merger Wave for Competition and Stability", in *Risk Measurement and Systemic Risk*, Proceedings of the Third Joint Central Bank Research Conference, 38-50
- [10] Cetorelli, N., 2002, "Does Bank Concentration Lead to Concentration in Industrial Sectors?" manuscript, Federal Reserve Bank of Chicago
- [11] Deneckere R. and C. Davidson, 1985, "Incentives to Form Coalitions with Bertrand Competition", *Rand Journal of Economics*, 16(4), 473-486
- [12] Dermine J., 2000, "Bank Mergers in Europe, the Public Policy Issues", *Journal of Common Market Studies*, 38(3), 409-425

- [13] Diamond D.W., 1997, "Liquidity, Banks, and Markets", *Journal of Political Economy*, 105(5), 928-956
- [14] Diamond D.W. and P.H. Dybvig, 1983, "Bank Runs, Deposit Insurance and Liquidity", *Journal of Political Economy*, 91(3), 401-419
- [15] European Central Bank, 2000, Mergers and Acquisitions Involving the EU Banking Industry - Facts and Implications, Frankfurt, December
- [16] European Central Bank, 2002, Developments in Banks' Liquidity Profile and Management, Frankfurt, May
- [17] Freixas X., B. Parigi and J.C. Rochet, 2000, "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank", *Journal of Money, Credit, and Banking*, 32(3-2), 611-640
- [18] Gertner R.H., D.S. Scharfstein and J.C. Stein, 1994, "Internal Versus External Capital Markets", *Quarterly Journal of Economics*, 109, 1211-1230
- [19] Graham J.R., M.L. Lemmon and J. Wolf, 2002, "Does Corporate Diversification Destroy Value?", *Journal of Finance*, 57, 695-720
- [20] Group of Ten, 2001, Report on Consolidation in the Financial Sector ("Ferguson Report"), BIS/IMF/OECD, January
- [21] Hanweck G.A. and B. Shull, 1999, "The Bank Merger Movement: Efficiency, Stability and Competitive Policy Concerns", *The Antitrust Bulletin*, 44(2), 251-284
- [22] Hartmann P., M. Manna and A. Manzanares, 2001, "The Microstructure of the Euro Money Market", *Journal of International Money and Finance*, 20(6), 895-948
- [23] Hellman T.F., K. Murdock and J. Stiglitz, 2000, "Liberalization, Moral Hazard in Banking and Prudential Regulation: Are Capital Requirements Enough?", *American Economic Review*, 90(1), 147-165
- [24] Holmström B. and J. Tirole, 1998, "Private and Public Supply of Liquidity", *Journal of Political Economy*, 106(1), 1-40
- [25] Houston J., C. James and D. Marcus, 1997, "Capital Market Frictions and the Role of Internal Capital Markets in Banking", *Journal of Financial Economics*, 46(2), 135-164
- [26] Kashyap A.K., Rajan R. and J. Stein, 2002, "Banks as Liquidity Providers: An Explanation for the Co-Existence of Lending and Deposit-Taking," *Journal of Finance*, 57(1), 33-73

- [27] Keeley, 1990, "Deposit Insurance, Risk and Market Power in Banking", *American Economic Review*, 80, 1183-1200
- [28] Manzanares, Andres, 2002, "On the Effects of Asymmetric Weights on the Distribution of the Sum of  $n$  Random Variables", mimeo, European Central Bank, Frankfurt
- [29] Matutes C. and X. Vives, 2000, "Imperfect Competition, Risk Taking and Regulation in Banking", *European Economic Review*, 44(1), 1-34
- [30] Organisation for Economic Co-operation and Development, 2000, Mergers in Financial Services, DAFFE/CLP(2000)17, Paris, September
- [31] Perotti E.C. and J. Suarez, 2002, "Last Bank Standing: What Do I Gain if You Fail", *European Economic Review*, 46(9), 1599-1622
- [32] Perry M. and R. Porter, 1985, "Oligopoly and the Incentive for Horizontal Merger", *American Economic Review*, 75(1), 219-227
- [33] Rajan R.G., 1992, "Insiders and Outsiders: The Choice Between Informed and Arm's Length Debt", *Journal of Finance*, 47, 1367-1400
- [34] Rajan R.G., H. Servaes and L. Zingales, 2000, "The Cost of Diversity: The Diversification Discount and Inefficient Investment", *Journal of Finance*, 55, 35-80
- [35] Rhoades S.A., 1998, "The Efficiency Effects of Bank Mergers: An Overview of Case Studies of Nine Mergers", *Journal of Banking and Finance*, 22, 273-291
- [36] Salop S., 1979, "Monopolistic Competition with Outside Goods", *Bell Journal of Economics*, 10, 141-156
- [37] Scharfstein D.S. and J.C. Stein, 2000, "The Dark Side of Internal Capital Markets: Divisional Rent Seeking and Inefficient Investment", *Journal of Finance*, 55, 2537-2564
- [38] Sharpe S.A., 1990, "Asymmetric Information, Bank Lending, and Implicit Contracts: A Stylized Model of Customer Relationships", *Journal of Finance*, 45, 1069-1087
- [39] Shubik M. and R. Levitan, 1980, *Market Structure and Behaviour*, Harvard University Press, Cambridge (MA)
- [40] Spagnolo, G., 2000, "Debt as a (Credible) Collusive Device," SSE Working Paper No. 243, Stockholm
- [41] Stein J.C., 1997, "Internal Capital Markets and the Competition for Corporate Resources", *Journal of Finance*, 52, 111-133

## Appendix

### Proof of Proposition 1

Using Leibniz's rule and (1), from (8) we obtain the first order conditions with respect to the choice variables  $r_i^L$  and  $R_i$ :

$$\frac{\partial \Pi_i}{\partial r_i^L} = L_i + (r_i^L - c) \frac{\partial L_i}{\partial r_i^L} - \left[ \frac{r^I}{2} \frac{L_i^2}{(L_i + R_i)^2} + \frac{r^D}{2} \right] \frac{\partial L_i}{\partial r_i^L} = 0, \text{ for } i = 1 \dots N, \quad (12)$$

$$\frac{\partial \Pi_i}{\partial R_i} = r^D (L_i + R_i)^2 - r^I L_i^2 = 0, \text{ for } i = 1 \dots N. \quad (13)$$

Solving (13) for  $R_i$  gives

$$R_i = \left( \sqrt{\frac{r^I}{r^D}} - 1 \right) L_i. \quad (14)$$

Solving (12) for  $r_i^L$  in a symmetric equilibrium where  $r_i^L = r_{sq}^L$  for  $i = 1 \dots N$  after substituting (2) and (14) gives

$$l + (r_{sq}^L - c - \sqrt{r^I r^D}) \left( -\gamma \frac{N-1}{N} \right) = 0,$$

from which  $r_{sq}^L$  and  $c_{sq}$  follow. Substituting then  $r_{sq}^L$  in (2) gives  $L_{sq}$ , and through (14)  $R_{sq}$ . Substituting  $R_{sq}$  and  $L_{sq}$  in (1), we obtain  $D_{sq}$ . Q.E.D.

### Proof of Corollary 1

Solving (3) and (4) gives  $\phi_i = 1 - \frac{R_i}{D_i}$  and  $\omega_i = \frac{(R_i)^2}{2D_i} - R_i + \frac{D_i}{2}$ . Substituting the expressions for  $R_{sq}$  and  $D_{sq}$ , we obtain  $\phi_{sq}$  and  $\omega_{sq}$  as in the corollary. Q.E.D.

### Proof of Lemma 1

We proceed in two steps. First, we show that the variance of the liquidity demand  $x_m$  of the merged banks is minimized when deposits are raised symmetrically in the two regions. Second, we show that the expected liquidity needs of the merged banks (and therefore their refinancing costs) are lower when deposits are symmetric.

