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Multi-layered interbank model for assessing systemic risk

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Abstract

In this paper, we develop an agent-based multi-layered interbank network model based on a sample of large EU banks. The model allows for taking a more holistic approach to interbank contagion than is standard in the literature. A key finding of the paper is that there are material non-linearities in the propagation of shocks to individual banks when taking into account that banks are related to each other in various market segments. The contagion effects when considering the shock propagation simultaneously across multiple layers of interbank networks can be substantially larger than the sum of the contagion-induced losses when considering the network layers individually. In addition, a bank “systemic importance” measure based on the multi-layered network model is developed and is shown to outperform standard network centrality indicators. The finding of non-linear contagion effects when accounting for the interaction between the different layers of banks’ interlinkages have important policy implications. For example, it provides an argument for separating banks’ trading activities from their other intermediation activities.

JEL Classification: C45, C63, D85, G21

Key words: Financial contagion, interbank market, network theory

Non-technical summary

In this paper, we develop an agent-based multi-layered interbank network model based on a sample of large EU banks. The model allows for taking a more holistic approach to interbank contagion than is standard in the literature, where bank-to-bank spillover effects are typically confined to specific segments. However, in reality banks are interrelated in several dimensions of their business activities. The basic notion promoted in the paper is that unless contagion risk across the many layers of interrelations between banks are taken into account, it is likely that contagion effects will be substantially underestimated.

Specifically, in this paper we consider three different layers of interbank relationships. These include a network of short-term interbank loans (i.e. less than 3-month maturity) to reflect funding risk and a network of longer-term bilateral exposures (i.e. above 3-month maturity) to reflect counterparty risk. In addition, we consider a third network layer of common exposures in banks' securities portfolios where contagion can spread when one bank is forced to sell those securities that may give rise to sharp revaluation effects. This last layer aims at capturing market risk.

On top of the multi-layered system we put an agent-based model where agents can interact with each other through the network structure. The introduction of agents enables us to investigate specific network structures in combination with plausible bank behaviors. In particular, in the model banks only adjust their balance sheets when endogenous or exogenous shocks bring their liquidity or their risk-weighted capital ratio below the minimum requirements.

Our dataset include a sample of 50 large EU banks. For each bank, we include information about capital, short-term and long-term interbank borrowing, deposits, short-term and long-term interbank loans, aggregate securities holdings, and cash. We do not have data on individual banks bilateral exposures, neither on the details of financial securities portfolios. Instead, we use this uncertainty as degree of freedom of the model, in order to investigate which multi-layered network structures are particularly prone to a systemic breakdown.

A key finding of the paper is that there are material non-linearities in the propagation of shocks to individual banks when taking into account that banks are related to each other in various market segments. In a nutshell, the contagion effects when considering the shock propagation simultaneously across multiple layers of interbank networks can be substantially larger than the sum of the contagion-

induced losses when considering the network layers individually. In addition, a bank “systemic importance” measure based on the multi-layered network model is developed and is shown to outperform standard network centrality indicators.

The finding of non-linear contagion effects when accounting for the interaction between the different layers of banks’ interlinkages have important policy implications. For example, it provides an argument for separating banks’ trading activities from their other intermediation activities.

1 Introduction

During the financial crisis that emerged in 2008 a large part of the global financial system came under stress with severe repercussions on the real economy.

A robust financial system should not amplify the propagation of idiosyncratic (or “local”) shocks to other parts of the system and ultimately to the real economy. In this paper, systemic risk exactly refers to the possibility that the financial system is in a configuration which makes it particularly prone to global breakdowns in case of an initial, local shock. The reasons driving the system to such unstable and fragile configurations are probably rooted in the duality among local and global properties of the financial system. As a matter of fact, each financial institution takes actions with the aim of maximizing its own profits and interests, while the impact of those actions on the stability of the system as a whole are hardly taken into account. Moreover, as we will show in this paper, also if banks were willing to minimize systemic risk when they take decisions, they would need to have sufficient information regarding the financial situations of the other banks, including the exposures each bank have on all the others. As an example, one can consider the direct exposures in an interbank market. If one bank wants to evaluate the riskiness associated with a loan to another bank, it should be able to know the exposures of its counterparty, which probability of default depends on its own counterparties, and so on. No bank is able to peer so deeply into the interbank credit network to evaluate the probability of defaults due to contagion effects.

A crucial role in ensuring financial stability is therefore played by information. If the ultimate goal is to reduce systemic risk, it is necessary to have a global view of the financial system in order to identify and monitor possible sources and channels of contagion. A robust framework for monitoring and assessing financial stability, and for managing it with interventions able to prevent the system from entering into critical configurations, must be able to evaluate the continuously evolving structure of the financial system.

Another important lesson emerging from the recent financial crisis that we try to account for in this paper is that the possible sources of systemic instability are multiple. For instance, direct bilateral exposures can create domino effects and propagate idiosyncratic (or local) shocks to the wider (global) financial system. In addition, institutions can be forced to sell part of their security portfolios. This can lead to strong asset price declines and can transmit losses through banks with common exposures and overlapping portfolios.

Furthermore, news about a firm's assets can signal that others with similar assets may also be distressed and thus create widespread market uncertainty. Moreover, the sudden interruption of a service provided by a bank to the financial system can constitute a threat in case other banks are not able to immediately substitute it. When all those dynamics work together, the result can be critical, although the initial shock was comparably small.

Against this background, the aim of this paper is to study systemic risk in highly interconnected financial systems. A natural way to represent and study an interbank market is network theory, nowadays commonly used in finance. In order to encapsulate the different kinds of possible connections among banks, we use a multi-layered network model. A multi-layered network is a system where the same set of nodes belong to different layers, and each layer is characterized by its own *kind* of edge (representing a particular kind of financial connection), by its own topology (so each node may have different neighbors in different layers), and its own rules for the propagation of eventual shocks. This holistic view of the financial system should enable us to study systemic risk in a more encompassing perspective, than the typical single-layered network structures focusing on individual segments.

On top of the multi-layered system we put an agent-based model where agents can interact with each other through the network structure. The standard approach in the literature to study systemic risk using network theory represents banks as passive entities (the *nodes* of the network) connected to each other by some kind of financial contract, generally being interbank loans (the *edges* of the network).¹ Those kinds of models are good at estimating the resilience of particular network structures against shocks, but they lack real dynamic effects, since shocks propagate through the system without incorporating the (likely) reaction of banks to those shocks. The introduction of agents enable us to investigate specific network structures in combination with a plausible bank behavior. In particular, in our model banks will only adjust their balance sheets when endogenous or exogenous shocks bring their liquidity or their risk-weighted capital ratio below the minimum requirements. In fact, if we assume that prior to the shock the system was in equilibrium, banks would just try to keep the same structure of their balance sheets also during the propagation of the shock.

The failure of a financial institution usually implies several repercussions on the system. The liquidation of a failed bank can push

¹A pioneering work in this direction was initially proposed by Nier *et al.* (2009), while a summary of the results coming from this branch of literature can be found in Upper (2011).

prices down, its counterparts can book losses from direct exposures, the financial services provided by that bank cannot always be replaced, at least not immediately, and the combination of such reactions can significantly amplify shocks and lead to dangerous spirals which could potentially collapse a substantial part of the financial system (Brunnermeier (2009)). The complete dynamics of such events is difficult to capture with analytical models and from this perspective an agent-based model is more suitable, since it enables studying also systems out of equilibrium.

The agent-based model combined with the multi-layered network representation of the financial system is subsequently used to design measures for the systemic importance of each bank in the system. Those measures rely on information regarding direct and indirect interbank connections, which can be inferred from network theory, and banks' balance sheet information. The basic notion is that standard network centrality measures alone cannot explain the systemic importance of individual financial institutions, since the high level of heterogeneity in banking systems can bring central capitalized nodes to stabilize the system, whereas network measures would just judge nodes depending on their centrality. Instead, it is necessary to combine information regarding the balance sheet structure of institutions with measures of centrality in order to understand the impact of each bank failure on the system.

This paper is organized as follows: section 2 reviews the main literature linked to our work, highlighting both the contributions in the multi-layered network theory and the agent-based interbank models; section 3 introduces the multi-layered interbank market and explains how the structure is calibrated on a real dataset; section 4 explains the model we use for investigating systemic risk; section 5 presents details about the implementation of the model and the results from our simulation engine; section 6 introduces our measures for the systemic importance banks, and shows how the measures can be used to monitor systemic risk in the system; section 7 concludes and provides some policy implications.

2 Literature Review

In the past years, especially after the last financial crisis, a large amount of studies have emerged analyzing the financial system, and in particular the banking sector, from a network perspective. An early, seminal contribution to this literature is Allen and Gale (2000). Starting from the model of Diamond and Dybvig (1983),

the authors introduce an interbank liquidity market which enables banks to insure each other against liquidity shocks. Although in normal conditions such an interbank market can improve the stability of the financial system, in case a large shock hits one of the banks, the bank may fail and induce losses to its counterparties. These losses can subsequently potentially cause other defaults, therefore creating a domino-effect. The authors show that when the underlying network structure is complete (each bank is connected to all the others) the system is much more resilient due to the risk sharing effect, while incomplete networks are much more fragile since banks find it more difficult to diversify their portfolio structure against idiosyncratic shocks.

Nier *et al* (2009) show in their work how the topological features of the interbank network can be related to the financial stability of the system. Surprisingly, the results highlight that the higher the risk-sharing among banks, the higher the size of the domino effect (up to a certain threshold value for the connectivity between banks) in case of a shock hits one of the banks in the system. Furthermore, they show that increasing the level of capitalization will reduce the number of defaults in case a shock hits the system, and this effect is strongly non linear. Other studies concerning the interbank network, e.g. Gai and Kapadia (2010), clearly show the dualism of interbank connections: on one side, they are necessary in order to pool idiosyncratic risk of single institutions and improve the efficiency of the banking sector. In Iori *et al* (2006), a dynamic model of the banking system where banks can interact with each other through interbank loans is used to show the stabilizing role of the interbank lending. On the other hand, interbank connections can turn to be channels for the propagation of local shocks through the whole system. A summary of the results coming from this branch of the literature can be found in Upper (2011).

From a supervisory and macroprudential viewpoint, it is therefore necessary to measure and monitor the stability of the banking system as a whole, in parallel to the situation of the single financial institutions. In this respect, different measures of systemic risk have been developed, and a taxonomy of these measures is provided for example in Bisias *et al* (2012). In this non-exhaustive literature review of systemic risk we focus only on some contributions based on network analysis and systemic financial linkages. In Eisenberg and Noe (2001) a recursive algorithm to find the clearing payment vector that clears the obligations of a set of financial firms is provided. In addition, the authors provide information about the systemic risk faced by each institution. In Battiston *et al* (2012) a measure

based on network feedback centrality is introduced, the so-called *DebtRank*; this measure is used to analyze a dataset concerning the FED emergency loans program to global financial institutions during the period 2008-2010. The results show how, at the peak of the crisis, all the largest institutions served by the FED program became systemically important at the same time. In Hałaj and Kok (2013) an approach to generate interbank networks with realistic topologies is presented. Furthermore, the authors expand the Eisenberg and Noe (2001) algorithm to include firesales effect. Delpini *et al* (2013) study the Italian electronic trading system (e-MID) with tools borrowed from statistical physics to find the key players on a liquidity overnight market. Interestingly, the drivers of the market (ie the nodes which are crucial for the functioning of the interbank market) are often not the *hubs* neither the largest lenders in the system. We highlight that in all these contributions, results are always restricted to contagion or spillover effects related to one particular segment of the interbank market, which usually is the interbank claims banks have on each other.

The branch of the literature closer to our contribution is probably the one concerning dynamic interbank models. These discrete-time models usually allow to include some realistic microeconomic behavior for the banks on top of the network structure. An example can be found in Bluhm and Krahnert (2011). The authors study systemic risk in a banking system where financial institutions are linked to each other through interbank lending, and fire sales by one institution can materialize losses in all the others, since the price of the (mark-to-market) assets in the secondary market is endogenous in the model, and driven by the liquidity needs of the banks. The authors also introduce a game-theoretical approach to identify the contribution of each bank to systemic risk, and use this measure to develop an optimal charge to reduce financial instability. Georg (2011) develops a dynamic banking system where banks are allowed to optimize their portfolios of investments and they are subject to random shocks to their deposits. Within this framework, the author shows how the topology of the interbank market affects the stability of the system. In particular, he shows that contagion effects are larger in random networks than in scale-free networks, the classical structure of real world systems. He also investigates the role of the central bank in the interbank market, and in particular how the level of collateral which is accepted by the central bank affects financial stability. The results show that an abundant provision of liquidity by the central bank leads to a reduction of the liquidity banks exchange with each other on the interbank market. Ladley

(2011) develops a model of a closed economy composed of households which can deposit their funds with the banking sector and take loans from the banks for their private investments, and banks which learn how to better allocate their resources in order to maximize their expected returns. Since banks can lend also among each other, bad investments taken by households can trigger domino effects among the banks in the system. Banks in the model are subject to regulation, and the aim of the model is to qualitatively show the link among regulation, interbank network structure, and the likelihood of a contagion. The results show that for high levels of connectivity the system is more stable when the shock is small, while the spillover effects are amplified in case of larger initial shocks. Hałaj and Kok (2015) similarly introduce an agent-based model where banks optimize their risk-adjusted returns. The model is used to study how the adjustment of some key macroprudential policy parameters influences the interbank network structure.

Despite the huge number of contributions in network theory aimed at the identification of *important* nodes in a graph, a lot of work still has to be done for what regards multi-layered (ML) networks which is the topic of this paper. In different fields, from telecommunication engineering to sociology, ML systems are a natural representation of the reality. Examples are the Open Systems Interconnections (OSI) model, used to abstract the real internal structure of a communication system into different functionality layers, or the several ML social network models which encapsulate in different layers the different natures of possible social connections among people. Financial systems are another example of multi-layered networks, given the several kinds of connections that can exist among banks and other financial institutions. Recently, Gomez *et al* (2012) showed that a diffusion process, modeled as a flow traveling on the network from node to node, can be extremely amplified in case the same set of nodes is connected through multiple layers. The linear equations they propose in order to analyze the model are hardly applicable to cases where the nodes have a non-trivial internal structure and the contagion mechanisms change from layer to layer, but the results clearly support the necessity to study ML systems from a more holistic perspective than their single-layered counterpart.

We contribute to the literature in two main dimensions. First, we study how different segments of the interbank markets, and the related risks arising from them, interact with each other in an holistic view of the financial system. Second, we introduce a new measure for systemic importance institutions which embodies information regarding both the network structure of the multi-layered financial

system, which can be extracted with classical tools from network theory, and the balance sheets of the banks.

3 Multi-Layered Financial Systems

A natural way to study highly interconnected systems is network theory. Network theory provides a rich set of tools to assess the centrality (or systemic importance) of the members of a network of nodes. In this paper, each node in the network represents a bank. Importantly, each node will be equipped with a non-trivial internal structure, representing the banks' balance sheets. This is crucial, since abstracting from a realistic internal structure for the node means to disregard the realistic and interesting effects linked to limited liabilities and capital absorption. Moreover, a key aspect of this paper is to analyze the interconnectedness between banks in a multi-dimensional space. Banks in reality are connected through several kinds of relationships, directed and undirected, with different maturities. In order to encapsulate this level of complexity, we use a multi-layered instead of a single-layered network. We formally denote a *multi-layered* network by a triple $\mathfrak{G} = (V, \mathbf{W}, L)$, where V is a set of nodes, common to all the layers, L is a set of labels indicating the different layers, $\mathbf{W} = (W^1, W^2, \dots, W^L)$ is a set of matrices $W^l \in \mathfrak{R}_{N \times N}$, with the same cardinality of L , representing the network topologies in the different layers.

We want to concentrate in particular on three layers, which represent three different kinds of dependencies among banks that were revealed to be fundamental during the last financial crisis: (i) long-term, direct bilateral exposures, reflecting the lending-borrowing network; (ii) short-term direct bilateral exposures, reflecting the liquidity network; and (iii) common exposures to financial assets, representing the network of overlapping portfolios.² Consequently, we will label layers l_1 and l_2 for the long-term and short-term bilateral exposures, respectively, and the layer l_3 for the network of common exposures. All the three networks are weighted and directed.³

In layer l_1 , a link from node i to node j represents an unsecured, long-term loan from bank i to bank j , and the load W_{ij}^1 on the edge represents the face value of the loan. If bank i defaults, losses in

²It should be noted that several other layers can be added to the multi-layered framework, for example the layers representing the network of collaterals and the network of derivatives exposures. Naturally, the inclusion and calibration of other layers require more data, not available to us, that would increase the correctness of the results.

³The kind of network arising in layer l_3 depends on the definition used to compute the amount of overlapping securities portfolios. Different definitions can bring to undirected and unweighted networks as well.

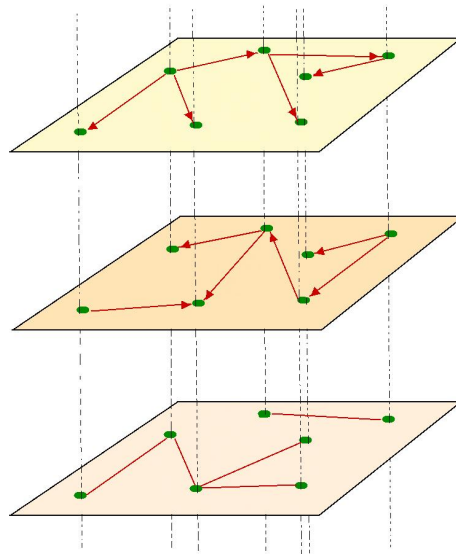


Figure 1: An example of a triple-layered network, where the same set of nodes belong to each of the three layers, characterized by its own topology. The first two layers contain directed networks, meanwhile the last one is undirected. The different neighbors in the different layers give the multi-layered networks completely different system dynamics during shock propagation, since the number of affected nodes can drastically be increased due to the multi dimensional structure of the system.

this layer are transmitted to its creditors, since its failure can potentially result in the inability of the bank to pay back (partially or totally) its outstanding loans. The losses thus incurred would directly affect the capital of the creditor banks. Layer l_1 therefore embodies interbank counterparty risk; differently from the case in which banks lend to isolated firms, when the borrower is a bank immerses in a network of credit relationships, its probability of default depends also on its own counterparties, which in turn depends on the conditions of their debtors, and so on. Interbank counterparty risk therefore is more complicated to estimate than risks related to non-bank counterparties, especially because banks usually do not have the complete information about the full network of exposures.

For what concerns layer l_2 , the global financial crisis illustrated that the short-term interbank funding market can play a crucial role in the propagation of shocks. Even well-capitalized financial institutions, which heavily rely on some form of short-term debt for financing their balance sheets, can get into trouble when the liquidity in the interbank markets suddenly evaporates. This happens if banks start (for whatever reason) to hoard liquidity instead of making it available on the market. The introduction of layer l_2 aims at capturing funding risk. A link from node i to node j represents an unsecured, short-term loan from bank i to bank j . The risk for bank j is that the debt will not be rolled over by its creditor bank i , exposing him to funding risk. We note the necessity to use different

layers in order to encapsulate different maturities in the interbank connections, which bring to different contagion mechanisms during a shock propagation.

The third layer l_3 is meant to reproduce the network of overlapping portfolios. When two banks invest in the same mark-to-market financial securities, their balance sheets can be correlated, since problems of one bank can force it to sell some securities, and the resulting price decline from such fire sales will affect the balance sheets of the banks which hold the same asset mark-to-market. Layer l_3 aims at reproducing such interdependencies among banks' balance sheets, and therefore embodies the liquidity risk banks face. A link between bank i and bank j exists if the two have some common mark-to-market assets in their balance sheets, and the load on the edge represents a measure of the strength of the correlation among them. In this layer, as already highlighted, shocks are transmitted through an indirect channel.

Funding risk and liquidity risk are intrinsically related to each other. Funding risk refers to the condition for which a bank is suddenly unable to raise liquidity, in this framework exemplified by the short-term interbank market. This can happen for several reasons: bad news about the financial institution leads to a deterioration of its creditworthiness, a common hoarding behavior by banks due to the fear of bad times ahead, or a real deterioration of the quality of the assets of the bank. If the bank is used to fund its assets through short-term loans, the inability of the bank to roll over its debt can force it to firesale some of its financial assets, which would have negative implications on the price of those assets. When asset prices fall down, deteriorating balance sheets may force firms which face capital ratio requirements to adjust their portfolios, perhaps by trying to hoard liquidity and capital. This mechanism can create liquidity spirals which amplify shocks (Brunnermeier (2009)).

4 Model for the Interbank Network

The model described in this section will be used for the analysis of systemic risk in this paper, and it is designed to capture important features of a real financial system. The model is composed of N interconnected financial institutions (hereafter, banks) and M financial securities. Banks' balance sheets are here composed of securities e_i , long-term interbank loans l_i^l , short-term interbank loans l_i^s , cash c_i , and other assets including all the other banks activity that will not be considered in our model, o_i^a ; i.e. total assets can be

expressed as follow: $a_i = e_i + l_i^l + l_i^s + c_i + o_i^a$. Liabilities include long-term interbank borrowing b_i^l , short-term interbank borrowing b_i^s , deposits d_i , and other liabilities not considered in the model, o_i^l . i.e. total liabilities can be expressed as: $l_i = b_i^l + b_i^s + d_i + o_i^l$. The balance sheets equality holds:

$$a_i = l_i + eq_i \quad (1)$$

where we call eq_i the equity of bank i . The securities portfolios of each bank are composed of a certain number of financial securities s_μ , $\mu = 1, 2, \dots, M$. So we can formally write for the mark-to-market value of the portfolio:

$$e_i = \sum_{\mu=0}^M s_\mu^i \cdot p_\mu \quad (2)$$

where p_μ is the price of the security μ and $s_\mu^i \geq 0$ is the notional amount of security μ in the portfolio of bank i . Banks' portfolios are assumed to be marked to market, and the price of the securities is endogenously determined in the model. The financial system can be mapped through the three weighted matrices described in section 3: W^1 describes the long-term interbank exposures, W^2 the short-term interbank exposures and W^3 the common exposures among banks.

Banks have to keep their risk-weighted capital ratio above a certain threshold value, and they have to fulfill a liquidity requirement. The risk-weighted capital ratio is computed as:

$$\gamma_i = \frac{a_i - l_i}{w^{ib} \cdot (l_i^l + l_i^s) + \sum_{\mu=0}^M w^\mu \cdot s_\mu^i p_\mu + CRWA_i} \quad (3)$$

where w^{ib} represents the weight for interbank assets, fixed here at 0.2, and w^μ are the weights for the financial assets, which are inferred from our data set; $CRWA_i$ represents the part of the risk-weighted assets which is not used in our model, and therefore is a constant. The first constraint banks have to fulfill is:

$$\gamma_i \geq \bar{\gamma} \quad (4)$$

where $\bar{\gamma}$ is the minimum capital requirement. The second constraint banks have to fulfill is:

$$c_i \geq \beta \cdot (d_i + b_i^s) \quad (5)$$

where β is the parameter representing the liquidity buffer.

In this model, a bank can suffer losses for two reasons: (i) some of its counterparts fail and are unable to pay back the debt, or (ii)

the price of some of its securities declines. The price of each security is endogenously determined in the model, and it is described by the following equation:

$$p_\mu = p_\mu^0 \cdot \exp \left\{ \frac{-\alpha_\mu \cdot \sum_i^N \text{sell}_\mu^i}{\sum_i^N s_\mu^i} \right\} \quad (6)$$

where $0 \leq \text{sell}_\mu^i \leq s_\mu^i$ is the amount of security μ sold by bank i , and α_μ is a positive constant representing the deepness of the market for that security.

If the bank's capital ratio in eq. (3) becomes lower than $\bar{\gamma}$ after it books some losses, the bank can increase it in two ways: (i) by reducing its short-term interbank exposure, or (ii) by selling securities. Since the cheapest way of increasing the risk-weighted capital ratio is to reduce interbank exposures, as long as $l_i^s > 0$ each bank first prefers to follow this way.⁴ Similarly, if the bank has to raise liquidity in order to fulfill the requirement expressed in eq. (5), it will first withdraw liquidity from the short-term interbank market, and if this is not enough, it will liquidate part of its portfolio. If a bank is not able to fulfill the capital requirement, it defaults. When a bank defaults, it is first liquidated, so all its securities are sold (if any) and it withdraws all its funds from the short-term interbank market, and then it tries to pay back its creditor banks. The failure of a bank involves, in the model, three risks for the other banks: (i) counterparty risk, associated with the possible losses from the interbank market, (ii) funding risk, associated with the possibility of losing funds from the short term interbank market, and (iii) liquidity risk, associated with firesales of mark-to-market financial securities.

4.1 Model Dynamics

The model dynamics is reported in Fig. 2. Starting from a particular configuration of the multi-layered network \mathfrak{G} of banks with heterogeneous balance sheets, we shock the system and then repeat the same sequence of events, representing a short-term financial period, until the number of defaults stops increasing.

⁴In this model, withdrawing funds from the short-term interbank market is the cheapest way to raise liquidity, since it does not involve any capital losses like the ones associated with firesales. Nevertheless, in reality a bank might prefer to sell assets if the market is deep enough to absorb the sales without resulting in large depreciation of the value of the assets. In any case, the dynamics reproduced in this model represents a possible series of events in case banks stop trusting each other inducing them to hoard liquidity rather than retain funds in the interbank market.

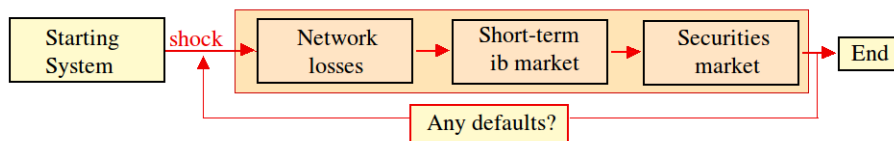


Figure 2: The Figure represents the dynamics of the model. Starting from the system at equilibrium, we shock it, usually by letting default one or more banks at the same time. Subsequently, the sequence of events in the shaded area of the figure is iterated till the number of defaults stops increasing; at the beginning of each (short-term financial) period, banks book losses coming from the default of their creditors during the previous period, if any; in a second step, they decide the percentage of debt to roll-over to their borrowers in the short-term interbank market; in the last step, banks which have liquidity needs liquidate part of their securities holdings.

At the beginning of each period, banks book losses from the interbank market, if any, due to the bankruptcy of their debtors in the previous period. Those losses immediately affect the capital of banks, and therefore their risk-weighted capital ratio described in eq. (3). If a bank's risk-weighted capital ratio remains above the threshold value $\bar{\gamma}$, then it will not react to the losses. Otherwise, it will first try to reduce its short-term interbank exposures. Indeed, during each period, banks have to decide which percentage of the short-term debt they want to roll-over to their debtors. This choice depends both on the internal needs of banks, due for example to losses coming from the long-term interbank market, which causes a reduction of the risk-weighted capital ratio of the bank under the threshold value $\bar{\gamma}$, or due to the fact that its own funding from other creditors bank is reduced, forcing it to withdraw money from the short-term market. This loop is properly described by the following map:

$$\vec{f} \cdot \vec{l}^s{}^\top = \min \left(\vec{r} + \max \left(W^2 \vec{f} - c_{buf}^\top; 0 \right); \vec{l}^s{}^\top \right) \quad (7)$$

where $\vec{f} = (f_1, f_2, \dots, f_N)$ is the percentage of funds withdrawn by each bank from the short-term interbank market ($f_i \in [0, 1]$, $i = 1, 2, \dots, N$); $\vec{r} = (r_1, r_2, \dots, r_N)$ is the amount each bank wants to withdraw for liquidity and capital reasons; $\vec{l}^s = (l_1^s, l_2^s, \dots, l_N^s)$ is the total short-term exposure of each bank; and $c_{buf}^\top = (c_{buf,1}, c_{buf,2}, \dots, c_{buf,N})$ is the total amount of cash each bank has out of its liquidity buffer, if any: $c_{buf,i} = \max [c_i - \beta(d_i + b_i^s); 0]$. The capital and liquidity needs are computed in order to restore the required level of cash and risk-weighted capital ratio according to the bank's constraints. We

have from equation (5):

$$r_i^{liq} = \min \left(l_i^s; \frac{\beta(d_i + b_i^s) - c_i}{1 + \beta} \right) \quad (8)$$

which is larger than zero as far as $c_i < \beta \cdot (d_i + b_i^s)$. If $c_i \geq \beta \cdot (d_i + b_i^s)$ the banks have no liquidity needs to fulfill, and therefore $r_i^{liq} = 0$. In the same spirit, we compute the amount to be withdrawn due to the risk-weighted capital ratio constraint; from equation (4) we have:

$$r_i^{cap} = \min \left(l_i^s - r_i^{liq}; \frac{\gamma_i(CRWA_i + \sum_{\mu=0}^M w^\mu \cdot s_\mu^i p_\mu) + \gamma_i w^{ib} \cdot (l_i^l + l_i^s - r_i^{liq}) - eq_i}{\gamma_i w^{ib}} \right) \quad (9)$$

r_i^{cap} is larger than zero as far as $\gamma_i < \bar{\gamma}$. If $\gamma_i \geq \bar{\gamma}$, then $r_i^{cap} = 0$. The final amount to withdraw will be $r_i = r_i^{liq} + r_i^{cap} \in [0, l_i^s]$. All in all, equation (7) simply states that each bank withdraws funds from the short-term interbank market only in case it has problems fulfilling its liquidity or risk-weighted capital ratio requirements, and in case other banks decide to withdraw their funds deposited with the bank and the cash it has is not enough to pay back those creditors.