*Step 1.* Define the liquidity demand of the merged banks as

$$x_m = \delta_1 \alpha D_m + \delta_2 (1 - \alpha) D_m,$$

where  $\alpha \in [0, 1]$  indicates the fraction of deposits that the merged banks raise in one region and  $(1 - \alpha)$  the fraction they raise in the other region. Since  $\delta_1$  and  $\delta_2$  are independent and  $Var(\delta_1) = Var(\delta_2)$ , the variance of  $x_m$  is simply

$$\begin{aligned} Var(x_m) &= \alpha^2 D_m^2 Var(\delta_1) + (1 - \alpha)^2 D_m^2 Var(\delta_2) \\ &= Var(\delta_1) [\alpha^2 D_m^2 + (1 - \alpha)^2 D_m^2]. \end{aligned}$$



Differentiating it with respect to  $\alpha$ , we obtain

$$\frac{\partial \text{Var}(x_m)}{\partial \alpha} = 2D^2 \text{Var}(\delta_1)(2\alpha - 1) = 0,$$

which has a minimum at  $\alpha = \frac{1}{2}$ .

*Step 2.* Define now the liquidity demand of the merged banks as

$$x_{ma} = \delta_1 \alpha D_m + \delta_2 (1 - \alpha) D_m,$$

when  $\alpha \neq \frac{1}{2}$ , and as

$$x_{ms} = \delta_1 \frac{D_m}{2} + \delta_2 \frac{D_m}{2}$$

when  $\alpha = \frac{1}{2}$ . Applying the general formula in Bradley and Gupta (2002) to our case, the density functions of  $x_{ma}$  and  $x_{ms}$  can be written as (assume  $\alpha < \frac{1}{2}$  without loss of generality):

$$f_{ma}(x_{ma}) = \begin{cases} \frac{x_{ma}}{\alpha(1-\alpha)D_m^2} & \text{for } x_{ma} \leq \alpha D_m \\ \frac{1}{(1-\alpha)D_m} & \text{for } \alpha D_m < x_{ma} \leq (1-\alpha)D_m \\ \frac{D_m - x_{ma}}{\alpha(1-\alpha)D_m^2} & \text{for } x_{ma} > (1-\alpha)D_m, \end{cases}$$

$$f_{ms}(x_{ms}) = \begin{cases} \frac{4x_{ms}}{D_m^2} & \text{for } x_{ms} \leq D_m/2 \\ \frac{4(D_m - x_{ms})}{D_m^2} & \text{for } x_{ms} > D_m/2. \end{cases} \quad (15)$$

Since  $\alpha < \frac{1}{2}$ ,  $f_{ma}(x_{ma})$  is steeper than  $f_{ms}(x_{ms})$  both for  $x_{ma} \leq \alpha D_m$  and for  $x_{ma} > (1 - \alpha)D_m$ . This implies that the two density functions do not cross in these intervals, whereas they do it in two points in the interval  $\alpha D_m < x_{ma} \leq (1 - \alpha)D_m$ . Given that they are symmetric around the same mean  $D_m/2$  with  $\text{Var}(x_{ma}) > \text{Var}(x_{ms})$ , it is:

$$F_{ma} > F_{ms} \text{ for } R_m < \frac{D_m}{2}, \quad (16)$$

$$F_{ma} < F_{ms} \text{ for } R_m > \frac{D_m}{2},$$

where  $F_{ma} = \Pr(x_{ma} < R_m)$  and  $F_{ms} = \Pr(x_{ms} < R_m)$ .

Denote now as  $\omega_{ma}$  and  $\omega_{ms}$  the expected liquidity needs of the merged banks with asymmetric deposits and symmetric deposits respectively. We have

$$\begin{aligned} \omega_{ma} - \omega_{ms} &= \int_{R_m}^{D_m} (x_{ma} - R_m) f_{ma}(x_{ma}) d(x_{ma}) - \int_{R_m}^{D_m} (x_{ms} - R_m) f_{ms}(x_{ms}) d(x_{ms}) \\ &= \int_{R_m}^{D_m} x_{ma} f_{ma}(x_{ma}) d(x_{ma}) - \int_{R_m}^{D_m} x_{ms} f_{ms}(x_{ms}) d(x_{ms}) \\ &\quad - R_m(1 - F_{ma}(R_m)) + R_m(1 - F_{ms}(R_m)). \end{aligned} \quad (17)$$

Differentiating (17) with respect to  $R_m$  gives

$$\begin{aligned} \frac{d(\omega_{ma} - \omega_{ms})}{dR_m} &= -R_m f_{ma}(R_m) + R_m f_{ms}(R_m) - (1 - F_{ma}(R_m)) \\ &\quad + R_m f_{ma}(R_m) + (1 - F_{ms}(R_m)) - R_m f_{ms}(R_m) \\ &= F_{ma}(R_m) - F_{ms}(R_m). \end{aligned}$$

From (16) it follows  $\frac{d(\omega_{ma} - \omega_{ms})}{dR_m} > 0$  for  $R_m < \frac{D_m}{2}$  and  $\frac{d(\omega_{ma} - \omega_{ms})}{dR_m} < 0$  otherwise. This, along with  $\omega_{ma} - \omega_{ms} = 0$  both for  $R_m = 0$  and for  $R_m = D_m$  implies  $\omega_{ma} - \omega_{ms} > 0$  for all  $R_m \in [0, D_m]$ . Q.E.D.

## Proof of Proposition 2

The demand for liquidity of the merged banks,  $x_m = \delta_1 \frac{D_m}{2} + \delta_2 \frac{D_m}{2}$ , has density function as in (15). Using Leibniz's rule, the equality  $D_m = R_m + L_1 + L_2$ , and the ratio  $k_m = \frac{R_m}{D_m}$ , from (10) we can express the first order condition  $\frac{\partial \Pi_m}{\partial R_m} = 0$  as

$$\begin{cases} \frac{8}{3}k_m^3 - 4k_m^2 + 1 = \frac{r^D}{r^I} & \text{for } k_m \leq 1/2 \\ \frac{8}{3}(1 - k_m)^3 = \frac{r^D}{r^I} & \text{for } k_m > 1/2. \end{cases} \quad (18)$$

The term on the LHS of the equalities is the marginal benefit of increasing the reserve-deposit ratio, that is the reduction in the expected need of refinancing induced by a marginal increase of the reserve ratio. The term on the RHS of the equalities is the ratio between the marginal cost of raising reserves  $r^D$  and the marginal cost of refinancing  $r^I$ . From (18), we obtain:

$$k_m = \begin{cases} z(r^I, r^D) & \text{for } r^I \leq 3r^D \\ 1 - \sqrt[3]{\frac{3}{8} \frac{r^D}{r^I}} & \text{for } r^I > 3r^D, \end{cases} \quad (19)$$

where  $z(r^I, r^D)$  is the solution of the equation  $z^3 - \frac{3}{2}z^2 + \frac{3}{8}(1 - \frac{r^D}{r^I}) = 0$  in the interval  $(0, \frac{1}{2}]$  increasing in the ratio  $\frac{r^I}{r^D}$ . Since  $f(0) > 0$ ,  $f(1/2) < 0$  and  $f'(z) < 0$ ,  $z(r^I, r^D)$  is the unique real solution.

To compare  $k_m$  with  $k_{sq}$ , we rearrange  $k_{sq}$  given in (9) as

$$(1 - k_{sq})^2 = \frac{r^D}{r^I}, \quad (20)$$

where, as before, the LHS is the marginal benefit of increasing the reserve-deposit ratio and the RHS is the ratio between the marginal cost of raising deposits and holding reserves  $r^D$  and the marginal cost of refinancing  $r^I$ .

Denote as  $f(k_m)$  the LHS of (18) and as  $f(k_{sq})$  the LHS of (20). Plotting  $f(k_m)$  and  $f(k_{sq})$  for  $k_{sq}$  and  $k_m$  between 0 and 1, we get Figure 4.

[FIGURE 4 ABOUT HERE]

The curves  $f(k_m)$  and  $f(k_{sq})$  cross only once at  $k_{sq} = k_m = \frac{5}{8}$ . Substituting this value in (18) or (20) gives  $k_{sq} = k_m$  when  $\frac{r^I}{r^D} = \frac{64}{9}$ . Thus,  $k_m > k_{sq}$  if  $\frac{r^I}{r^D} < \frac{64}{9}$ , and  $k_m < k_{sq}$  otherwise. Q.E.D.

### Proof of Lemma 2

From the last two terms in (8), we can express the financing costs of competitors as

$$\frac{r^I}{2} \frac{L_c^2}{(R_c + L_c)} + \frac{r^D}{2} (R_c + L_c). \quad (21)$$

Using  $\frac{R_c}{D_c} = k_c$  and  $\frac{L_c}{D_c} = 1 - k_c$  in (21) and rearranging terms, we obtain

$$\frac{r^I(1 - k_c)^2 + r^D}{2(1 - k_c)}. \quad (22)$$

Analogously, from the last two terms in (10), using  $\frac{R_m}{D_m} = k_m$  and  $\frac{L_m}{D_m} = 1 - k_m$ , we obtain the financing costs of the merged banks as

$$\begin{cases} \frac{r^I(3-6k_m+4k_m^3)+3r^D}{6(1-k_m)} & \text{for } r^I \leq 3r^D \\ \frac{4r^I(1-k_m)^3+3r^D}{6(1-k_m)} & \text{for } r^I > 3r^D. \end{cases} \quad (23)$$

It is easy to check that when the merged banks set  $k_m$  at the level which is optimal for competitors, the financing costs of the merged banks are always lower than the ones of the competitors. A fortiori this must be true when they set  $k_m$  to minimize their financial costs. Q.E.D.

### Proof of Proposition 3

The merged banks choose  $r_1^L$  and  $r_2^L$  to maximize (10) while competitors choose  $r_i^L$  to maximize (8) where the subscript  $i$  is now  $c$ . Define from the financing costs in Lemma 2 ((22) and (23)) the total marginal costs of the competitors and the merged banks as

$$c_c = c + \frac{r^I(1 - k_c)^2 + r^D}{2(1 - k_c)} \quad (24)$$

and

$$c_m = \begin{cases} \beta c + \frac{r^I(3-6k_m+4k_m^3)+3r^D}{6(1-k_m)} & \text{for } r^I \leq 3r^D \\ \beta c + \frac{4r^I(1-k_m)^3+3r^D}{6(1-k_m)} & \text{for } r^I > 3r^D, \end{cases} \quad (25)$$

respectively. Using the expressions for  $k_m$  and  $k_c$  in (19) and (20), those for  $c_c$  and  $c_m$  in (24) and (25),  $D_m = R_m + L_1 + L_2$  and  $D_c = R_c + L_c$ , we can write the expected profits for the merged banks and competitors when reserves are chosen optimally as

$$\Pi_m = r_1^L L_1 + r_2^L L_2 - c_m(L_1 + L_2)$$

$$\Pi_c = (r_1^L - c_c)L_c,$$

where

$$L_m = L_1 + L_2 = \left[ l - \gamma \left( r_1^L - \frac{1}{N} \sum_{j=1}^N r_j^L \right) \right] + \left[ L - \gamma \left( r_2^L - \frac{1}{N} \sum_{j=1}^N r_j^L \right) \right], \quad (26)$$

and  $L_c$  is given by (2). The first order conditions are then given by

$$\frac{\partial \Pi_m}{\partial r_h^L} = L_h + (r_1^L - c_m) \frac{\partial L_1}{\partial r_h^L} + (r_2^L - c_m) \frac{\partial L_2}{\partial r_h^L} = 0 \text{ for } h = 1, 2 \quad (27)$$

$$\frac{\partial \Pi_c}{\partial r_i^L} = L_c + (r_i^L - c_c) \frac{\partial L_c}{\partial r_i^L} = 0 \text{ for } i = 3 \dots N. \quad (28)$$

We look at the post-merger equilibrium where  $r_1^L = r_2^L = r_m^L$  and  $r_i^L = r_c^L$ . Substituting (26) in (27) and (2) in (28), we obtain the best response functions as

$$r_m^L = \frac{l}{2\gamma(\frac{N-2}{N})} + \frac{c_m}{2} + \frac{r_c^L}{2}. \quad (29)$$

$$r_c^L = \frac{l}{\gamma(\frac{N+1}{N})} + \left(\frac{N-1}{N+1}\right)c_c + \frac{2}{N+1}r_m^L. \quad (30)$$

Solving (29) and (30) gives the post-merger equilibrium loan rates  $r_m^L$  and  $r_c^L$ . Substituting  $r_m^L$  and  $r_c^L$  respectively in (26) and in (2) gives the equilibrium  $L_m$  and  $L_c$ . Analogously, we derive  $D_m$  and  $D_c$ . Q.E.D.

## Proof of Corollary 2

Using (15), we can express the liquidity risk for the merged banks as

$$\phi_m = \Pr(x_m > R_m) = \begin{cases} 1 - \int_0^{R_m} \frac{4x_m}{D_m^2} dx_m & \text{for } r^I \leq 3r^D \\ \int_{R_m}^{D_m} \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I > 3r^D. \end{cases}$$

Solving the integrals, we obtain  $\phi_m = 1 - 2\frac{R_m^2}{D_m^2}$  for  $r^I \leq 3r^D$  and  $2 - 4\frac{R_m}{D_m} + 2\frac{R_m^2}{D_m^2}$  for  $r^I > 3r^D$ . Substituting  $k_m = \frac{R_m}{D_m}$  implies

$$\phi_m = \begin{cases} 1 - 2k_m^2 & \text{for } r^I \leq 3r^D \\ 2(1 - k_m)^2 & \text{for } r^I > 3r^D. \end{cases}$$

Substituting  $k_m$  as in (19), we can express the merged banks' resiliency as

$$1 - \phi_m = \begin{cases} 2[z(r^I, r^D)]^2 & \text{for } r^I \leq 3r^D \\ 1 - 2\left(\sqrt[3]{\frac{3}{8} \frac{r^D}{r^I}}\right)^2 & \text{for } r^I > 3r^D. \end{cases}$$

Similarly, from Corollary 1 we can write a bank's individual resiliency in the status quo as  $1 - \phi_{sq} = k_{sq} = 1 - \sqrt{\frac{r^D}{r^I}}$ . Plotting these expressions as a function of the ratio  $\frac{r^I}{r^D}$ , one immediately sees that  $1 - \phi_m > 1 - \phi_{sq}$  always holds, so that  $\phi_m < \phi_{sq}$ . The plot is available from the authors upon request. Q.E.D.

### Proof of Corollary 3

Using (15), we can express the expected liquidity needs for the merged banks as

$$\omega_m = \begin{cases} \int_{R_m}^{\frac{D_m}{2}} (x_m - R_m) \frac{4x_m}{D_m^2} dx_m + \int_{\frac{D_m}{2}}^{D_m} (x_m - R_m) \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I \leq 3r^D \\ \int_{\frac{D_m}{2}}^{D_m} (x_m - R_m) \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I > 3r^D. \end{cases}$$

Solving the integrals, we obtain  $\omega_m = \frac{D_m}{2} - R_m + \frac{2}{3} \frac{R_m^3}{D_m^2}$  for  $r^I \leq 3r^D$  and  $\frac{2}{3} \frac{(D_m - R_m)^3}{D_m^2}$  for  $r^I > 3r^D$ . Substituting  $k_m = \frac{R_m}{D_m}$ , we obtain

$$\omega_m = \begin{cases} \left( \frac{1}{2} - k_m + \frac{2}{3} k_m^3 \right) D_m & \text{for } r^I \leq 3r^D \\ \frac{2}{3} (1 - k_m)^3 D_m & \text{for } r^I > 3r^D. \end{cases}$$

To compare  $\omega_m$  with  $2\omega_{sq}$ , we substitute (19) in the above expression for  $\omega_m$  and (20) in the expression for  $\omega_{sq}$  as in Corollary 1. We obtain:

$$\omega_m - 2\omega_{sq} = \begin{cases} \left( \frac{1}{2} - k_m + \frac{2}{3} k_m^3 \right) D_m - (1 - k_{sq})^2 D_{sq} & \text{for } r^I \leq 3r^D \\ \frac{r^D}{r^I} \left( \frac{D_m}{4} - D_{sq} \right) & \text{for } r^I > 3r^D. \end{cases}$$

For  $r^I > 3r^D$  it is immediate to see that  $\omega_m - 2\omega_{sq} < 0$  if  $\frac{D_m}{D_{sq}} < 4$ . For  $r^I \leq 3r^D$ ,  $\omega_m - 2\omega_{sq}$  can be rearranged as

$$\omega_m - 2\omega_{sq} = (1 - k_{sq})^2 D_{sq} \left[ \frac{\left( \frac{1}{2} - k_m + \frac{2}{3} k_m^3 \right) D_m}{(1 - k_{sq})^2 D_{sq}} - 1 \right].$$