Once banks decide about how much to withdraw from the interbank market, they may still need to sell securities in order to pay back eventual creditors and to restore the required levels of liquidity and capital buffers. As described by eq. (7), banks first use their available liquidity to pay back creditors, and if this is not enough they withdraw funds from the short-term interbank market. In case they still need liquidity, they have to liquidate some securities. We can indicate with $Z \in \mathbb{R}_{N \times M}$ the matrix whose entries $Z_{i\mu} \geq 0$ indicate how many securities of kind μ bank i has to sell in order to fulfill its needs. Since the securities prices are adjusting according to eq (6), we use a modified version of the map introduced by Eisenberg and Noe (2001) in order to compute both matrix Z and the clearing vector \vec{p} which resolves the system. We have:

$$\vec{p} = \min \left[\vec{l}; \Pi^\top \cdot \vec{p} + \vec{c} + Z \cdot \vec{v} \right] \quad (10)$$

where we denoted with Π the matrix with the relative obligations among banks, that is:

$$\Pi_{ij} = \frac{w_{ji}^2 f_j}{\sum_j w_{ji}^2 f_j} \quad (11)$$

The vector \vec{l} represents the total obligations of the banks towards the other institutions, that is:

$$l_i = \sum_j w_{ji}^2 f_j \quad (12)$$

and \vec{v} is the vector indicating the value of each security, according to eq. (6).

In turn, the matrix Z is computed as the sum of three components, which are the liquidity needs driven by obligations towards other banks in the system, the liquidity needs driven by the requirement expressed in eq. (5), and the liquidity needs driven by the capital requirement expressed in eq. (4). In more details, they can be formalized as follows: suppose there is only one security in the system, the generalization to the case of several securities is then straightforward; in this case, the matrix Z becomes a vector, again composed by three parts; the first part is:

$$Z^{ib} = \min \left[\max \left[0; \frac{\vec{l} - \vec{c} - \Pi^T \cdot \vec{p}}{p_\mu} \right]; \vec{s} \right] \quad (13)$$

where we indicated with $\vec{s} = (s_1, s_2, \dots, s_N)$ the amount of securities each bank still have in its portfolio. This is the component driven by the credit line reduction in the short-term interbank market.

The second component is:

$$Z^{liq} = \min \left[\frac{\max [0; \vec{c} - \alpha(\vec{d} + \vec{b}^s)]}{p_\mu}; \vec{s} \right] \quad (14)$$

This component takes into account the liquidity requirements of banks.

Eventually, there is the component due to the necessity of fulfilling capital requirements, which is larger than zero if also by withdrawing all their funds from the short-term interbank market they still need to increase their risk-weighted capital ratio:

$$Z^{cap} = \min \left[\frac{w^{ib} \vec{l}^{ib} + w^\mu p_\mu - \frac{e\vec{q}}{\bar{\gamma}}}{w^\mu}; \vec{s} \right] \quad (15)$$

The sum of these three components represents the total amount which appears in eq. (10) : $Z = Z^{ib} + Z^{liq} + Z^{cap}$. The generalization to the case of multiple securities is simply derived as follow: each bank tries to sell the first type of security in its portfolio; if the bank sells all those securities, it moves to the second type of security, and so on, up to the point when it fulfills its liquidity needs. Alternatively, if its liquidity needs cannot be fulfilled the bank will have to sell all its securities.

After the payment vector \vec{p} is computed, banks which are not able to pay back their creditors or to fulfill their Risk-Weighted Capital Ratio (hereafter RWCR) are declared in default, they are liquidated and eventual losses are transmitted through the long and short-term interbank market at the beginning of the next period. The dynamic is repeated until the cumulated number of defaults, namely the sum of the number of defaults in each short-term financial period, stops increasing. It should also be noted here that in our framework a bank can default for two different reasons: first, it can be unable to fulfill liquidity or capital requirements, second, it may be illiquid and become unable to pay back its debtors.

4.2 Data Set

Our dataset consists of a sample of 50 large EU banks. For each bank, we include information about capital, short-term and long-term interbank borrowing, deposits, short-term and long-term interbank loans, aggregate securities holdings⁵, and cash. The distinction between short and long-term interbank assets reflects the maturity of the loan which can be below or above three months. We also know the RWCR of banks, from which we can reconstruct the mean weights for the financial securities of each bank. The data sources are the banks' annual financial reports, and Bureau van Dijk's Bankscope; the balance sheet data refer to the end of 2011. Figure 3 shows the total capital across the banks in the sample, and their Risk-Weighted Capital Ratios, revealing a high level of heterogeneity. The horizontal red line in the lower panel of the figure represents the standard Risk-Weighted Capital Ratio requirement equals to 8%, as specified in the Basel standards. The aggregate short-term interbank exposures in the system amount to about €1.2tn and the aggregate long-term interbank assets amounts to €900bn.

We do not have data on individual banks' bilateral exposures, neither on the details of financial securities portfolios. Instead, we use this uncertainty as a degree of freedom of the model, in order to investigate which multi-layered network structures are particularly prone to a systemic breakdown. In principle, every possible network in each of the three layers represents a plausible configuration for the multi-layered network structure; in order to focus only on the interbank networks which are the most probable in the real financial system, we extract the network topologies for the short and long-

⁵As securities holdings, we use the sum of *Securities Held for Trading*, *Securities Held at Fair Value* and *Available for Sale Securities*.

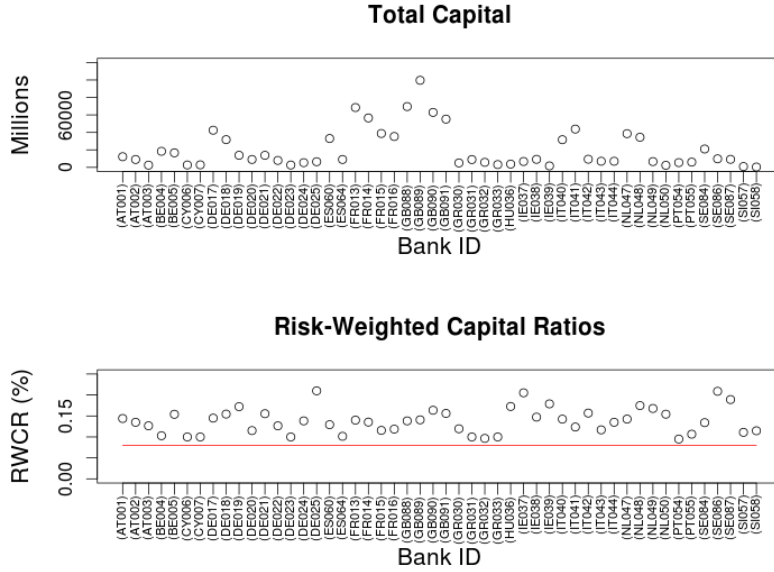


Figure 3: In the upper panel, the equities of the 50 banks in our sample, in millions. In the bottom panel, the Risk-weighted Capital Ratio of the banks; the horizontal red line represents the standard Basel capital requirement of 8%. The figure highlight a high level of heterogeneity in the sample, both in term in total equity and in term of Risk-weighted Capital Ratio.

term interbank exposures according to a probability matrix, with the only restriction that each bank is exposed to other entities at most 20% of its total interbank assets. A probability matrix P^G is a matrix which entries p_{ij}^G specify the probability of existing of the directed link $i \rightarrow j$, representing a loan from bank i to bank j . The probability matrix is built upon the European Banking Authority (EBA) disclosures on the geographical breakdown of individual banks' activities as disclosed in the context of the EU-wide roll stress test. The methodology is based on Hałaj and Kok (2013), and networks in layers l_1 and l_2 are generated as follow: banks are randomly extracted from the sample, and for each bank we sequentially generate links according to the probability matrix; for each link, a random number from a uniform distribution on $[0, 1]$ is extracted, indicating what percentage of the residual interbank assets of the first bank is deposited in the interbank liabilities of the second. Formally, for links in layer l_1 we have:

$$l_{ij}^1 = \epsilon_{ij} \left(l_i^l - \sum_{k=1}^m l_{ik}^1 \right) \quad (16)$$

where $\epsilon_{ij} \sim U(0, 1)$, and $\{l_{i1}^1, l_{i2}^1, \dots, l_{ik}^1\}$ are the links in layer l_1 starting from node i generated in the previous m steps of the algorithm. A similar expression can be written for layer l_2 . The amount in eq. (16) is properly truncated to take into account the limited liabilities of the borrowing bank, and the constraint that each bank is exposed to no more than 20% of its total interbank assets to each other bank. This constraint excludes network realizations where a bank lends all its interbank assets to a single counterparty.

In contrast, the network in layer l_3 is randomly generated, since we do not have sufficiently granular data or statistics concerning the securities portfolio structures of the banks in the sample. We only have information about individual banks' aggregate amount of securities. This random network generation is conducted by first choosing the number M of securities to use in the simulations, and subsequently building a random bipartite network between the N nodes and the M securities: in this network a link from a bank i and a security μ means that the bank has in its portfolio that particular security, and the amount of the shares is represented through the weight of the edge. Each link in this bipartite network has the same probability p to exist. In the baseline setting we assume that, for each bank, all the out-coming links have the same weight. Starting from this random bipartite network, there are different ways to build the network of the overlapping portfolios, and an example is:

$$W_{ij}^3 = \sum_{\mu=1}^M \frac{s_j^\mu}{s_j^{tot}} \cdot \left[\max \left[1, \frac{s_i^\mu}{s_j^\mu} \right] \right] \quad (17)$$

In this setting, the weight of the directed link from bank i to bank j is the proportion of the portfolio of bank i that overlaps with the portfolio of bank j .

We note that the topology of the multi-layered network is the only degree of freedom in the simulations, since banks' balance sheets are always kept fixed and calibrated according to our data. Therefore, all the degrees of randomness would be completely removed in case of full knowledge of direct bilateral exposures for the long-term interbank market, exposures on the short-term interbank market, and more granular information on banks' portfolios.

4.3 Topological properties

The networks in layers l_1 and l_2 generated with the algorithm proposed in the previous section have link weights which depend on the order of drawn linkages. For a given bank i , the first drawn

link (i, j) would on average carry 50% of bank i 's interbank assets, the second drawn link 25%, and so on. Since we do not have data for a proper calibration of link weights, we are implicitly assuming that banks trade more loan volumes with their more frequent counterparties. We note that if the number of simulations is large enough several different scenarios will be generated, including situations where nodes have many linkages of similar size. Moreover, the use of a probability matrix to randomly generate the networks in the different layers does not take into account the possible statistical dependency of two links to exist in the same network. Again, without proper data, it can be difficult to reproduce such a correlation structure in the links formation. Networks produced in this way nevertheless show some of the most common statistical regularities found in real interbank networks, as documented in Boss *et al.* (2004), Iori *et al.* (2008), Fricke and Lux (2012) and Bargigli *et al.* (2013). Such regularities are heterogeneity of nodes' degree, disassortative mixing, i.e. the tendency of high degree nodes to connect with low degree nodes, sparsity, and a Jaccard similarity among different layers similar to the one found in real multi-layered interbank networks,

More in detail, Fig. 4 shows the total degree distributions for layers l_1 and l_2 ; the two graphs highlight a high level of heterogeneity in the nodes' degree, meaning that most of the nodes have very few connections, and few nodes have many connections to the other banks in the system.⁶

One way to capture assortative mixing in a network is by examining the properties of the average nearest neighbor degree as function of vertex degrees, usually indicated as $\langle K_{nn} \rangle$, and defined as:

$$\langle K_{nn}(k) \rangle = \sum_{k'} P(k'|k) \cdot k' \quad (18)$$

where $P(k'|k)$ is the conditional probability that an edge of node degree k has a neighbor of degree k' . If the above function is increasing, the network shows an assortative mixing, since node with high degree tend (on average) to connect to nodes with high degree. Alternatively, in case function 18 is decreasing, the network shows a disassortative mixing, since nodes with low degree tend to connect with high degree nodes, and vice versa. Figure 5 shows a clear disassortative mixing in the structure of layers l_1 and l_2 .⁷ Finally, we

⁶Also if we use the same probability matrix P^G for the two layers l_1 and l_2 , the final topologies can be different due to the role played in the generating algorithm by short and long-term interbank exposures.

⁷The non-monotonic trend, observed in the left panel of the figure, arises directly from the combination of the probability matrix P^G and the banks balance sheets.

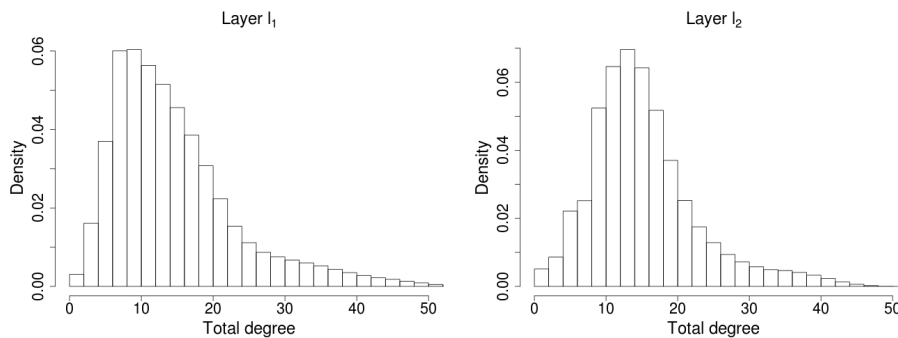


Figure 4: The LHS figure shows the total degree distribution of layer l_1 , and the RHS figure shows the distribution for layer l_2 . A clear level of heterogeneity among nodes' degree is evident in both the layers.

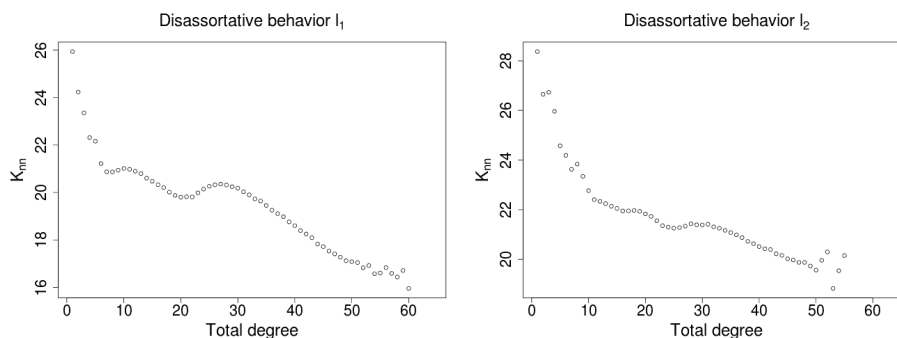


Figure 5: The LHS figure shows the disassortative behavior for layer l_1 , while the RHS shows the same for layer l_2 . As means to capture the assortative mixing is by plotting the average nearest neighbor degree as function of vertex degrees. A decreasing trend means that the network is dissortative, since nodes of high degree tend to connect to nodes of lower degree.

report a mean density for layers l_1 and l_2 equal to respectively 14% and 12%.