Suppose for a moment  $k_m = k_{sq}$  and  $D_m = 2D_{sq}$ . Then, the expression simplifies to  $k_{sq}^2 D_{sq} \left( \frac{4}{3} k_{sq} - 1 \right)$ , which is negative because  $k_{sq} < 1/2$ . To see that this holds also for  $k_m > k_{sq}$ , we use (20) and rewrite  $\omega_m - 2\omega_{sq}$  as

$$\omega_m - 2\omega_{sq} = \frac{r^D}{r^I} D_{sq} \left[ \frac{r^I}{r^D} \left( \frac{1}{2} - k_m + \frac{2}{3} k_m^3 \right) \frac{D_m}{D_{sq}} - 1 \right].$$

Denote now  $A = \left( \frac{1}{2} - k_m + \frac{2}{3} k_m^3 \right)$ . Since  $A$  is decreasing in  $k_m$  and  $k_m > k_{sq}$  for  $r^I \leq 3r^D$ , it follows  $\omega_m - 2\omega_{sq} < 0$  when  $D_m = 2D_{sq}$ . The same holds for  $\frac{D_m}{D_{sq}} < 2$ . By plotting the expression  $\left( \frac{r^I}{r^D} A \frac{D_m}{D_{sq}} - 1 \right)$  for  $\frac{D_m}{D_{sq}} > 2$  and  $\frac{r^I}{r^D} \in (1, 3]$ , one sees that there is a level  $h \in (2, 4)$

of the ratio  $\frac{D_m}{D_{sq}}$  such that  $\omega_m \leq 2\omega_{sq}$  if  $\frac{D_m}{D_{sq}} \leq h$ , and  $\omega_m > 2\omega_{sq}$  otherwise. The plot is available from the authors upon request. Q.E.D.

#### Proof of Proposition 4

This proof is a generalization of that of Lemma 1. Let  $D_{tot}$  denote the total deposits  $ND_{sq} = D_m + (N-2)D_c$ , and let  $R_{tot}$  denote the total reserves  $NR_{sq} = R_m + (N-2)R_c$ . Applying the general formula for the distribution of a weighted sum of uniformly distributed random variables in Bradley and Gupta (2002) to our model, we obtain the density functions of the aggregate liquidity demands in the status quo  $f_{sq}(X_{sq})$  and after the merger  $f_m(X_m)$  as

$$f_{sq}(X_{sq}) = \frac{1}{(N-1)!(D_{sq})^N} \sum_{i=0}^N \left[ (-1)^i \binom{N}{i} (X_{sq} - iD_{sq})_+^{N-1} \right],$$

$$f_m(X_m) = \frac{\sum_{i=1}^{N-2} \left[ (-1)^i \binom{N-2}{i-1} (X_m - D_m - (i-1)D_c)_+^{N-2} + \binom{N-2}{i} (X_m - iD_c)_+^{N-2} \right]}{(N-2)!D_m(D_c)^{N-2}}.$$

The two density functions are plotted in Figure 3. The density  $f_{sq}(X_{sq})$  is more concentrated around the mean than  $f_m(X_m)$ . To verify that this is always the case, we compare the variances of  $X_{sq}$  and  $X_m$ , which are given by

$$Var(X_{sq}) = \sum_{i=1}^N D_{sq}^2 Var(\delta_i),$$

$$Var(X_m) = \frac{D_m^2}{4} Var(\delta_1) + \frac{D_m^2}{4} Var(\delta_m) + \sum_{i=3}^N D_c^2 Var(\delta_i)$$

$$= Var(\delta_i) \left[ \frac{D_m^2}{2} + \sum_{i=3}^N D_c^2 \right]$$

because  $Var(\delta_1) = Var(\delta_2) = Var(\delta_i)$ . Since  $D_m + \sum_{i=3}^N D_c = \sum_{i=1}^N D_{sq}$ , one obtains  $\left[ \sum_{i=1}^2 \frac{D_m^2}{4} + \sum_{i=3}^N D_c^2 \right] > \sum_{i=1}^N D_{sq}^2$  by Lagrangian maximization. Hence, it is always  $Var(X_m) > Var(X_{sq})$ . Since  $f(X_{sq})$  and  $f(X_m)$  are well behaved (they approach a normal distribution), they intersect only in two points.<sup>10</sup> This, along with the symmetry of the two density functions around the same mean  $E[X_m] = E[X_{sq}] = \frac{D_{tot}}{2}$  and  $Var(X_m) > Var(X_{sq})$ , implies

$$\Phi_{sq} = \Pr(X_{sq} > R_{tot}) > \Phi_m = \Pr(X_m > R_{tot}) \text{ for any } R_{tot} < \frac{D_{tot}}{2},$$

and vice versa for  $R_{tot} > \frac{D_{tot}}{2}$ . Using Proposition 1,  $R_{tot} = NR_{sq}$ , and (1), we obtain that  $R_{tot} < \frac{D_{tot}}{2}$  if  $\frac{r^I}{r^D} < 4$ . The first statement follows.

<sup>10</sup>A formal proof that this is the case is in Manzanares (2002).

Using the definition in (7), we have

$$\begin{aligned}\Omega_m - \Omega_{sq} &= \int_{R_{tot}}^{D_{tot}} (X_m - R_{tot})f_m(X_m)d(X_m) - \int_{R_{tot}}^{D_{tot}} (X_{sq} - R_{tot})f_{sq}(X_{sq})d(X_{sq}) \\ &= \int_{R_{tot}}^{D_{tot}} X_m f_m(X_m)d(X_m) - \int_{R_{tot}}^{D_{tot}} X_{sq} f_{sq}(X_{sq})d(X_{sq}) \\ &\quad - R_{tot}(1 - F_m(R_{tot})) + R_{tot}(1 - F_{sq}(R_{tot})).\end{aligned}$$

Deriving it with respect to  $R_{tot}$  gives

$$\begin{aligned}\frac{d(\Omega_m - \Omega_{sq})}{dR_{tot}} &= -R_{tot}f_m(R_{tot}) + R_{tot}f_{sq}(R_{tot}) - (1 - F_m(R_{tot})) \\ &\quad + R_{tot}f_m(R_{tot}) + (1 - F_{sq}(R_{tot})) - R_{tot}f_{sq}(R_{tot}) \\ &= F_m(R_{tot}) - F_{sq}(R_{tot}).\end{aligned}$$

As showed earlier,  $F_m(R_{tot}) - F_{sq}(R_{tot}) > 0$  for  $R_{tot} < \frac{D_{tot}}{2}$  and  $F_m(R_{tot}) - F_{sq}(R_{tot}) < 0$  for  $R_{tot} > \frac{D_{tot}}{2}$ . Also,  $F_m(0) = F_{sq}(0) = 0$  and  $F_m(R_{tot}) = F_{sq}(R_{tot}) = 0$ . This implies  $\Omega_m - \Omega_{sq} > 0$  for all  $R_{tot} \in [0, D_{tot}]$ . The second statement follows. Q.E.D.

### Proof of Lemma 3

Suppose first  $\frac{r^I}{r^D} < \frac{64}{9}$ . In this range, the aggregate reserve/deposit ratio in the status quo (which coincides with the individual banks' deposit ratio) is smaller than the one after merger; i.e.,

$$k_{sq} = \frac{R_{sq}}{D_{sq}} = \frac{\sum_{i=1}^N R_{sq}}{ND_{sq}} < K_m$$

because  $k_m > k_c = k_{sq}$ . Consider now the aggregate liquidity risk. When  $D_m = 2D_c$ , this is given by

$$\Phi_{sq} = \text{prob} \left( \sum_{i=1}^N \delta_i D_{sq} > \sum_{i=1}^N R_{sq} \right) = \text{prob}(X' < k_{sq})$$

in the status quo, and by

$$\Phi_m = \text{prob} \left( \sum_{i=1}^N \delta_i D_c > R_m + \sum_{i=3}^N R_c \right) = \text{prob}(X' < K_m),$$

after the merger, where  $X' = \sum_{i=1}^N \frac{\delta_i}{N}$ . Since  $K_m > k_{sq}$ , it follows  $\Phi_m < \Phi_{sq}$ .

We can then express the expected aggregate liquidity needs in the status quo as

$$\Omega_{sq} = \int_{k_{sq}ND_{sq}}^{ND_{sq}} (X_{sq} - k_{sq}ND_{sq})f(X_{sq})d(X_{sq}) = ND_{sq} \int_{k_{sq}}^1 (X' - k_{sq})f(X')d(X').$$

Applying the same logic, the post-merger expected aggregate liquidity needs are

$$\begin{aligned}\Omega_m &= ND_c \int_{K_m}^1 (X' - K_m)f(X')d(X') \\ &= ND_{sq} (1 + (K_m - k_{sq})) \int_{K_m}^1 (X' - K_m)f(X')d(X'),\end{aligned}$$

where we have used  $D_m = 2D_c$  and  $D_m + (N - 2)D_c = ND_c = ND_{sq} + (K_m - k_{sq})ND_{sq}$ . Given  $K_m > k_{sq}$ , we can write the expected aggregate liquidity needs as

$$\begin{aligned}\Omega_{sq} &= ND_{sq} \left[ \int_{K_m}^1 (X' - k_{sq})f(X')d(X') + \int_{k_{sq}}^{K_m} (X' - k_{sq})f(X')d(X') \right] \\ &= ND_{sq} \left[ \int_{K_m}^1 (X' - K_m)f(X')d(X') + (K_m - k_{sq}) \int_{K_m}^1 f(X')d(X') + \int_{k_{sq}}^{K_m} (X' - K_m)f(X')d(X') \right]\end{aligned}$$

and, after rearranging and simplifying, we have

$$\Omega_m - \Omega_{sq} = ND_{sq} \left[ \begin{aligned} &(K_m - k_{sq}) \int_{K_m}^1 (X' - K_m - 1)f(X')d(X') \\ &- \int_{k_{sq}}^{K_m} (X' - K_m)f(X')d(X') \end{aligned} \right] < 0$$

because  $(X' - K_m - 1) < 0$ . Analogous steps can be followed for the case  $\frac{r^I}{r^D} > \frac{64}{9}$ . Q.E.D.

### Proof of Proposition 5

Proposition 4 implies that if  $k_m = k_{sq}$ , then  $\Phi_m > \Phi_{sq}$  and  $\Omega_m > \Omega_{sq}$  for any  $\frac{r^I}{r^D} > \frac{64}{9}$ . A fortiori this must be true in equilibrium where  $k_m < k_{sq}$  ( $\Phi_m$  and  $\Omega_m$  are decreasing in  $K_m$ , which falls with  $k_m$ ). Q.E.D.

### Proof of Proposition 6

Statement 1. From the proof of Proposition 4,  $K_m = k_{sq}$  implies  $\Phi_m = \Phi_{sq}$  when  $\frac{r^I}{r^D} = 4$ , and  $\Phi_m < \Phi_{sq}$  when  $\frac{r^I}{r^D} < 4$ . Since  $K_m > k_{sq}$  in the range  $\frac{r^I}{r^D} < \frac{64}{9}$ , it is  $\Phi_m < \Phi_{sq}$  when  $\frac{r^I}{r^D} = 4$ . The strict inequality and continuity imply that there must exist a neighborhood where  $\frac{r^I}{r^D} > 4$  and  $\Phi_m < \Phi_{sq}$ . For  $\frac{r^I}{r^D} > \frac{64}{9}$ ,  $\Phi_m > \Phi_{sq}$  (from Proposition 5); hence, there must exist a critical level  $g \in (4, \frac{64}{9})$  such that as  $\Phi_m < \Phi_{sq}$  if  $\frac{r^I}{r^D} < g$ , and  $\Phi_m > \Phi_{sq}$  otherwise. The first statement follows.

Statement 2. From Proposition 2,  $k_m = k_{sq}$  for  $\frac{r^I}{r^D} = 1$  and  $\frac{r^I}{r^D} = \frac{64}{9}$ , and  $k_m > k_{sq}$  for  $1 < \frac{r^I}{r^D} < \frac{64}{9}$ . This induces the same relation between  $K_m$  and  $k_{sq}$ , so that  $K_m - k_{sq}$  is first increasing and then decreasing in the interval  $\frac{r^I}{r^D} \in (1, \frac{64}{9})$ . By Proposition 4, when  $D_m \neq 2D_c$  there is a neighborhood of  $\frac{r^I}{r^D} = 1$  where  $\Omega_m - \Omega_{sq} > 0$ . Also, when  $\frac{r^I}{r^D} = \frac{64}{9}$  and  $D_m \neq 2D_c$ ,  $\Omega_m > \Omega_{sq}$ . When  $\frac{r^I}{r^D} = 1$ , it is always  $\Omega_m = \Omega_{sq} = \frac{D_{tot}}{2}$ . From Lemma 3, when  $D_m = 2D_c$  it is  $\Omega_m - \Omega_{sq} < 0$  for all  $\frac{r^I}{r^D} \in (1, \frac{64}{9})$  and  $\Omega_m = \Omega_{sq}$  when  $\frac{r^I}{r^D} = \frac{64}{9}$ . By continuity, if one fixes a sufficiently small level of asymmetry in the deposit bases across banks ( $D_m - 2D_c$  sufficiently small), then  $\Omega_m - \Omega_{sq} > 0$  in an immediate neighborhood of  $\frac{r^I}{r^D} = 1$ . Given that  $K_m - k_{sq}$  is increasing around  $\frac{r^I}{r^D} = 1$ , there will be a higher ratio  $\frac{r^I}{r^D}$ , named  $\underline{g}$ , such that if the merger generates that asymmetry when  $\frac{r^I}{r^D} = \underline{g}$ , then  $\Omega_m - \Omega_{sq} = 0$  and  $\Omega_m - \Omega_{sq} < 0$  in the immediate right neighborhood. Again by continuity,  $\Omega_m - \Omega_{sq} > 0$  in an immediate neighborhood of  $\frac{r^I}{r^D} = \frac{64}{9}$ . Given that  $K_m - k_{sq}$  is decreasing



around  $\frac{r^I}{r^D} = \frac{64}{9}$ , there will be a smaller ratio  $\frac{r^I}{r^D}$ , named  $\bar{g}$ , such that, when  $\frac{r^I}{r^D} = \bar{g}$ , then  $\Omega_m - \Omega_{sq} = 0$  and  $\Omega_m - \Omega_{sq} < 0$  in the immediate left neighborhood. The second statement follows. Q.E.D.

#### **Proof of Lemma 4**

Consider the parameter  $\beta$ . From Proposition 3, it is easy to check  $\partial r_m^L / \partial \beta > 0$  and  $\partial r_c^L / \partial \beta > 0$ . Since banks compete in strategic complements, it is also  $\partial r_m^L / \partial \beta > \partial r_c^L / \partial \beta$  and consequently  $\partial L_m / \partial \beta > \partial L_c / \partial \beta$ . Given  $D_m = \frac{1}{1-k_m} L_m$  and  $D_c = \frac{1}{1-k_c} L_c$ , it follows  $\partial(D_m/2D_c) / \partial \beta < 0$ . Analogous reasoning applies for the parameters  $\gamma$  and  $N$ . Q.E.D.

#### **Proof of Proposition 7**

When  $\frac{r^I}{r^D} < \frac{64}{9}$ , the aggregate reserve/deposit ratio  $K_m$  increases (reserve channel). By Lemma 3, this implies lower aggregate liquidity risk and lower expected aggregate liquidity needs. By Lemma 4, a decrease in  $\beta$  (an increase in  $N$  or  $\gamma$ ) increases the ratio  $D_m/2D_c$ , which, in the range  $D_m < 2D_c$ , reduces the asymmetry in the deposit bases, and, consequently, the variance of the aggregate liquidity demand (asymmetry channel). This last effect tends to reduce expected aggregate liquidity needs for all  $\frac{r^I}{r^D} < \frac{64}{9}$ . The statement follows. Q.E.D.