We next introduce a measure for the similarity among the topologies in the different layers, since this measure will be used to analyze the results from the simulation engine. Generally speaking, given two networks G_1 and G_2 , we use the Jaccard index $J_{12} \in [0; 1]$ to describe the similarity among the networks (see Appendix for a formal definition). This index will be equal to 0 when G_1 and G_2 have no links in common, and it will be equal to 1 when the two networks are identical. As documented in Bargigli *et al.* (2013), values of the Jaccard index for different layers in the same interbank market range roughly about between 0.1 and 0.3, depending on the kind of transaction (secured or unsecured) and on the point in time the index is measured. As comparison, we can compute the Jaccard

index J_{12} among layers l_1 and l_2 of our multilayer network, the Jaccard index J_{23} among layers l_2 and l_3 , and the Jaccard index J_{13} among layers l_1 and l_3 . The mean values and the standard deviations of these three indexes, computed over 10^5 different multilayer network structures generated according to our simulation engine, are reported in Table 1. The Jaccard index J_{12} for layers l_1 and l_2 is comparable to the one found in reality. Obviously, since layers l_3 are generated from a random bipartite network, we cannot expect realistic values also for the indexes J_{13} and J_{23} , which we are not able to measure in reality. We will use those indexes again when we study the results of our simulation engine.

Table 1: Jaccard indexes. The table reports the mean values of the Jaccard indexes J_{12} , J_{13} and J_{23} for the multilayer networks generated with the algorithm proposed in Section 4.2, together with their standard deviations.

J	mean	sd
J_{12}	0.27	0.03
J_{13}	0.09	0.04
J_{23}	0.10	0.04

We stress again that the choice to use three layers for the structure of the financial system is also driven by data availability. Introducing further layers without having proper data to calibrate them, would result in the introduction of a large number of parameters, which can drastically complicate the analysis of the results. Instead, we prefer to use layers that can be (at least partially) calibrated, and at the same time that were revealed to play a fundamental role in the last financial crisis.

5 Simulation Results

Systemic risk in interbank markets depends on numerous factors regarding both the financial status of the members of the banking system, their balance sheets, and the disposition of the linkages among them. In this paper, we keep a defined and realistic structure of banks' balance sheets, as described in section 4, and we investigate how the different structures for the interconnections among the agents affect the financial stability of the whole system. This is interesting for various reasons. First, it gives indications about the impact of different network structures on financial stability; second, by using classical tools from network theory, it enables us to assess each bank's contribution to systemic risk; third, it sheds light on the role of banks' capitalization on the resilience of the system.

In the baseline specification of the model, parameters are set in a way to reproduce realistic regulatory requirements on banking systems and a plausible price elasticity for the securities market. The minimum risk-weighted capital ratio requirement is fixed, according to the Basel standard, to $\bar{\gamma} = 8\%$. The minimum required liquidity buffer is fixed through the parameter $\beta = 5\%$.

The price of all M securities is initially fixed at 1: $p_{\mu}^0 = 1$ ($\mu = 1, 2, \dots, M$). The elasticity factors, α_{μ} , are fixed at 0.2, and the number of securities is $M = 30$. In this way, banks do not have preferences about which securities to liquidate first in case of need, and the bipartite network banks-securities, which represents banks' securities holdings, is built with a Erdős-Rényi index $p = 0.2$. We will investigate later how the number of securities and the topology of the network in layer l_3 affect the results.

The initial shocks are assumed to derive from the failure of one of the 50 banks in the sample. The failure of the bank implies the liquidation of all its securities holdings, the transmission of losses on the long-term interbank market, if any, and the withdrawn of all the funds it provides in the short-term interbank market. The risk for the system hence materializes via the lack of the funding services provided by the targeted bank, together with the risk of losses transmitted through the exposure channel and the securities market. How the system reacts to this initial shock strongly depends on the topological structure of the underlying multi-layered network.

5.1 Systemically Important Banks

The importance of a bank in a banking system does not depend only on its financial situation. In fact, contagion is a process involving two main steps: the default of one or more components of the system, which in turn depends on the financial situation of the entities, and the propagation of the shock through interbank linkages. In this paper, we are interested in this second effect, namely how the network structure can affect the stability of the system after an idiosyncratic shock hits one of the banks, and part of our task is to determine which structures are more prone to financial breakdowns.

A first result from our simulation engine is a test of the impact of each bank's failure on the whole system. For this purpose, we first shock one initial bank, we call it bank b_0 , and then we let the system evolve according to the scheme in Fig. 2 up to when the cumulated number of defaults stops increasing. The impact of each bank on the financial stability of the system is measured through the total number of defaults its failure produces. This number of

defaults is the random variable we want to estimate the distribution of. In fact, even if the banks' balance sheets are always the same, including also the aggregate exposures of each bank towards all the others, the degree of randomness left in the structure of the financial multi-layered system produces a level of uncertainty on the number of defaults following the bankruptcy of bank b_0 .⁸

In order to highlight the role of each bank in the system, we present the disentangled effects from the three layers, together with the effects coming from the complete multi-layered network's structure. To this end, we first run the simulations when all the banks are only connected through the long-term interbank market, meaning that the only layers presenting some edges is l_1 ; the only risk present in this system is therefore the counterparty risk. Then we run the same simulations with only layer l_2 activated, meaning that the only risk present in the system is the funding risk.⁹ In the third scenario, we run the simulations with layer l_3 as the only active layer¹⁰, representing the case where the only risk banks face is liquidity risk. Finally, we present the case where all the three layers are activated simultaneously.

As a benchmark example, we start to show the dynamics of the contagion process when a particular bank defaults, for one specific configuration of the multi-layered network. In particular, the red bold line in Fig. (6) represents the evolution of the number of defaults when all the three layers are working together. The other lines in the graph represent all the possible other combinations of active contagion channels. Simply by eye-balling, it is easy to discern that the sum of the number of defaults in the single-channel scenarios never reaches the total number of defaults for the whole system. A deeper examination reveals that this phenomenon is actually due to spiral effects: in case only one of the three layers is active, the contagion process is dampened (see Fig. 6). Yet when more than one channel of contagion is present, the contagion process is much more probable, and liquidity needs of one bank can result in a capital reduction of others, which have to increase their capi-

⁸It should be recalled that when the bank b_0 defaults at the beginning of the simulation, it is liquidated, implying that it withdraws all its funds from the short-term interbank market, it sells all its available for sale securities, and it tries to pay back its creditors on the short and long-term interbank market.

⁹In those two scenarios, each bank is assumed to have a securities portfolio which is completely independent from all the other banks' portfolio in the system. Nevertheless, price is still driven by eq. (6), and therefore firesales can still be costly for the banks, also if there are no contagion effects due to common exposures.

¹⁰In this third scenario, all the interbank assets of the institutions in our sample are supposed to be directed to an external node, and all the liabilities in the interbank market are provided by this node, which does not play any other role in our financial simulator, in the sense that it never withdraws funds and it cannot fail or transmit any losses.

tal ratio by withdrawing further short-term funds or by liquidating their securities portfolio.

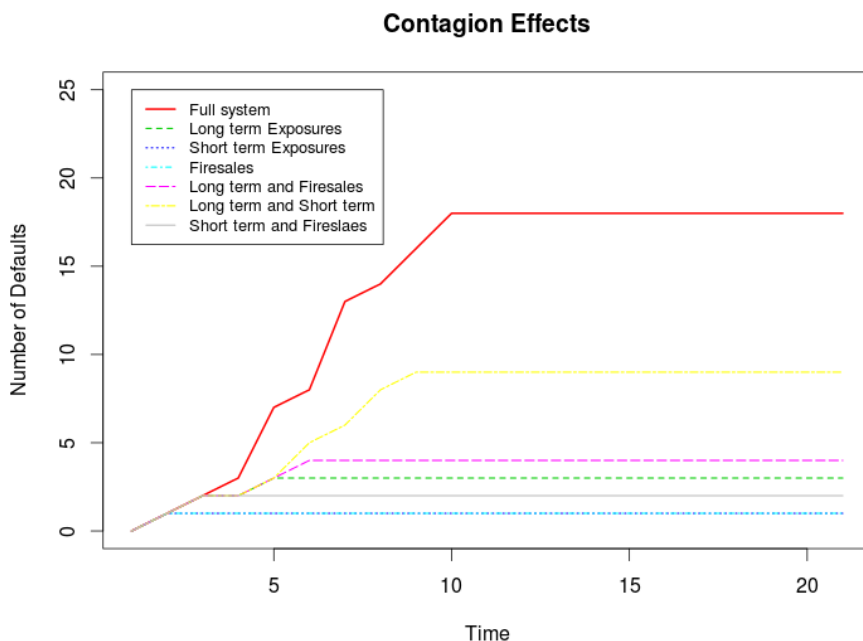


Figure 6: The figure shows the dynamic process when the bank fails for one particular realization of the multi graph. The horizontal axes represents the time, and the vertical axes represents the total number of defaults.

To clarify the importance of taking into account the interactions among different layers, Fig. (7) reports the results for bank FR014 (as example) in a more statistical fashion.¹¹ The four panels in the figure show the distributions of the number of defaults in the four scenarios described above, namely when only layer l_1 is activated (top left panel), when only layer l_2 is activated (bottom left panel), when only layer l_3 is activated (top right panel), and finally when the three layers are simultaneously activated (bottom right panel). The red line in the bottom right panel represents the quantitative convolution of the three single-layered network distributions: it basically represents the linear superimposition of the three effects, and it is interesting to compare it with the distribution for the total number of defaults in the case of three active layers. In fact, the differences among the two have to be attributed to the interaction of the three layers.

As one can see from the figure, the default of bank FR014 results

¹¹Similar graphs for the other important banks in the sample are available upon request.

in contagion effects via only one channel, namely the short-term interbank exposures represented in the bottom left panel of the figure. Interestingly, however, the systemic importance of the bank is amplified by the presence of the other two layers in the multi-layered network. In fact, when the single layers are considered separately, the largest number of defaults is 12, reported when only layer l_2 is activated, meaning that bank FR014 is an important short-term liquidity provider. No defaults are reported when only layer l_1 is activated, and a maximum of 5 defaults can be seen when only layer l_3 is activated. Yet, when we consider the three layers working together, the largest number of defaults reported in the simulations is 42, and the distribution is much more fat tailed. As one can see from the bottom right panel of Fig. (7), the distribution of the number of default for the case where the three layers are simultaneously activated differs from its convolution counterpart (red line in the same panel) in the way that the three layers working together produce more mass in the tail. We will show in section 6 that the risk transformation process implicitly performed in banks' balance sheet activities is at the core of the generation of high level of systemic risk, and this will clarify the importance to study the financial stability from a more holistic lens.

Overall, for the great majority of the banks there is no substantial contagion effects when they fail, indicating a certain resilience of the financial system against random defaults of its members. At the same time, there are a few banks whose default could have considerable contagion effects in at least one of the three layers, and this importance is extremely amplified when considering all the three layers in conjunction. The main lesson from these results is the limitations of measures of systemic risk based on single-layered networks' configurations. Single-network measures run the risk of heavily underestimating the systemic importance of banks, since they usually take into account only the counterparty risk associated with a particular segment of the interbank relations. The simulations performed with only layer l_2 activated, on the other hand, show the importance of funding risk in banking activities, as also highlighted during the last financial crisis, and how it can materialize if banks start hoarding liquidity instead of making short-term funds available on the interbank market. Moreover, the amplification of the shock due to fire sales and to non-perfectly liquid markets can greatly amplify local shocks, leading to much more dangerous configurations in which a large portion of the banking system can break down. We also note that with the selected parameters, the layer l_3 representing common exposures usually just works as amplifier for the propagation of an

(FR014)

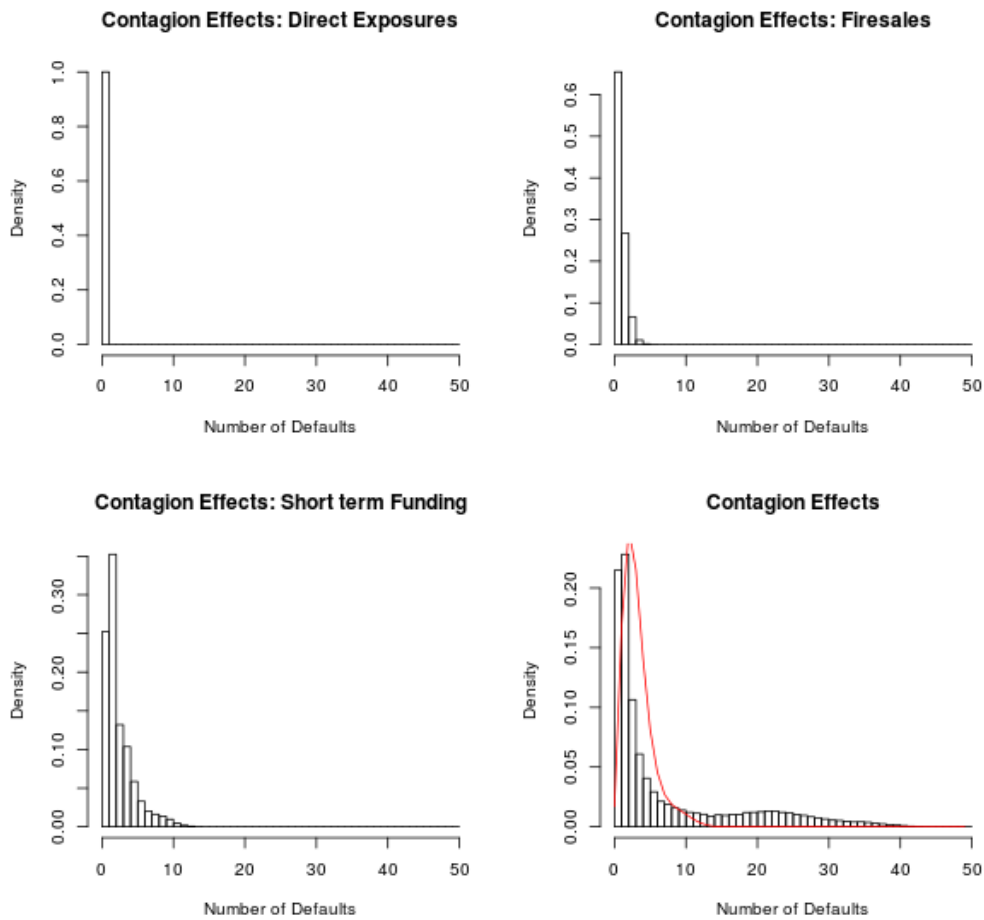


Figure 7: In the top left panel, the distribution of the total number of defaults when the bank FR014 defaults in our simulation engine in the first scenario, namely when the only active layer is l_1 . The distribution shows the counterparty risk that the bank represents to the whole system. In the bottom right panel, the distribution of the number of defaults when the only active layer is l_2 . In the top right panel the same distribution is presented for the case of layer l_3 , which represents the contribution of the bank to the liquidity risk of the system. In the bottom right panel, the distribution of the total number of defaults in the case of all the three layers are active at the same time. The red line represents the quantitative convolution of the other three distributions, representing the linear sum of the three effects. Each graph is the result of 50000 realizations of the banking system.

initial shock.

5.2 Systemically Important Topologies

The previous subsection showed that, given an initial defaulting bank, different topologies for the multi-layered network imply different results with respect to the stability of the financial system. In particular, for some banks there exist critical configurations for the system such that it becomes prone to systemic breakdowns. Those configurations are the ones which populate the fat tails of the distributions of the total number of defaults highlighted in the previous subsection.

An interesting question which can be addressed with the simulation engine is whether there exist some configurations which are critical for all the banks at the same time. This is not a trivial issue. In fact, also if a topology of the multi-layered graph can make the system very vulnerable to the failure of one particular institution, we cannot so far say anything about the systemic importance of the other banks in exactly the same network structure. In case a very important bank for the system in terms of the financial services it provides to the other banks, assumes a central position in the network structure, systemic risk is high, since the bankruptcy of this bank can create contagion effects which affects a large number of other financial institutions. If substantial contagion occurs only in some of the simulated network structures we generate in our simulations, it means that, in those cases, the idiosyncratic risk assumed by the defaulting bank was badly distributed among the other institutions in the system. We therefore speak about systemic risk, and systemically important institutions. Moreover, the possibility that more large banks become systemically important at the same time is a much riskier situation for the entire system. Given the probability matrix P^G , we are interested in investigating the possibility of existence of systemically important topologies; formally, given a certain multi-layered graph \mathfrak{G} , we can compute the systemic risk associated with the structure as follow:

$$R_{\mathfrak{G}} = \frac{\sum_{i=1}^N d(i)}{N} \quad (19)$$

where we indicate with $d(i)$ the number of defaults caused by the bankruptcy of bank i , computed as the result of our simulation engine.

In order to explore the possibility and the frequency of extremely critical configuration for the banking system, we generate 10^6 multi-layered network topologies, and for each of these configurations we compute the mean value of the number of defaults produced by the

initial failure of each of the 50 banks in the system, according to eq. (19). In this way, we associate to each network structure produced its systemic relevance, indicating the mean level of systemically importance across the banks. Obviously, since most of the banks do not produce any contagion effects upon their failure, the mean number of defaults will be relatively low. Figure 8 shows the results of this exercise. In the left panel of the picture the distribution of the systemic relevance $R_{\mathcal{G}}$ of 10^6 multi graphs produced following the methodology described in section 4 is shown. It can be observed from the figure that, most of the network structures are only relevant in the case where one of the largest banks default. There exist, nevertheless, some topologies which make the financial system particularly prone to a financial breakdown. To clearly illustrate this idea, in the right-side panel of Fig. 8 two extreme cases are shown: in the multi network structure represented by the blue crosses, the initial bankruptcy of almost all the banks does not produce any contagion effects, apart from the case of bank 34 which triggers two other defaults. The systemic relevance for this structure will therefore be close to zero. By contrast, the red triangles in the same picture show a very risky configuration for the system, since the initial failure of 11 financial institutions would trigger a lot of other defaults, highlighting the financial weakness of the entire system.

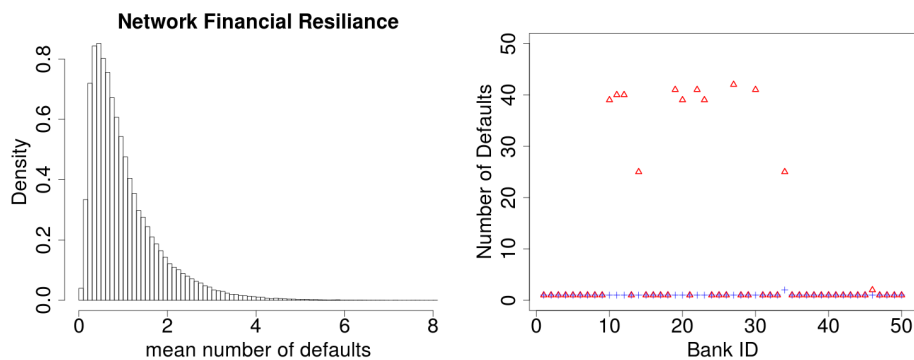


Figure 8: On the left panel, the distribution of the systemic relevance is plotted for 10^6 different network topologies. Each systemic relevance parameter is built by generating the same network N times, where in our case $N = 50$, and for each of this realizations we shock one of the banks in the system and we count the number of defaults: the mean value of those numbers is then used as systemic relevance for that configurations. The tail of the distribution highlights the existence of some critical configurations for the financial system. As example, we present in the right panel of the figure two cases: the network described by the blue crosses is a resilient configuration, since the defaults of all the banks does not produce any considerable effects. The network described by the red triangles, on the other hand, is extremely unstable, since the failure of one of the largest bank trigger a lot of subsequent defaults.

Hence, fig. 8 illustrates that network structures matter for the financial resilience and the proper functioning of the banking system. It should be recalled that in all the simulations the banks' balance sheets are kept constant, and therefore also the aggregate short and long-term interbank exposures. It is clear that configurations like the one in the tail of the distribution in the left side panel of Fig. 8 have to be avoided. In this framework, the multi-layered networks are extracted according to a particular distribution specified by the probability matrix P^G for layer l_1 and l_2 and by a random portfolios generator for layer l_3 , and they are all plausible networks, in the sense that there is a certain probability for the real system to be in those configurations. In reality, however, the multi-layered network structure arises as the result of the local behaviors of a multitude of economic agents, which (supposedly) have as target the maximization of their personal interests. The experiments we performed highlights once again the necessity of having more granular data regarding banks' direct and indirect interconnections, in order to monitor the system from a global perspective and avoid it to evolve through configurations extremely prone to large breakdowns.

Consequently, a key objective of our analysis is to identify dangerous configurations. We introduced in Section 4.3 the Jaccard index as measure of similarity between two different networks, and we characterized its basic statistical properties for the networks generated in our simulation engine. We now study the correlation between the indexes J_{12} , J_{13} and J_{23} and the systemic relevance parameter $R_{\mathfrak{G}}$ introduced above. Fig. 9 shows the result. In particular, the four panels plot the Jaccard index against the systemic relevance parameter, where $J^* = J_{12} + J_{13} + J_{23}$ is used to take into account possible crossed correlations among the three layers which could potentially bring high level of systemic risk. As one can see from the figure, simple similarity measures like the one we use is not able to explain the formation of critical configurations. To solve the problem, we will introduce a numerical algorithm in section 6 that allows taking into account the real roots of the systemic risk generated in our model, which is the intrinsic nature of banks' balance sheet management that can cause the various kinds of financial risks to which banks are exposed to interact with and reinforce each other.

5.3 The Systemic Importance of the Securities Portfolios

In the previous sections the initial shock to the financial system was always the bankruptcy of one single bank. In this section, we investigate how the system reacts when instead the shock consists of the

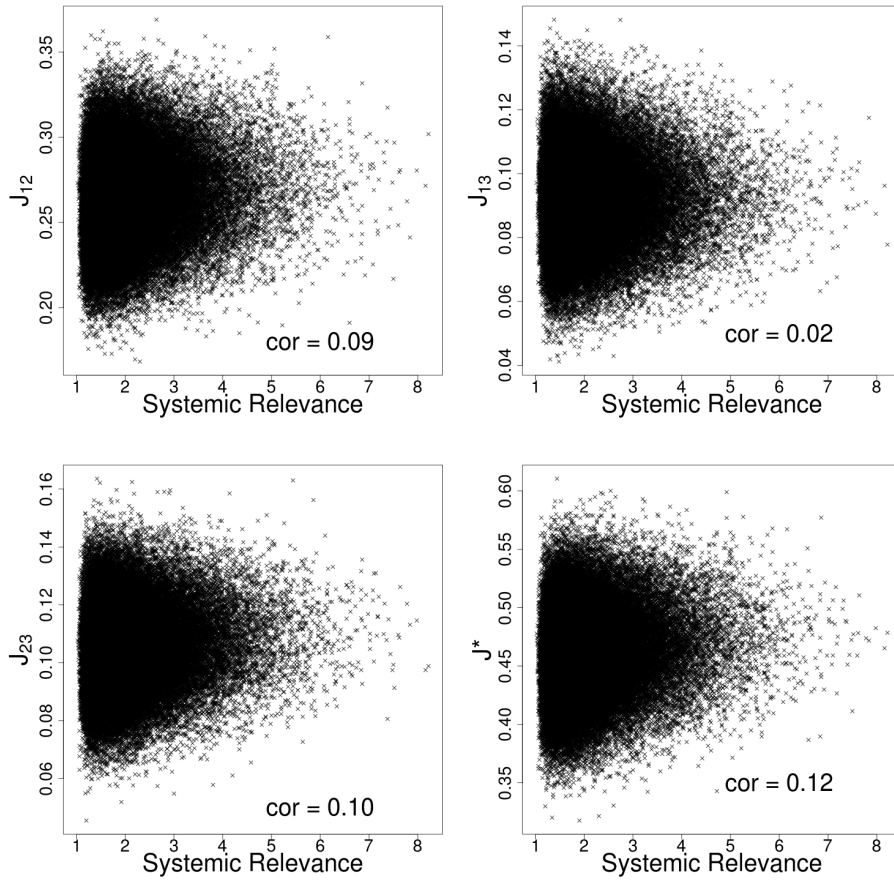


Figure 9: The four panels show the correlation between the Jaccard indexes J_{12} , J_{13} , J_{23} , their sum $J^* = J_{12} + J_{13} + J_{23}$, and the systemic relevance parameter $R_{\mathcal{G}}$. In particular, the points in each panel represent a multi-layer network structure extracted according to the algorithm presented in Section 4.2. For each structure, we measure its systemic relevance parameter (reported on the horizontal axes) and the Jaccard indexes (reported on the vertical axes). Correlations between the two quantities are also reported in the graph. Results are reported for 10^5 different network topologies.

depreciation of the value of one or more securities. It should be recalled that in the model banks are endowed with random portfolios. All the securities, moreover, are characterized by the same price at the beginning of the simulations, which for sake of simplicity is fixed to $p_{\mu}(0) = 1$, and the same elasticity factor $\alpha_{\mu} = 0.2$. In the previous subsections, the number of securities was fixed to $M = 30$.¹² Keeping fixed this initial configuration, we first investigate how the

¹²Since the initial bipartite network is random, where a link between any bank i and any security μ has a probability to exist equal to p , it is easy to see that the corresponding network l_3 of overlapping portfolios is also random, with an Erdős' coefficient equals to $p' = 1 - (1 - p^2)^M$, where M is the number of securities.

banking system absorbs a price reduction of one or more securities. Fig. 10 shows the results. In the left side panels, the number of defaults following a certain percentage of reduction of the securities' price is shown, respectively when the price reduction affects only one security (top left panel), two securities (top right panel), three securities (bottom left panel) and ten securities (bottom right panel). In each of the graphs are reported the mean number of defaults corresponding to different shock sizes, where the solid line represents the situation when all the three layers are activated, while the dashed line represents the situation when the only active layer is l_3 . It is observed that if banks were completely independent from each other in the layers l_1 and l_2 , there would be very few defaults, especially for price shocks which are not abnormally large.¹³ Consider, for example, the case when 10 securities are shocked at the same time by reducing their value of 15%. Without any other connections among banks apart from the common exposures, the mean number of defaults is around 7. Meanwhile this number drastically increases to 38 if banks are also connected through layers l_1 and l_2 . We note that since all the securities have the initial same price, and are all characterized by the same elasticity factor, in this random portfolio scenario it does not play a role which securities are shocked, since the effects are averaged out when the number of simulations is large enough. Eventually, as one can see from the figure, for values of the shock smaller than 5% no defaults are observed, indicating an adequate capital buffer level for small losses in banks' securities portfolios.

On the right-side panel of Fig. 10 we report the tails of the distributions of the number of defaults for a shock to the securities equal to 15%, for the cases of one, two, three and ten initial shocked securities, respectively. The blue areas highlighted in the graphs represent the last fifth quantile of the distributions. In the cases of one, two and three shocked securities, the great part of the mass of these distributions is concentrated in values close to zero, highlighting a considerable financial resilience of the banking system for random assets depreciations. Nevertheless, one can see in the graphs that, also in the scenario of one security shocked by 15% of its initial value, the shock can be amplified to destroy a large part of the banking system.¹⁴ These findings highlight that also if

¹³We report in the graphs all the possible values for a shock, so from 0% to 100% of reduction of the asset's value; of course, this is only an illustrative simulation exercise, since in reality depreciations larger than 20% are extremely rare.

¹⁴We note that those fat tails disappear as far as the layers l_1 and l_2 are deactivated. We do not report here also those distributions, but one can see from Fig. 10 that the mean values of the number of defaults is exactly zero for shocks equal to 15% (dashed lines in the

the initial shock derives from a depreciation of the mark-to-market banks' portfolios, the multi-layered network structure is playing the crucial role of shock amplifier.

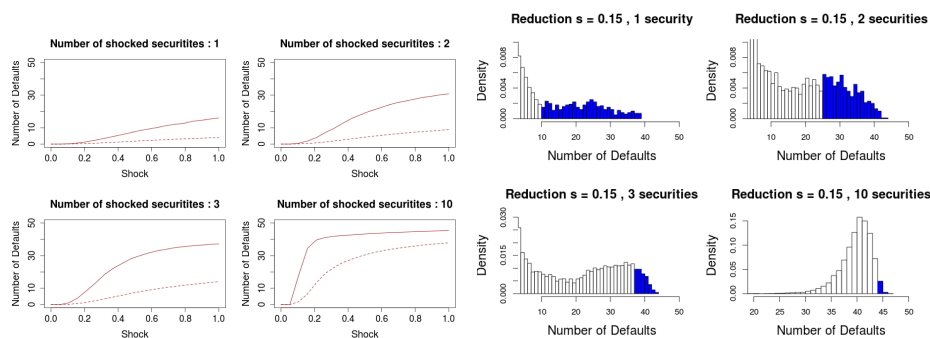


Figure 10: On the left side of the figure, the four panels show the number of defaults when one, two three and ten securities are shocked; the solid lines represent the number of defaults when all the three layers are active at the same time, while the dashed lines represents the same results when only the layer l_3 is activated (firesales contagion effects). On the right side, the tails of the distributions of the total number of defaults are reported, when the percentage of securities' reduction is equal to 15%; results are here reported for the case of one, two, three and ten initially shocked securities. The blue areas highlighted represent the last fifth quantile of the distributions.