Table 1: Bank concentration ratios in industrial countries, 1980-1998

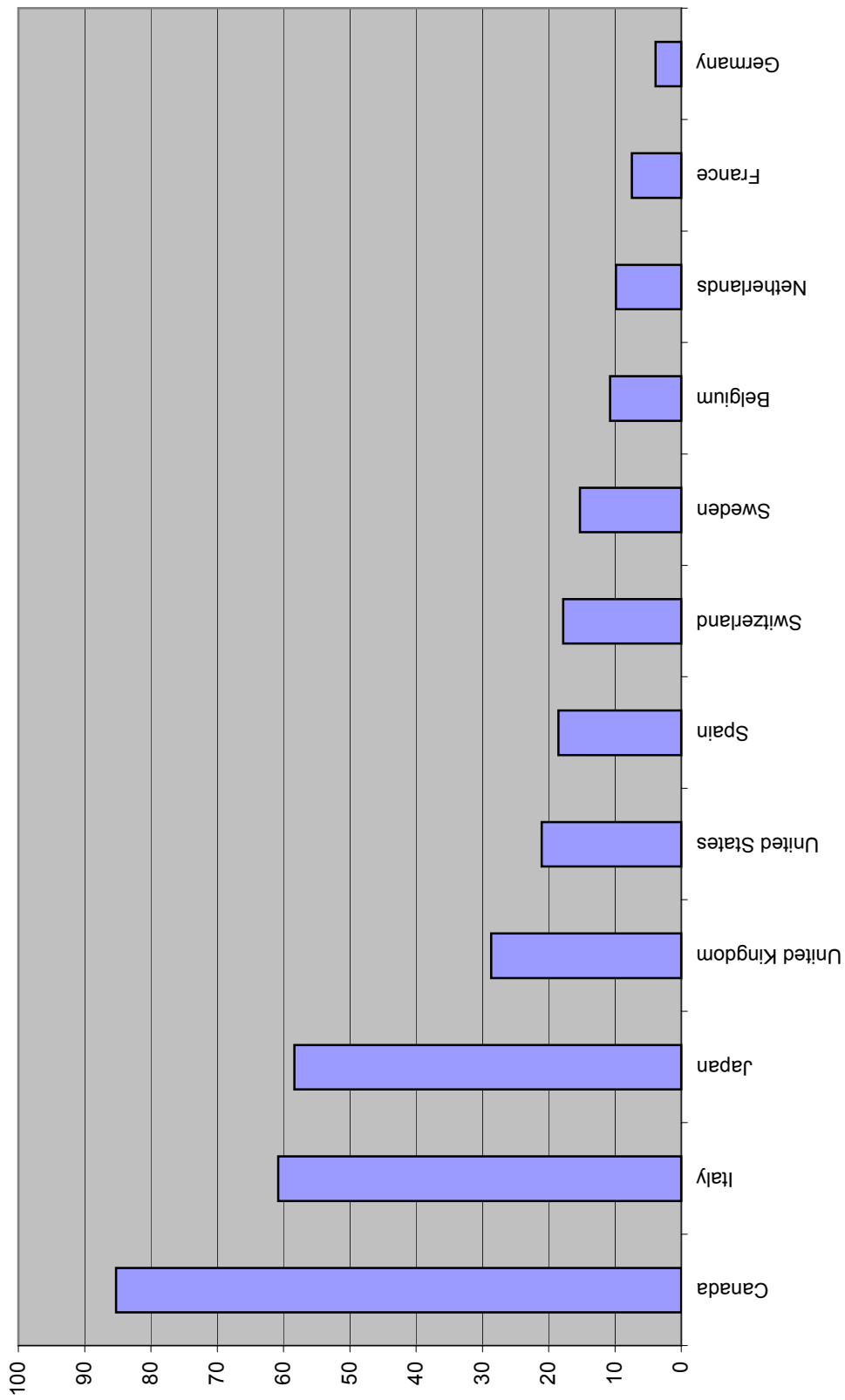
	1980	1990	1998	Change 1980-1990	Change 1990-1998
<i>Europe</i>					
Belgium	53.4	48.0	66.7	-5.4	18.7
France	n.a.	51.9	70.2	n.a.	18.3
Germany	n.a.	17.1	18.8	n.a.	1.7
Italy	n.a.	(25.9)	38.3	n.a.	12.4
Netherlands	n.a.	73.7	81.7	n.a.	8.0
Spain	38.1	38.3	(47.2)	0.2	8.9
Sweden	n.a.	62.0	84.0	n.a.	22.0
Switzerland	n.a.	53.2	(57.8)	n.a.	4.6
United Kingdom	n.a.	43.5	35.2	n.a.	-8.3
<i>North America</i>					
Canada	n.a.	60.2	77.7	n.a.	17.5
United States	14.2	11.3	26.2	-2.9	14.0
<i>Pacific Rim</i>					
Australia	76.5	72.1	73.9	-4.4	1.8
Japan	28.5	31.8	30.9	3.3	-0.9

*Notes:* Concentration ratios are defined as the share of the five largest banks in total bank deposits (in %). Values in parentheses are for 1992 (Italy) or 1997 (Spain, Switzerland). Changes are in percentage points (Spain and Switzerland 1990-1997, Italy 1992-1998). n.a.=not available. *Source:* Group of Ten, 2001

Table 2: Effects of a merger on loan rates and expected aggregate liquidity needs

	$\frac{r^I}{r^D}$ low	$\frac{r^I}{r^D}$ high
$\frac{c_m}{c_c}$ high	I $\Omega \uparrow\downarrow$ $r^L \uparrow$	III $\Omega \uparrow$ $r^L \uparrow$
$\frac{c_m}{c_c}$ low	II $\Omega \uparrow\downarrow$ $r^L \downarrow$	IV $\Omega \uparrow$ $r^L \downarrow$

**Figure 1: Banking consolidation in industrial countries, 1990-99**



*Note* Number of domestic M&As between banks (1990-99) divided by the average number of banks (1990-99) times 100.  
*Australia has been excluded for data consistency.*

*Source* Group of Ten, 2001.

Figure 3: Aggregate liquidity risk before merger,  $\Phi_{sq}$ , and after merger,  $\Phi_m$