A particular aspect related to the banks' portfolio structures should be highlighted. In all the previous results, the securities portfolios were built according to the random algorithm described in section 4.2. It should be noted however, that since all the securities in our framework are equivalent, banks maximize their utilities by simply allocating their funds in equal measure in all the possible available securities. In this configuration the system results in a maximum degree of overlap of banks' portfolios, which implies a fully connected (i.e. complete) network in the layer l_3 . The diametric opposite of this configuration happens when banks invest all in different securities, which translates in an empty network in the layer l_3 . In order to illustrate the impact that the degree of overlapping portfolios has on systemic risk, we use now a number of securities M equal to N , the number of banks. This allows for comparing situations ranging from banks having maximum overlapping portfolios (precisely, when all the banks equally share their funds among all the possible M securities), to situations where banks invest their funds in only one security and there are no common exposures among them. The results of this exercise are shown in figure 11. We assume that the shock is a reduction of the value of all the M securities in the system, respectively of 5% (black line), 7% (red left side panels), a part of the case when ten securities are shocked at the same time.

line), 10% (blue line) and 15% (green line). In this way, for a given shock size, all the banks have to book the same losses (in percentage points) in all the portfolios' configuration we examine. The horizontal axes of the graph reports the number n_s of securities each bank is investing in, and the portfolios are built in a way to always minimize the degree of overlap among different banks. When n_s is equal to one, each bank has only one security in its portfolios, each different from all the others (so there is a correspondence one-to-one between the N banks and the $M = N$ securities in the system). When n_s is equal to N , each bank invest its funds in all the possible securities, and all the banks have the same portfolio structure. It is interesting to note that moving along the horizontal axes from left to right maximizes banks' portfolio diversification (and hence reduces their vulnerability to idiosyncratic risk) but at the same time minimizes financial stability (it maximizes the number of defaults, and therefore, roughly speaking, the systemic risk). Our model highlights the interesting duality between maximization of banks' utility and minimization of systemic risk, a concept already highlighted in Beale *et al* (2011) who argue that banks' portfolios optimization can lead to higher level of systemic risk, thereby emphasizing the necessity to supervise systemic risk from a more global perspective.¹⁵

6 Systemic Importance Measure

A multi-graph financial structure reveals its fragility only in case a shock hits the system; part of our task is to show when the system is in a critical configuration, namely a configuration which is able to amplify a local shock to the entire financial system. We recall that, in this paper, systemic risk reflects the possibility that a single major events triggers a series of defaults among financial institutions within a short time period. Among the different methodologies developed in the last years to identify systemically important banks and their contribution to systemic risk¹⁶, network-based measures are receiving more and more attention, although there is no a standard measure so far which can be considered universally accepted in the literature. The main reason for the inconsistency among systemic risk measures is that they rely on different microeconomic models for the specification of banks' behavior and the mechanisms through which a shock can propagate within the financial system. At the same time, network-based measures have the advantage of

¹⁵See also Tasca and Battiston (2012) for similar results.

¹⁶See e.g. Upper (2011) and Bisias *et al* (2012).

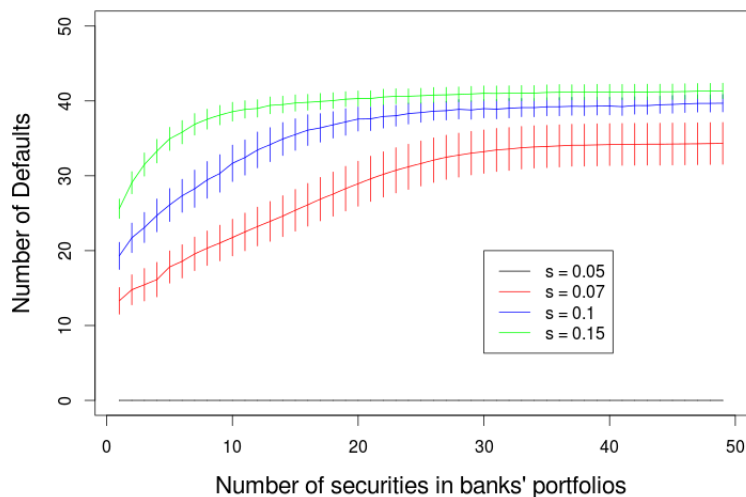


Figure 11: The horizontal axes represents the number of securities in banks' portfolios; banks portfolios are built in a way to minimize their overlapping. The vertical axes represent the mean number of defaults when all the securities are shocked by 5% (black line), 7% (red line), 10% (blue line) and 15% (green line). The vertical ticks represent the standard deviations computed over 10^5 simulations.

compressing a lot of information regarding direct and indirect bank interconnections, which appeared to be crucial during the last financial crisis. A network-based representation of the banking system is therefore crucial to understand how the single institutions share their idiosyncratic risks with the others, and to which extent this risk-pooling is dangerous for the system.

It is important to note that a comprehensive study of the systemic risk generated from the presence of interbank connections cannot rely only on the network structure of the financial system. The interconnections in an interbank market provide a way for banks to pool the unavoidable risks linked to their activities, and the interbank market should in principle play a stabilizing role for the banking system. A bank which is very connected to a major part of the others can have a crucial positive role in this scenario if its level of capitalization is large enough, as it can be able to absorb the local shocks of its neighbors. Such a bank will be considered as central in terms of spillover potential to other part of the system, but from the economic point of view its presence is beneficial for the system, since it reduces idiosyncratic risks of other institutions. Figure 12 clearly illustrates this notion. The panels in the figure represents a comparison between some classical network centrality measures and

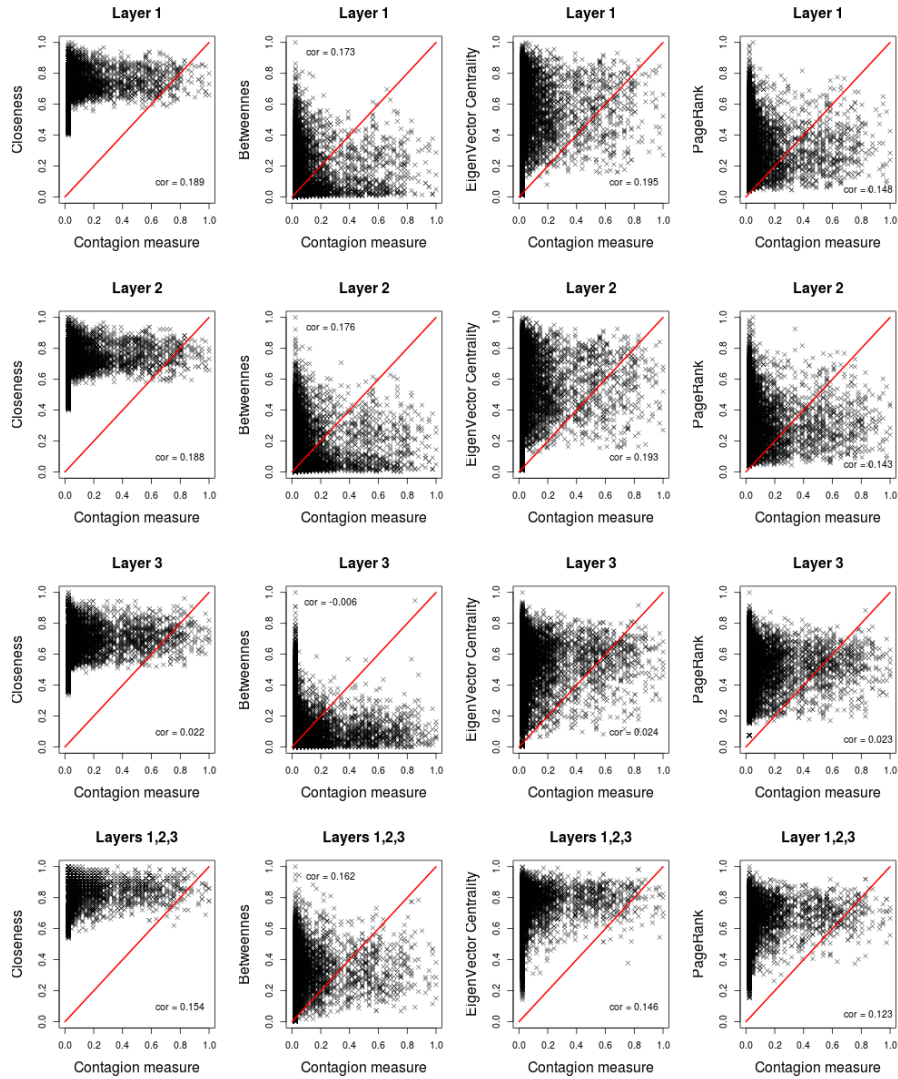


Figure 12: The panels show a comparison between some classical network centrality measures, and the number of defaults reported in our simulation engine following the defaults of one particular financial institution. Each tick in the panels represents a bank in a random-generated multi-layered network structure; the vertical axes represents a measure of centrality of that bank in layer l_1 (first row of panels), layer l_2 (second row of panels), layer l_3 (third row of panel) and the superimposition of the three layers (last row of panels); the horizontal axes represents the number of defaults triggered by the bankruptcy of that particular bank, according to our simulation engine. All the value are normalized to one, and the panels also show the correlation among the two indexes. Results are reported for 10^5 random replications of the system.

the number of defaults reported in our simulation engine following the bankruptcy of one bank. The number of defaults can be used as a proxy for the systemic importance of a bank in the system. Since we are dealing with a multi-layered framework, we compute four different centrality measures (which are *closeness*, *betweenness*, *eigenvector centrality* and *PageRank*) for all the three layers separately, and the same measures when the three layers are projected in a single one. As can be seen from the panels, there is basically no correlation among those network measures and the number of defaults we obtain from our simulations. This result highlights the necessity to develop more sophisticated measures to assess the systemic contribution of each institution to the financial system, and those measures have to take into account the articulated internal structure of the nodes in the network (in other words, banks' balance sheets) as well as the different mechanisms of contagion and risk-sharing present in the banking system.

This notwithstanding, considering only banks' balance sheets information to assess the level of systemic risk in the banking sector is extremely restrictive. Prior to the recent financial crisis micro-prudential supervision was based on the notion that it was sufficient to ensure the stability of the banking sector to require institutions to operate with an adequate level of capitalization. The recent financial crisis, if anything, revealed that focusing only on individual banks' soundness is a necessary but not sufficient condition for safeguarding the financial system. In fact, as we will show later, the risk-pooling mechanism, which is at the core of an interbank market, can increase the chances of multiple failures to occur following an initial shock. Since the process of contagion among financial institutions, as we already highlighted, is composed of two parts, which are an initial triggering events (for example the failure of one single institution), and the propagation of losses and distress in the financial system, the extent to which a local shock can propagate and be amplified from bank to bank greatly depends also on the structure of the banking system as a whole. To illustrate this point, figure 13 shows a comparison between some balance sheet-related quantities and the number of defaults following the bankruptcy of a single institution. The figure shows that classical quantities like banks' total assets, total interbank liabilities, total interbank assets and risk-weighted capital ratios do not necessarily provide useful information regarding the systemic importance of the bank, as measured by the number of defaults its bankruptcy can trigger. In particular, one can see from the picture that the failure of small-sized banks usually does not trigger too many other defaults. On

the other hand, regarding large-sized banks we find mixed results in the sense that some of them trigger domino effects, while others do not. Eventually, the last panel on the right-hand side shows that there is no link between the banks risk-weighted capital ratios and their systemic importance.

To account for the fact that neither classical centrality measures nor balance sheet indicators are sufficient for assessing the systemic importance of an institution, the next subsection introduces an algorithm to derive the systemic contribution of each bank to the financial system. The framework will take into account both network and balance sheets information, with the final aim of (i) reproducing the results we obtained with the simulation engine; and (ii) visualizing the network structure in a way to highlight how the idiosyncratic risk of each bank is distributed among the other institutions, and when this risk-sharing brings the system to an unstable configuration.

6.1 The aggregation algorithm

The algorithm we propose in this section to study the multi-layered financial network is based on the concept of *critical link*. In each of the three layers we introduced, a link starting from node i and pointing to node j is said to be critical if the bankruptcy of bank i results in the bankruptcy of bank j . We note immediately that, without critical links in the three layers, no contagion effect is possible, although losses can be transmitted to the direct neighbors of the failed bank. In fact, in case the default of a single bank does not imply any other failures, the direct and indirect counterparties of that bank were assuming an acceptable amount of risk with respect to their own capital buffer, and we speak about counterparty risk (or liquidity risk, or funding risk) but not about systemic risk. We can distinguish the conditions for a link to be critical in the three different layers. While the detailed approach to identify critical links is reported in the Appendix, the general ideas are presented here.

Following the definition, a link in layer l_1 between bank i and bank j is critical if the default of bank j will induce losses that bank i is not able to absorb without violating the RWCR requirement. In the computation of the threshold value for the link weight, one has therefore to take into account, among other factors, the losses-given-default of bank j , the available capital of bank i , together with all the items in bank i 's balance sheets which can be used by the bank to increase its RWCR. In the same spirit, a link in layer l_2 between two banks i and j is said to be critical if the interruption by bank

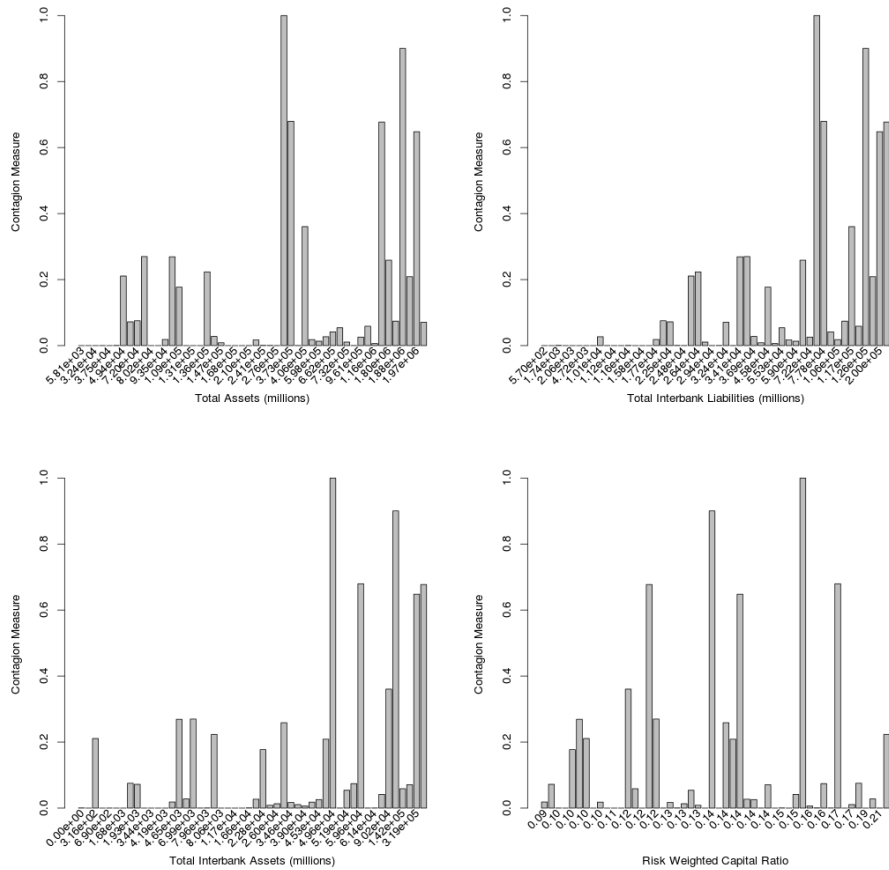


Figure 13: The four panels show a comparison between some banks' balance sheets characteristics, (namely, total assets, interbank liabilities, interbank assets, and risk-weighted capital ratios) and a contagion index, computed as the mean value of the number of defaults triggered after the bankruptcy of the bank with that particular characteristics. Mean values, taken over 10^5 realizations of the multi-layered network, are here used as proxy for the systemic importance of the single institutions. The values are normalized to the maximum number of defaults reported in simulations.

i of the credit line to bank j will induce the failure of the borrower bank. Eventually, a link in layer l_3 between banks i and j is said to be critical if the liquidation by bank i of its whole securities portfolio will produce losses to bank j which is not able to cope with.

We stress here that a link criticality depends on the micro behavioral rules assumed to drive the banks into the dynamic model. This implies that changing the banks' behavior in the model will change the threshold values for the link weights necessary to identify critical links. Nevertheless, the algorithm we propose can still be used to simplify the multi layer network structure and to identify systemic important banks and critical configurations in the financial system.

Before introducing the algorithm for the simplification of the multi-layered financial network, we need to introduce the following notation: given a square-real-matrix $A_{N \times N}$ and a set of indexes $I = \{i_1, i_2, \dots, i_K\}$ ($0 < i_1 < i_2 < \dots < i_K \leq N$), we indicate with A_I the $(N - K + 1) \times (N - K + 1)$ square-real-matrix obtained by summing the rows and columns indicated in the set I , and by putting the row and column arising from the sum first in the new matrix. If the matrix A is the weighted matrix of a network, the reduction operation just described is the aggregation of the nodes in the set $I = \{i_1, i_2, \dots, i_K\}$ in one single node; this new super-node has links to all other nodes that were connected to the original subset absorbed into the super-node, and the weights on the links are summed accordingly.

We can finally introduce the aggregation algorithm for the simplification of a multi-layered financial network. We start with a multi-layered structure \mathfrak{G} and an initial bank b_0 for which we want to compute its systemic importance. In the first step, $s = 0$, we consider the node b_0 as the only one in the super-node, and in each step $s = 1, 2, \dots$ we perform the following operations:

1. We build up the matrices $W_{I_{s-1}}^1$, $W_{I_{s-1}}^2$ and $W_{I_{s-1}}^3$, where I_{s-1} are the nodes belonging to the super-node the step before. We note that this is equivalent to introduce a new bank in the system, instead of the banks in the set I_{s-1} , whose balance sheet is the aggregation of the K suppressed banks' balance sheets, and whose links are the aggregation of the in-coming and out-coming links of the nodes in I_{s-1} .
2. We identify the critical links in each of the three layers l_1 , l_2 and l_3 and we build up three new matrices A_s^1 , A_s^2 and A_s^3 which

entries are:

$$A_{s,ij}^l = \begin{cases} 1 & \text{if there is a critical link from } i \text{ to } j \text{ in layer } l \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

3. We find the directed tree in the unweighted, directed network characterized by the adjacency matrix $A_s = A_s^1 + A_s^2 + A_s^3$ starting from the super-node; the nodes belonging to this tree will constitute the set I_s , while its edges are recorded in the set C_s .

The algorithm ends when the size of the super-node stops increasing and it happens in at most N steps, since in the worst case each node is absorbed in the super-node in a different step. The first output of the algorithm is a series of sets of nodes I_s ($s = 1, 2, \dots$) which can be used to extremely simplify the multi-layer network structure. In fact, nodes absorbed in the super-node in step s are all characterized by the following property: they will fail if all the nodes belonging to the set I_{s-1} fail simultaneously, but not if any single node in I_{s-1} fails separately. The second output of the algorithm is the series C_s of links belonging to the spanning trees starting from the super-nodes. This series of critical links helps us in the identification of critical paths in the system, namely multi-dimensional paths which can bring the losses from one node in the network to a remote region of the same network.

A multidimensional critical path has actually a meaning which is deeper than only being a channel for the transmission of losses through the financial system. The presence of multidimensional paths in interbank network represents a way of risk sharing that goes beyond the knowledge of the single banks. The idiosyncratic risk of one single institution is shared not only with its direct counterparties, which are aware of the risk taken, but also with other players not directly connected to the institution, and which cannot be fully conscious of the risk-transfer represented by the critical paths in the network. Without full knowledge of the multi-layered network structure no banks will be in a position to exactly estimate its exposure to the idiosyncratic risk of the other banks. Moreover, critical multi-dimensional paths highlight the risk transformation process. In fact, the deepness with which financial stress can propagate in a financial system is extremely amplified by the ability of a bank to absorb a risk, transform it, and share it with its counterparties under a different shape.

Those concepts are illustrated in the following subsection where

we show how the aggregation algorithm can be used to identify systemic banks.

6.2 Results

To better clarify the working and the outputs of the aggregation algorithm, we analyze one particular scenario, and we show how it is possible to simplify the financial structure of the banking network. This benchmark example also illustrates the origins of the non-linear behavior in such propagation within the network.

We consider a multi-layered financial network \mathfrak{G} , and a bank b_0 for which we want to know the systemic importance in \mathfrak{G} . The two outputs of the algorithm, $\{I_s\}$ and $\{C_s\}$, can be used to simplify the network structure as illustrated in Fig. 14. The figure shows the three steps involved in the algorithm for this particular configuration \mathfrak{G} (the first step $s = 0$, where the super-node is composed only by the initial failed node, is not reported in the figure). In each step, the super-node is highlighted in red color, and it contains all the nodes involved in the previous steps, including the previous super-node. The figure represents also the critical links reported by the algorithm (blue links represent critical links in layer l_1 , green links in layer l_2 and purple links in layer l_3). The algorithm reports a final number of defaults equal to 18. In the left part of the figure one can see the initial failing bank, $b_0 = 11$, which is the only member of the super-node in step $s = 0$; in step $s = 1$, one can see the multi-dimensional tree on the three layers involving additional 8 defaults as a result of the default of $b_0 = 11$. In step $s = 2$, the super-node aggregates all the 9 nodes already defaulted, whose simultaneous failures in turn produce 5 further defaults. Finally, in the last step, one can see how the simultaneous failures of the previous 14 banks results in 4 more defaults.

Figure 14 clearly shows the non-linear nature of the contagion problem when accounting for multiple layers of interconnectedness. It is clear from the picture that if we repeat the same exercise but only with layer l_1 activated, the total number of defaults triggered by the failure of bank 11 will be no larger than 5 (namely banks 9, 10, 13, 21 and 7), meanwhile no defaults at all would be triggered in case of only layer l_2 or l_3 are active. Therefore, the non-linearity which appears for example in Fig. 7 is due to the creation of critical paths in the multi-dimensional space, which amplifies the range of propagation of the initial shock. This highlights also the fact that when considering the three single layers in isolation the systemic risk in the banking system would be heavily underestimated. As the large

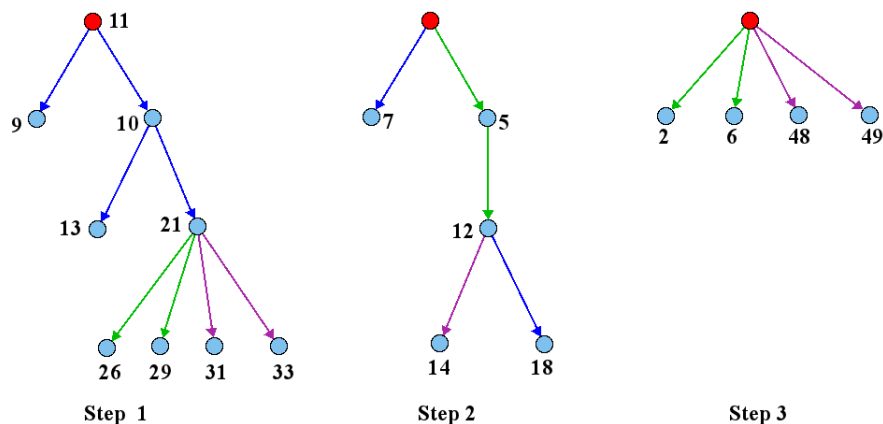


Figure 14: The figure shows a representation of the outputs of the aggregation algorithm for one particular multi-layered financial system \mathfrak{G} and the initial defaulting bank $b_0 = 11$. The color of the edges reflects their nature (blue edges belong to layer l_1 , green edges to layer l_2 and purple edges to layer l_3). Three steps are involved in this process; in the first one on the left, the tree shows how the failure of bank 11 can bring to default of banks 9, 10, 13 and 21 because of the losses transmitted through layer l_1 , banks 26, 29 and 31 fail become illiquid, and bank 33 fails because of its common exposures with bank 21. All these 9 nodes are then aggregated into the super-node of step 2 (red node); the defaults of this super-node triggers other 5 failures. In the last step (last tree on the right) the 5 banks (5,7,12,14,18), aggregated into the super-node, bring to the failure of other 4 banks.

number of defaults in the complete scenario (when all the three layers are activated simultaneously) is due to multi-dimensional critical paths that can reach also remote banks in the system, the removal of one layer can interrupt these critical paths and so underestimate the number of banks involved in the propagation process. Moreover, the identification of critical paths is necessary in order to understand how the idiosyncratic risk taken by the single institutions can affect the stability of the system. It is evident that there is a strong interaction among the different risks embedded in our model: a well working interbank market has to be able to properly share these risks among the different financial institutions in such a way that the system can absorb local shocks without propagating them to the entire system.

We highlight here a fundamental point of the whole paper, made clear by the example reported in Fig. 14. In the first step of the algorithm, bank 21 plays a fundamental role in increasing the extent to which the shock can propagate in the financial system. In fact, losses materialize for node 21 in the form of interbank counterparty risk. Nevertheless, the bank transmit stress to other institutions in the form of funding and market risk. The banks behavior assumed

in the model enables the risk to be transformed from one shape into an other, and this transformation-and-sharing risk process is at the very core of the high level of systemic risk we report in our simulations.

A natural measure of systemic importance for a bank in the system is immediately achieved through the aggregation algorithm. A bank becomes systemically important if its failure materializes in substantial losses for the other institutions, leading to other defaults and eventually a significant impact on the real economy. The aggregation algorithm has the advantage that it does not take into account the reasons why a bank fails: once it does, it is aggregated into the super-node. The size of the super-node when the algorithm converges therefore reflects the order of magnitude of the spillovers produced by that particular bank, which in turn depends both on the composition of the banking system (i.e. balance sheet information are included when computing the threshold values for the critical links) and on the multi-layered network structure itself. The size of the super-node, which should reproduce the number of defaults obtained from the simulation engine, is an approximation in two main respects: (i) losses directly affecting the capital from different layers (for example layer l_1 and layer l_3) are not summed up together to trigger the default of a bank, but the bank will fail only if losses from separate layers trigger the threshold for that particular layer. This shortcut can be avoided at the price of a more complicated algorithm, while we prefer to keep a good trade-off between simplicity and interpretability, and correctness. (ii) Liquidity spirals are only partially reproduced with the algorithm: if a bank fails at some point in the algorithm, its borrowers in the short-term interbank market will experience a liquidity shock, that can in turn trigger their defaults, and so on. However, in reality (and also in our simulations) banks start withdrawing liquidity before they fail, because of liquidity needs or because they have to fulfill their Risk-weighted Capital Ratio. This mechanism of *precautionary* withdrawal of liquidity is not captured by the algorithm, and it is difficult to include if we want to keep its iterative nature, which has the advantage to be easily tractable. In light of these observations, we cannot expect that the number of defaults in the simulations will be exactly reproduced by the size of the super-node. Nevertheless, to its advantage, the algorithm is able to simplify the network structure and to reproduce the non-linearity we find in the simulations.

To assess the validity of the aggregation algorithm, Fig. 15 show the comparison between the results from the simulation engine (number of defaults) and the size of the super-node computed

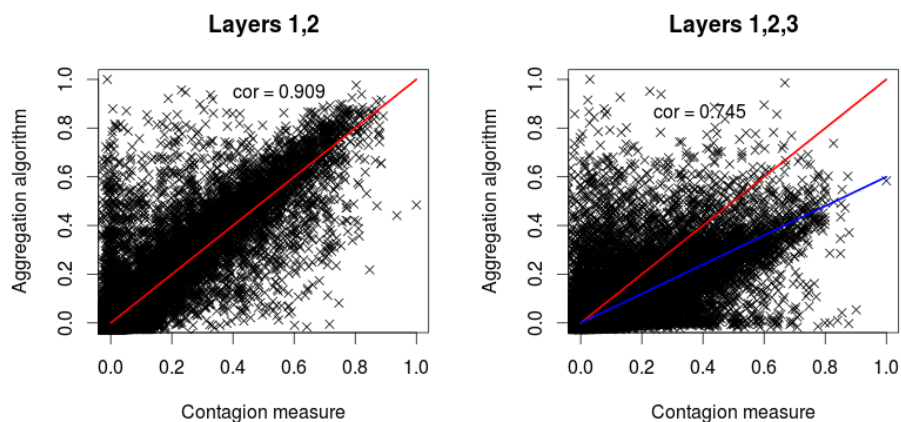


Figure 15: In the left-side panel of the figure we report the comparison between the number of defaults obtained from the simulation engine (horizontal axes) and the size of the super-node as output of the aggregation algorithm (vertical axes), for 10^5 random realizations of the multi-layered interbank network. For each realization, we randomly select one of the 50 banks as initial defaulting bank. The red line is the unitary slope dependency $y = x$. On the right-side panel of the figure, we report the same results when all the three layers are activated simultaneously, and the blue line is the best linear regression $y = a \cdot x$, where $a = 0.59$. All the values are normalized to the maximum number of defaults reported in the simulations.

with the aggregation algorithm. In particular, on the left-side panel there is the comparison when only two layers are activated (namely layer l_1 and l_2), and in the right-side panel the same comparison is reported when all the layers are activated simultaneously. In both cases, there is a significant level of correlation among the two measures, highlighting the good performance of the aggregation algorithm, especially if compared to the classical network measures reported in Fig. 12, or the balance sheet-based measures shown in Fig. 13. The larger accordance in the case of just two active layers has already been explained in point (i) above. In fact, the differences in the number of defaults can be attributed to those banks who fail because they receive losses from different layers, a mechanism which is absent in the aggregation algorithm, that instead aggregates losses from different counterparties only within the same layers.