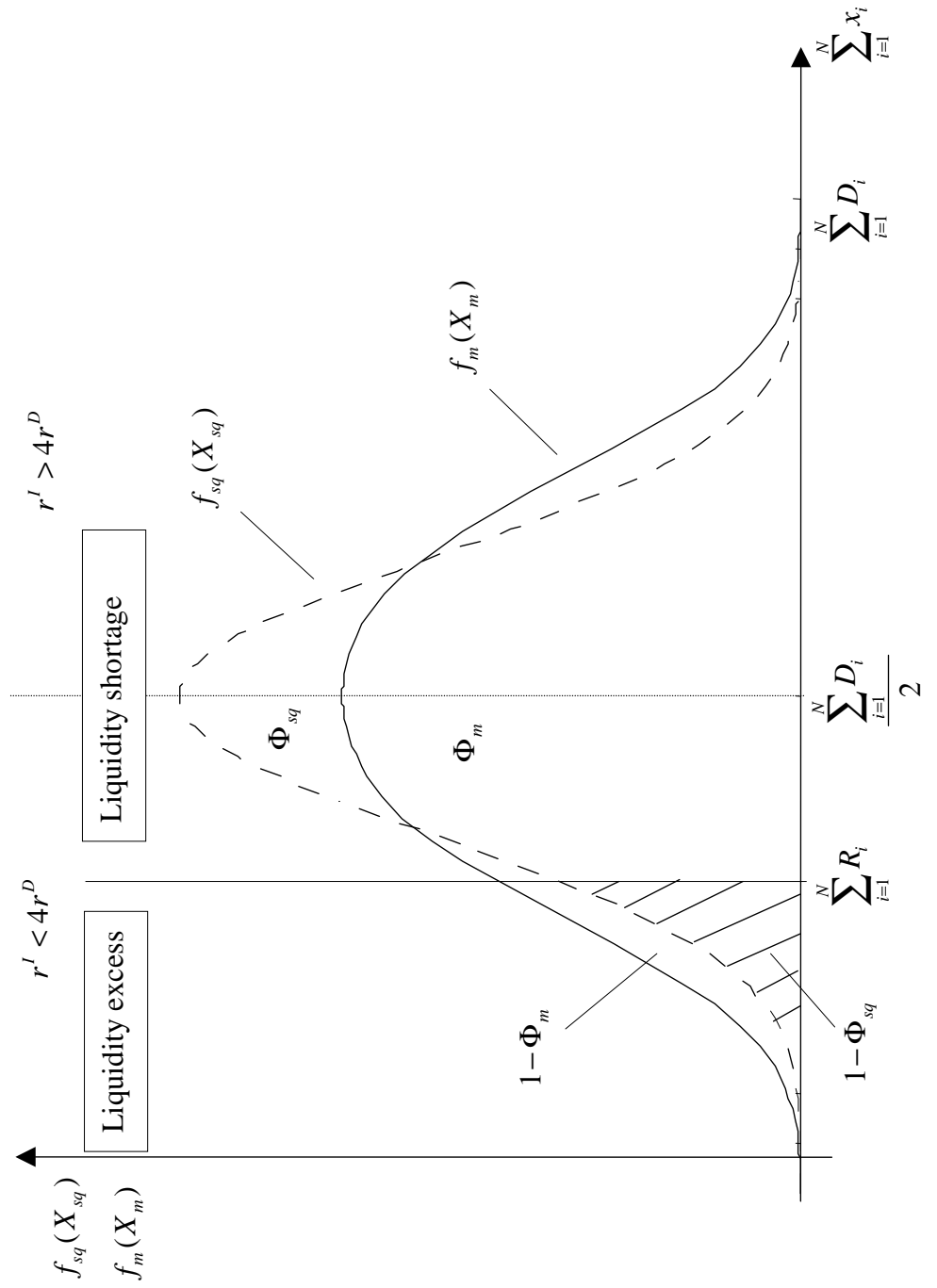
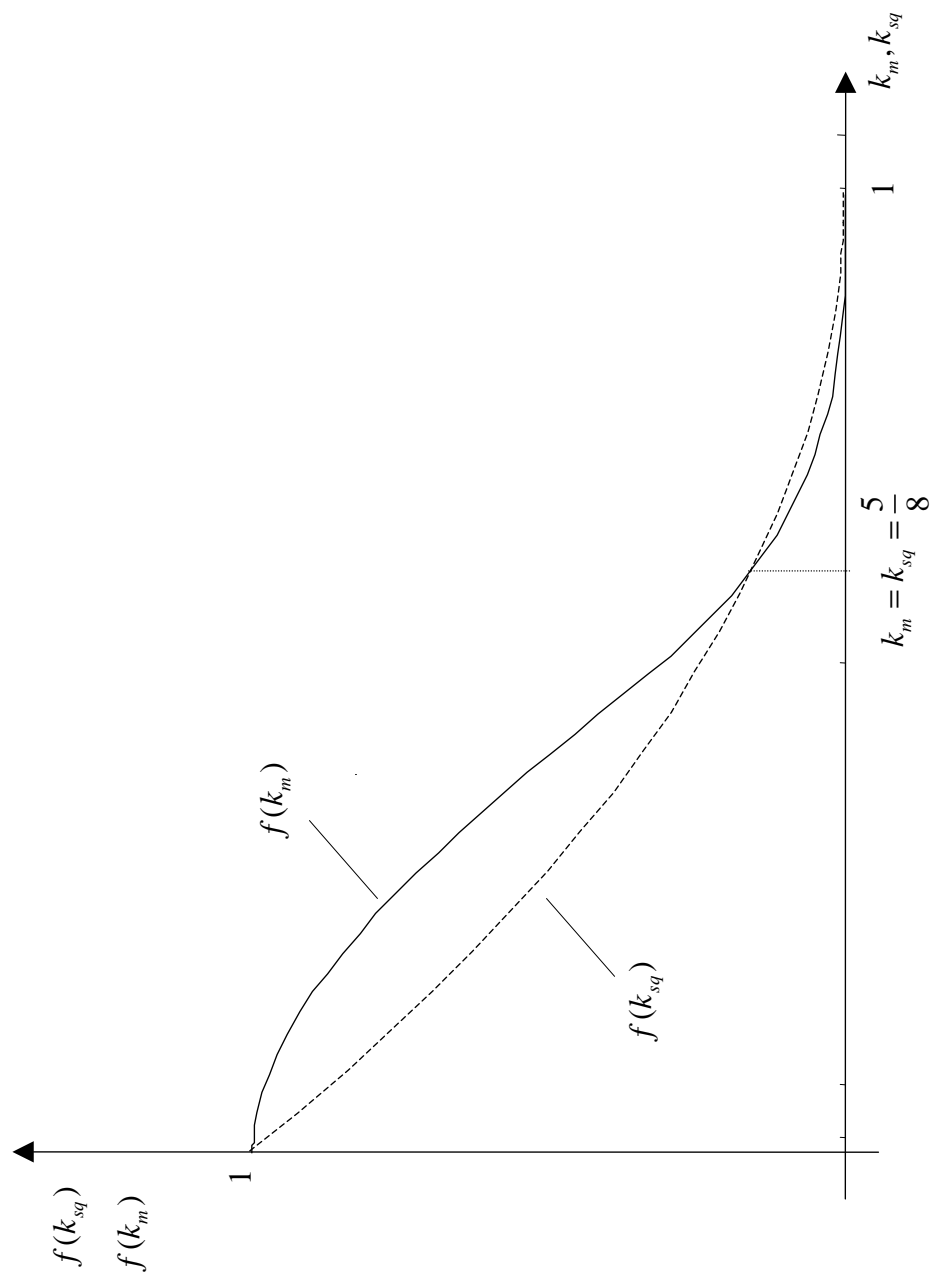


Figure 4: Marginal benefits of higher reserve-deposit ratios for the merged banks,  $f(k_m)$ , and for banks in the status quo,  $f(k_{sq})$ .



## European Central Bank working paper series

For a complete list of Working Papers published by the ECB, please visit the ECB's website (<http://www.ecb.int>).

- 202 "Aggregate loans to the euro area private sector" by A. Calza, M. Manrique and J. Sousa, January 2003.
- 203 "Myopic loss aversion, disappointment aversion and the equity premium puzzle" by D. Fielding and L. Stracca, January 2003.
- 204 "Asymmetric dynamics in the correlations of global equity and bond returns" by L. Cappiello, R.F. Engle and K. Sheppard, January 2003.
- 205 "Real exchange rate in an inter-temporal n-country-model with incomplete markets" by B. Mercereau, January 2003.
- 206 "Empirical estimates of reaction functions for the euro area" by D. Gerdesmeier and B. Roffia, January 2003.
- 207 "A comprehensive model on the euro overnight rate" by F. R. Würtz, January 2003.
- 208 "Do demographic changes affect risk premiums? Evidence from international data" by A. Ang and A. Maddaloni, January 2003.
- 209 "A framework for collateral risk control determination" by D. Cossin, Z. Huang, D. Aunon-Nerin and F. González, January 2003.
- 210 "Anticipated Ramsey reforms and the uniform taxation principle: the role of international financial markets" by S. Schmitt-Grohé and M. Uribe, January 2003.
- 211 "Self-control and savings" by P. Michel and J.P. Vidal, January 2003.
- 212 "Modelling the implied probability of stock market movements" by E. Glatzer and M. Scheicher, January 2003.
- 213 "Aggregation and euro area Phillips curves" by S. Fabiani and J. Morgan, February 2003.
- 214 "On the selection of forecasting models" by A. Inoue and L. Kilian, February 2003.
- 215 "Budget institutions and fiscal performance in Central and Eastern European countries" by H. Gleich, February 2003.
- 216 "The admission of accession countries to an enlarged monetary union: a tentative assessment" by M. Ca'Zorzi and R. A. De Santis, February 2003.
- 217 "The role of product market regulations in the process of structural change" by J. Messina, March 2003.