It should be noted that the main scope of the aggregation algorithm is not to reproduce the number of the defaults we obtain in the simulation engine, but approximate it with the advantage of having some more clues about how the network structure propagates local shocks to a global scope. Given the correlation between the simulation results and the recursive algorithm, and given that there is no

other way for the algorithm to produce non-linear effects a part of the creation of multi-dimensional paths, we can conclude that also in the simulations the non-linear effects are generated through the same mechanism. We note, moreover, that the algorithm is easily customizable to take into account different choices for the banks' micro-behavior; in fact, the good performance of the algorithm reported in Fig. 15 is also due to the choice of the criticality conditions appearing in eq.s (22)-(26), which reflect the micro behavior of banks in the system. Changing the banks' micro-behavior will reflect in different condition for the links criticality, but the algorithm can still be used to simplify the financial network structure.

7 Conclusions and policy implications

The agent-based, multi-layered interbank network model presented in this paper illustrates the importance of taking a holistic approach when analysing the contagion risks related to the interconnections between banks. The main finding is that looking at segments of banks' interconnections in isolation, without considering the interactions with other layers of banks' interrelationships, can lead to a serious underestimation of interbank contagion risk. In other words, by taking into account the various layers of interbank relations and the interactions between them the contagion effects of a shock to one layer can be significantly amplified, compared to the situation where contagion risks are assumed to be confined within the specific layer where the initial shock arose. This finding points to the existence of important non-linearities in the way bank-specific shocks are propagated throughout the financial system.

Another important finding of the paper is that the structure of the network and the underlying balance sheet positions of the banks (nodes) in the network matter in terms of resilience to shocks. In many, in fact the majority, of our simulated network structures financial contagion is likely to be limited. However, in certain network constellations, also depending on the financial soundness of the central players in those networks, contagion risk is substantially more pronounced.

Furthermore, by considering not only contagion via direct bilateral exposures but also via banks' common exposures (through their securities holdings) we are able to demonstrate a trade-off between risk diversification decisions and financial stability. In other words, due to the potential contagion risks related to banks' common exposures decisions to diversify their investments in securities that may

be optimal at the individual bank level can in fact imply higher contagion risks for the system as a whole.

In view of these findings, the paper proposes a “systemic importance” measure that accounts for the multi-dimensional aspect of banks’ interrelations. That is, based on our multi-layered network model and taking into account individual banks’ balance sheet structure the approach provides a single measure of banks’ systemic importance that outperforms standard network centrality measures as well as typical balance sheet indicators.

The observation that unless a holistic view of banks’ interrelations is taken the analysis of interbank contagion risk is likely to underestimate the true contagion risk has major policy implications. From both a micro-prudential and in particular a macroprudential perspective the findings of this paper suggest that it is insufficient to analyze contagion within specific market segments in isolation. Indeed, according to the findings presented here, a major component of the propagation mechanism that transmits losses in one bank to the rest of the system derives from the interactions between the multiple layers of interactions that banks have with each other. On this basis, an immediate policy prescription emerging from this analysis is the importance of collecting adequate supervisory and other micro level data that allows for assessing in a holistic way the interconnectedness of the banking system and thus account for the non-linearities that the existence of multi-layered interbank networks may induce. An even more important policy implications is that the finding of non-linear contagion potential arising due to the multitude of interactions between across different types of activities (e.g. market making, trading, funding markets, etc.) is that to mitigate and minimise the amplitude of such contagion effects might warrant some form of institutional separation between key bank activities (e.g. proprietary trading).

References

- [1] Allen, F. and Gale, D., ‘Financial contagion’, *Journal of Political Economy*, Vol. 108, pages 1-33, 2000
- [2] Bargigli, L., di Iaso, G., Infante, L., Lillo, F., Pierobon, F., ‘The multiplex structure of interbank networks’, arXiv:1311.4798v1, 2013
- [3] Battiston, S., Puliga, M., Kaushik, R., Tasca, P., Caldarelli, G., ‘DebtRank: Too Central to Fail? Financial

- Networks, the FED and Systemic Risk', *Scientific Reports* 2, Article number 541, 2012
- [4] Beale, N., Rand, D. G., Battey, H., Crosson, K., May, R. M., Nowark, M. A., 'Individual versus systemic risk and the Regulator's Dilemma', *PNAS* 2011 108(31), 2011
- [5] Bluhm, M. and Krahenen, J. P. 'Default Risk in an interconnected Banking System with Endogenous Asset Markets', CFS Working Paper Series No. 2011/19
- [6] Bisias, D., Flood, M. D., Lo, A. W., Valavanis, S., 'A Survey of Systemic Risk Analytics', U.S. Department of Treasury, Office of Financial Research No. 0001, 2012
- [7] Boss, M., Elsinger, H., Summer, M., and Thurner, S., 'Network topology of the interbank market.', *Quantitative Finance*, 4, p. 677-684, 2004
- [8] Brunnermeier, M. K., 'Deciphering the Liquidity and Credit Crunch 2007-2008', *Journal of Economic Perspectives*, Volume 23, Number 1, p. 77-100, 2009
- [9] Diamond, D. W. and Dybvig P. H., 'Bank Runs, Deposit Insurance, and Liquidity', *The Journal of Political Economy*, Vol. 91, No. 3, pp. 401-419, 1983
- [10] Delpini, D., Battiston, S., Riccaboni, M., Gabbi, G., Pammolli, F., Caldarelli, G. 'Evolution of Controllability in Interbank Networks', *Scientific Reports* 3, Article number 1626, 2013
- [11] Eisenberg, L., Noe, T. H., 'Systemic Risk in Financial System', *Management Science*, vol. 47 no. 2 236'249, 2001
- [12] Fricke, D., Lux, T., 'Core-periphery structure in the overnight money market: Evidence from the e-mid trading platform', Kiel Working Papers 1759, Kiel Institute for the World Economy, 2012
- [13] Gai, P. and S. Kapadia, 'Contagion in Financial Networks', Working paper no 383/2010, Bank of England, 2010

- [14] Georg, C. 'The effect of the interbank network structure on contagion and common shocks', *DeutscheBank Discussion Paper*, Series 2: Banking and Financial Studies No 12/2011
- [15] Gómez, S., Diaz-Guilera, A., Gómez-Gardeñes, J., Pérez-Vicente, C.J., Moreno, Y., Arenas, A. 'Diffusion dynamics on multiplex networks', arXiv:1207.2788 [physics.soc-ph]
- [16] Hałaj, G. and Kok, C. S., 'Interbank contagion using simulated interbank networks', *Computational Management Science*, Vol. 10(2), pp. 157-186, 2013
- [17] Hałaj, G. and Kok, C. S., 'Modelling the emergence of interbank networks: an agent-based model for macroprudential policy assessments', *Quantitative Finance*, 2015
- [18] Haldane, A. and R. May, 'Financial Systems: Ecology and Economics', *Nature* 469, 2011.
- [19] Iori, G., Saqib, J., Francisco, G. P., 'Systemic risk on the interbank market', *Journal of Economic Behavior & Organization*, Vol. 61 (2006) 525-542
- [20] Iori, G., De Masi, G., Precup, O.V., Gabbi, G., and Caldarelli, G., 'A network analysis of the Italian overnight money market', *Journal of Economic Dynamics and Control*, 32, p. 259-278, 2008
- [21] Ladley, D. 'Contagion and risk-sharing on the interbank market', Discussion paper in Economics 11/10, Department of Economics, University of Leicester, 2011
- [22] May R., and N. Arinaminpathy, 'Systemic Risk: The Dynamics of Model Banking Systems', *Journal of the Royal Society Interface* 2, 2010.
- [23] Nier, E., Yang, J., Yorulmazer, T., Alentorn, A., 'Network models and financial stability', Working paper no 346/2008, Bank of England, 2008
- [24] Tasca, P., and Battiston, S., 'Diversification and Financial Stability', CCSS Working Paper CCSS-11.001, 2012

- [25] Upper, C., 'Simulation methods to assess the danger of contagion in interbank markets', *Journal of Financial Stability* doi:10.1016/j.jfs.2010.12.001, 2011
- [26] Gai, P., Haladane, A., Kapadia, S., 'Complexity, concentration and contagion', *Journal of Monetary Economics*, Volume 58, Issue 5, Pages 453–470, July, 2011

A Jaccard index

Among the several measures that can be introduced to measure similarity among set of numerical or binary data (see for example Bargigli *et al.* (2013)), we use in this paper the so called Jaccard index. Given two networks g_1 and g_2 , described by the weighted matrix W^1 and W^2 , we introduce the following quantities¹⁷:

- M_{11} : number of entries (i, j) which have non null values both in the matrix W^1 and W^2 ;
- M_{10} : number of entries (i, j) which have non null values in the matrix W^1 and null value in the matrix W^2 ;
- M_{01} : number of entries (i, j) which have null values in the matrix W^1 and non null value in the matrix W^2 ;
- M_{00} : number of entries (i, j) which have null values both in the matrix W^1 and W^2 .

We have $M_{11} + M_{10} + M_{01} + M_{00} = N^2$. The Jaccard index is then defined as:

$$J_{12} = \frac{M_{11}}{M_{10} + M_{01} + M_{11}} \quad (21)$$

and its value ranges in the interval $[0, 1]$. In particular, J_{12} is equal to 0 if the two networks do not have a single common link, and it is equal to 1 if the two networks are identical.

B Computation of the critical links

The identification of the thresholds for a link weight to be defined critical is at the core of the aggregation algorithm proposed in the

¹⁷For simplicity we assume here that both the matrices are $N \times N$, with entries (i, j) belonging to the real space \mathfrak{R} .

paper. In this section we report the details necessary to compute them. We note that the threshold values compute here depend on the micro behavioral rules assumed for the banks. Changing the banks' behavior will of course change threshold values, but the aggregation algorithm will still work as tool for the simplification of the multi-layer network structure.¹⁸

- Layer l_1 : given the matrix W^1 whose entries represent the long-term direct exposures among banks, there exists a critical link in layer l_1 between two banks i and j if:

$$W_{ji}^1 \cdot LGD_i > \frac{eq_j - \bar{\gamma} \left[RWEA_j + \sum_{\mu=0}^M p_\mu w^\mu s_\mu^j + w^{ib} l_j^l \right]}{1 - \bar{\gamma} w^{ib}} \quad (22)$$

Despite the complicated form of eq. (22), its meaning is simple: a critical link between nodes i and j exists if node j is not able to absorb the losses transmitted in case of the defaults of node i . In the above equation we introduce the losses-given-default (LGD) of bank i , computed as an estimation of the percentage of loans that bank i is not able to repay in case of its default¹⁹. We note that the use of LGD is fundamental in order to replicate a more realistic scenario in the simulations, and the quality of its estimation depends on the available data.

- Layer l_2 : given the matrix W^2 whose entries represent the short-term direct exposures among banks, there exists a critical link in layer l_2 between two banks i and j if:

$$W_{ij}^2 > \left(c_j + l_j^s + \sum_{\mu=0}^M \bar{s}_\mu^j \cdot \exp \left\{ -\alpha_\mu \frac{\bar{s}_\mu^j}{s_\mu^{tot}} \right\} \right) \quad (24)$$

¹⁸The computation of the thresholds necessary to identify critical links represents the tricky part of the algorithm. In fact, a part of layer l_1 for which one can easily compute the maximum losses each bank can absorb without going below the capital requirements, for the other layers approximations are necessary.

¹⁹In our framework, this amount to:

$$LGD_i = 1 - \min \left[\max \left[\frac{c_i + \sum_{\mu=0}^M s_\mu^i p_\mu + l_i^s - b_i^s}{l_i^l}; 0 \right]; 1 \right] \quad (23)$$

Of course, better calibrations are possible depending on data availability and the dynamics used in the model.

The sequence $\{\bar{s}_1^j, \bar{s}_2^j, \dots, \bar{s}_M^j\}$ are the roots of the equation:

$$\frac{eq_j + \sum_{\mu=0}^M \bar{s}_\mu^j \cdot \left[1 - \exp\left\{-\alpha_\mu \frac{\bar{s}_\mu^j}{s_\mu^{tot}}\right\}\right]}{CRWA_j + w^{ib}l_j^l + \sum_{\mu=0}^M (s_\mu^j - \bar{s}_\mu^j) \exp\left\{-\alpha_\mu \frac{\bar{s}_\mu^j}{s_\mu^{tot}}\right\}} - \bar{\gamma} = 0 \quad (25)$$

Those roots have to be found numerically since we have to impose the pecking order, as in the simulator engine, and the nonlinearities appearing both in the numerator and in the denominator of eq. (25) make it impossible to find analytical solutions.

Equation (24) states that a critical link between i and j exists if bank i can force bank j to liquidate an amount of assets, by withdrawing all its short-term funding, which will reduce the RWCR of bank j beyond the threshold value $\bar{\gamma}$. In other words, bank j is relying too heavily on the funding services provided by bank i . We note that the link between illiquidity and insolvency, in the simulator engine, was properly expressed through the map in eq. (10).

- Layer l_3 : given the matrix of the portfolios $S_{N \times M}$, whose entries s_μ^i represent the securities μ in the portfolio of bank i , there exists a critical link in layer l_3 between two banks i and j if the liquidation of the whole bank i 's portfolio results in the default of bank j , namely when:

$$\frac{eq_j - \sum_{\mu=0}^M (1 - p_\mu^*) s_\mu^j}{CRWA_j + w^{ib}(l_j^s + l_j^l) + \sum_{\mu=0}^M w^\mu p_\mu^* s_\mu^j} < \bar{\gamma} \quad (26)$$

Where we indicated with p_μ^* the price of the security μ after bank i liquidates its portfolio, according to eq. (6).

Macprudential Research Network

This paper presents research conducted within the Macprudential Research Network (MaRs). The network is composed of economists from the European System of Central Banks (ESCB), i.e. the national central banks of the 27 European Union (EU) Member States and the European Central Bank. The objective of MaRs is to develop core conceptual frameworks, models and/or tools supporting macro-prudential supervision in the EU.

The research is carried out in three work streams: 1) Macro-financial models linking financial stability and the performance of the economy; 2) Early warning systems and systemic risk indicators; 3) Assessing contagion risks.

MaRs is chaired by Philipp Hartmann (ECB). Paolo Angelini (Banca d'Italia), Laurent Clerc (Banque de France), Carsten Detken (ECB), Simone Manganelli (ECB) and Katerina Šmídková (Czech National Bank) are workstream coordinators. Javier Suarez (Center for Monetary and Financial Studies) and Hans Degryse (Katholieke Universiteit Leuven and Tilburg University) act as external consultants. Fiorella De Fiore (ECB) and Kalin Nikolov (ECB) share responsibility for the MaRs Secretariat.

The refereeing process of this paper has been coordinated by a team composed of Gerhard Rünstler, Kalin Nikolov and Bernd Schwaab (all ECB).

The paper is released in order to make the research of MaRs generally available, in preliminary form, to encourage comments and suggestions prior to final publication. The views expressed in the paper are the ones of the author(s) and do not necessarily reflect those of the ECB or of the ESCB.

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