- 218 “The zero-interest-rate bound and the role of the exchange rate for monetary policy in Japan” by G. Coenen and V. Wieland, March 2003.
- 219 “Extra-euro area manufacturing import prices and exchange rate pass-through” by B. Anderton, March 2003.
- 220 “The allocation of competencies in an international union: a positive analysis” by M. Ruta, April 2003.
- 221 “Estimating risk premia in money market rates” by A. Durré, S. Evjen and R. Pilegaard, April 2003.
- 222 “Inflation dynamics and subjective expectations in the United States” by K. Adam and M. Padula, April 2003.
- 223 “Optimal monetary policy with imperfect common knowledge” by K. Adam, April 2003.
- 224 “The rise of the yen vis-à-vis the (“synthetic”) euro: is it supported by economic fundamentals?” by C. Osbat, R. Ruffer and B. Schnatz, April 2003.
- 225 “Productivity and the (“synthetic”) euro-dollar exchange rate” by C. Osbat, F. Visselaar and B. Schnatz, April 2003.
- 226 “The central banker as a risk manager: quantifying and forecasting inflation risks” by L. Kilian and S. Manganelli, April 2003.
- 227 “Monetary policy in a low pass-through environment” by T. Monacelli, April 2003.
- 228 “Monetary policy shocks – a nonfundamental look at the data” by M. Klaeffing, May 2003.
- 229 “How does the ECB target inflation?” by P. Surico, May 2003.
- 230 “The euro area financial system: structure, integration and policy initiatives” by P. Hartmann, A. Maddaloni and S. Manganelli, May 2003.
- 231 “Price stability and monetary policy effectiveness when nominal interest rates are bounded at zero” by G. Coenen, A. Orphanides and V. Wieland, May 2003.
- 232 “Describing the Fed’s conduct with Taylor rules: is interest rate smoothing important?” by E. Castelnuovo, May 2003.
- 233 “The natural real rate of interest in the euro area” by N. Giammarioli and N. Valla, May 2003.
- 234 “Unemployment, hysteresis and transition” by M. León-Ledesma and P. McAdam, May 2003.
- 235 “Volatility of interest rates in the euro area: evidence from high frequency data” by N. Cassola and C. Morana, June 2003.

- 236 “Swiss monetary targeting 1974-1996: the role of internal policy analysis” by G. Rich, June 2003.
- 237 “Growth expectations, capital flows and international risk sharing” by O. Castrén, M. Miller and R. Stiegert, June 2003.
- 238 “The impact of monetary union on trade prices” by R. Anderton, R. E. Baldwin and D. Taglioni, June 2003.
- 239 “Temporary shocks and unavoidable transitions to a high-unemployment regime” by W. J. Denhaan, June 2003.
- 240 “Monetary policy transmission in the euro area: any changes after EMU?” by I. Angeloni and M. Ehrmann, July 2003.
- 241 “Maintaining price stability under free-floating: a fearless way out of the corner?” by C. Detken and V. Gaspar, July 2003.
- 242 “Public sector efficiency: an international comparison” by A. Afonso, L. Schuknecht and V. Tanzi, July 2003.
- 243 “Pass-through of external shocks to euro area inflation” by E. Hahn, July 2003.
- 244 “How does the ECB allot liquidity in its weekly main refinancing operations? A look at the empirical evidence” by S. Ejerskov, C. Martin Moss and L. Stracca, July 2003.
- 245 “Money and payments: a modern perspective” by C. Holthausen and C. Monnet, July 2003.
- 246 “Public finances and long-term growth in Europe – evidence from a panel data analysis” by D. R. de Ávila Torrijos and R. Strauch, July 2003.
- 247 “Forecasting euro area inflation: does aggregating forecasts by HICP component improve forecast accuracy?” by K. Hubrich, August 2003.
- 248 “Exchange rates and fundamentals” by C. Engel and K. D. West, August 2003.
- 249 “Trade advantages and specialisation dynamics in acceding countries” by A. Zaghini, August 2003.
- 250 “Persistence, the transmission mechanism and robust monetary policy” by I. Angeloni, G. Coenen and F. Smets, August 2003.
- 251 “Consumption, habit persistence, imperfect information and the lifetime budget constraint” by A. Willman, August 2003.
- 252 “Interpolation and backdating with a large information set” by E. Angelini, J. Henry and M. Marcellino, August 2003.
- 253 “Bond market inflation expectations and longer-term trends in broad monetary growth and inflation in industrial countries, 1880-2001” by W. G. Dewald, September 2003.



- 254 “Forecasting real GDP: what role for narrow money?” by C. Brand, H.-E. Reimers and F. Seitz, September 2003.
- 255 “Is the demand for euro area M3 stable?” by A. Bruggeman, P. Donati and A. Warne, September 2003.
- 256 “Information acquisition and decision making in committees: a survey” by K. Gerling, H. P. Grüner, A. Kiel and E. Schulte, September 2003.
- 257 “Macroeconomic modelling of monetary policy” by M. Klaefferling, September 2003.
- 258 “Interest rate reaction functions and the Taylor rule in the euro area” by P. Gerlach-Kristen, September 2003.
- 259 “Implicit tax co-ordination under repeated policy interactions” by M. Catenaro and J.-P. Vidal, September 2003.
- 260 “Aggregation-theoretic monetary aggregation over the euro area, when countries are heterogeneous” by W. A. Barnett, September 2003.
- 261 “Why has broad money demand been more stable in the euro area than in other economies? A literature review” by A. Calza and J. Sousa, September 2003.
- 262 “Indeterminacy of rational expectations equilibria in sequential financial markets” by P. Donati, September 2003.
- 263 “Measuring contagion with a Bayesian, time-varying coefficient model” by M. Ciccarelli and A. Rebucci, September 2003.
- 264 “A monthly monetary model with banking intermediation for the euro area” by A. Bruggeman and M. Donnay, September 2003.
- 265 “New Keynesian Phillips Curves: a reassessment using euro area data” by P. McAdam and A. Willman, September 2003.
- 266 “Finance and growth in the EU: new evidence from the liberalisation and harmonisation of the banking industry” by D. Romero de Ávila, September 2003.
- 267 “Comparing economic dynamics in the EU and CEE accession countries” by R. Süppel, September 2003.
- 268 “The output composition puzzle: a difference in the monetary transmission mechanism in the euro area and the US” by I. Angeloni, A. K. Kashyap, B. Mojon and D. Terlizzese, September 2003.
- 269 “Zero lower bound: is it a problem with the euro area?” by G. Coenen, September 2003.
- 270 “Downward nominal wage rigidity and the long-run Phillips curve: simulation-based evidence for the euro area” by G. Coenen, September 2003.
- 271 “Indeterminacy and search theory” by N. Giammarioli, September 2003.

- 272 “Inflation targets and the liquidity trap” by M. Klaefferling and V. López Pérez, September 2003.
- 273 “Definition of price stability, range and point inflation targets: the anchoring of long-term inflation expectations” by E. Castelnuovo, S. Nicoletti-Altimari and D. Rodríguez-Palenzuela, September 2003.
- 274 “Interpreting implied risk neutral densities: the role of risk premia” by P. Hördahl and D. Vestin, September 2003.
- 275 “Identifying the monetary transmission mechanism using structural breaks” by A. Beyer and R. Farmer, September 2003.
- 276 “Short-term estimates of euro area real GDP by means of monthly data” by G. Rünstler, September 2003.
- 277 “On the indeterminacy of determinacy and indeterminacy” by A. Beyer and R. Farmer, September 2003.
- 278 “Relevant economic issues concerning the optimal rate of inflation” by D. R. Palenzuela, G. Camba-Méndez and J. Á. García, September 2003.
- 279 “Designing targeting rules for international monetary policy cooperation” by G. Benigno and P. Benigno, October 2003.
- 280 “Inflation, factor substitution and growth” by R. Klump, October 2003.
- 281 “Identifying fiscal shocks and policy regimes in OECD countries” by G. de Arcangelis and S. Lamartina, October 2003.
- 282 “Optimal dynamic risk sharing when enforcement is a decision variable” by T. V. Koepl, October 2003.
- 283 “US, Japan and the euro area: comparing business-cycle features” by P. McAdam, November 2003.
- 284 “The credibility of the monetary policy ‘free lunch’” by J. Yetman, November 2003.
- 285 “Government deficits, wealth effects and the price level in an optimizing model” by B. Annicchiarico, November 2003.
- 286 “Country and sector-specific spillover effects in the euro area, the United States and Japan” by B. Kaltenhaeuser, November 2003.
- 287 “Consumer inflation expectations in Poland” by T. Łyziak, November 2003.
- 288 “Implementing optimal control cointegrated I(1) structural VAR models” by F. V. Monti, November 2003.
- 289 “Monetary and fiscal interactions in open economies” by G. Lombardo and A. Sutherland, November 2003.

- 290 “Inflation persistence and robust monetary policy design” by G. Coenen, November 2003.
- 291 “Measuring the time-inconsistency of US monetary policy” by P. Surico, November 2003.
- 292 “Bank mergers, competition and liquidity” by E. Carletti, P. Hartmann and G. Spagnolo, November 2003.