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INTERNATIONALLY CORRELATED JUMPS

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Abstract

Stock returns are characterized by extreme observations, jumps that would not occur under the smooth variation of a Gaussian process. We find that jumps are prevalent in most countries. This has been little investigation of whether the jumps are internationally correlated. Their possible inter-correlation is important for investors because international diversification is less effective when jumps are frequent, unpredictable and strongly correlated. Public supervisors may also mind about widely correlated jumps, as they could bring down certain financial intermediaries. We investigate using daily returns on broad equity indexes from 82 countries and for several statistical measures of jumps. Various jump measures are not in complete agreement but a general pattern emerges. Jumps are internationally correlated but not as much as returns. Although the smooth variation in returns is driven strongly by systematic global factors, jumps are more idiosyncratic and most of them are found in Europe. Some pairs of correlated jumps occur simultaneously but not to the extent of correlated returns.

JEL CLASSIFICATION: G11, G15

KEYWORDS: Diversification; Jumps; correlation

Non-Technical Summary

Stock returns exhibit jumps relative to the rather smooth variation typical of a normal distribution. Jumps might be caused by sudden changes in the parameters of the conditional return distribution, extreme events such as political upheavals in a particular country, shocks to some important factor such as energy prices, global perturbation of recessions.

The ubiquity of jumps has important implications for investors, who must rely on diversification for risk control. If jumps are idiosyncratic to particular firms or even countries, they might be only a second-order concern. But if jumps are broadly systematic, unpredictable, and highly correlated, diversification provides scant solace for even the best-diversified portfolio. Jumps that affect broad markets are also headaches for policy makers such as financial supervisors.

Little has been previously documented about the international nature of jumps. To this end, we compare their prevalence and severity across 82 countries. We did not weight to countries and stock markets by their size and our jumps are not limited to political events and natural disasters. While jumps do not span around the globe, many correlated jumps we found occurred in the G-20 countries. We also tabulate calendar periods that had the most influence on jump correlations and compare them with the most influential periods for return correlations. We perform some robustness tests including simulation. Our general finding is that jumps are less correlated across countries than raw returns. In other words, jumps are less systematic than the smoother (non-jump) component of country price indexes. Almost all the monthly return correlations are positive and almost 80% are statistically significant at the 1% level; this is for 3,321 individual correlation coefficients computed with returns from 82 countries. But jumps are less correlated. For some of the jump measures, the correlation is very weak and is statistically significant in only a few pairs of countries. This is based on the Barndorff-Nielsen and Shephard (BNS) (2006) jump measure.¹ Simulations in Section 4 of our paper show that BNS performs very well; it does not indicate the presence of correlated jumps when there are actually none and it has good power to reject a false null hypothesis of no correlated jumps. A few pairs of countries (which we identify) jumps are relatively idiosyncratic. This suggests that

¹ We also apply the other jump measures including Lee and Mykland (2008), Jiang and Oomen (2008), Jacod and Todorov (2009). The results from applying these four jump measures remain intact.

jumps are mainly induced by country-specific events such as political events or natural disasters.² They are not often induced by shocks to global factors such as energy or investor confidence.

We also document two other interesting features of jumps: first, we display particular calendar periods that contribute the most to international jump correlations. Perhaps surprisingly, these are not usually the same months that are most influential for return correlations, though again, there are some differences among the jump measures. Second, we provide information on particular pairs of countries that are most influenced by extreme jumps.

Another surprise is that the most jump-correlated countries are larger and more developed and are mainly in Europe. Because jumps are more correlated among European neighbors, international diversification is less effective in that region. In contrast, jump co-movement is uncommon among developing countries or in non-European developed countries. The rarity of international correlation among jumps suggests they are mostly caused by local influences such as political events and not by common global factors such as energy prices.

Although jumps are frequent in all countries and are probably hard to predict, they are not as correlated internationally as returns themselves. Returns seem to be more driven by global systematic influences while jumps are somewhat more idiosyncratic. Diversification might provide reasonably satisfactory insurance against jumps; nonetheless, policy makers should *not* be complacent from our results because future crises might be broad and be associated with contagion.

² This conclusion is in full agreement with the recent paper by Lee (2012), who reports that U.S. jumps are mostly attributable to events such as Federal Reserve announcements or initial jobless claims (which are mainly idiosyncratic from a global perspective) or else are due to clearly idiosyncratic firm-specific events such as earnings reports.

1 Introduction

Stock returns exhibit jumps relative to the rather smooth variation typical of a Gaussian distribution.³ Jumps might arise for a number of different reasons; to name a few: sudden changes in the parameters of the conditional return distribution, extreme events such as political upheavals in a particular country, shocks to some important factor such as energy prices, global perturbation of recessions.

The ubiquity of jumps has important implications for investors, who must rely on diversification for risk control. If jumps are idiosyncratic to particular firms or even countries, they might be only a second-order concern. But if jumps are broadly systematic, unpredictable, and highly correlated, diversification provides scant solace for even the best-diversified portfolio. Eraker et al. (2003) find that the jumps command larger risk premiums than continuous returns. Das and Uppal (2004) examine the portfolio choice problem of an international investor when returns are categorized by jumps, leading to systemic risks. Using monthly return data for a few developed markets, they measure diversification benefits and the home bias. They do not consider a large number of markets and do not apply the jump technology in their paper. Asgharian and Bengtsson (2006) find significant jumps in large markets that lead to jumps in other markets. They conclude that markets in the same region and with similar industry structures tend to experience jump contagion. Jumps might be more prominent in emerging market returns where skewness and kurtosis are widely documented (Bekaert, et al. (1998a, b).

Hartmann, Straetmans, and de Vries (2004) derive nonparametric estimates for the expected number of market crashes given that at least one market crashes. Their approach does not rely on a specific probability law for the returns and thus has an advantage over the often used correlation. They apply their measure to study the comovements of stocks and government bond markets during periods of stress. Instead of studying contagion or joint crashes of stocks, they investigate the phenomena of flight to quality or a crash in stock markets followed by a

³ See, inter alia, Chernov, et al. (2003), Eraker, et al. (2003), and Huang and Tauchen (2005).

boom in government bond markets. Similar to Pukthuanthong and Roll (2009), they agree correlation is not a good measure of market integration as it is predisposed toward the multivariate normal distribution, which normally underestimates the frequency of extreme market spillovers. Similar to this study, they conclude the financial market contagion phenomenon may have been overestimated in the literature on financial crisis (see also Forbes and Rigobon, 2002). Policymakers should not be complacent from these results since the next crisis might be broad and associated with contagion. Poon, Rockinger and Tawn (2004) develop tail dependence measure document the widespread asymptotic independence among stock market returns, which has been ignored in the finance literature. The omission of asymptotic independence can cause estimation errors of portfolio risk and thus suboptimal portfolio choice. Consistent with the extant literature, they find dependence between volatilities is strong during bear markets than in bull markets. Consistent with our study, the dependence between volatilities has increased over time to produce asymptotically dependent stock markets within Europe but still asymptotically independent stock markets among other regions. Hartmann, Straetmans, and de Vries (2007) apply a multivariate extreme value techniques applied by Hartmann et al (2004) and Poon et al (2004) to estimate the strength of banking system risks. Specifically, they apply extreme value theory to evaluate the extreme dependence between bank stock returns and measure banking system risk.

These studies apply the novel multivariate extreme value approach to assess the extreme dependence between stock returns and to measure system risk. That is, they focus on crisis propagations or relations between extremely large negative returns over time while we focus on the simultaneous effects of common shocks or jumps on a single day. Our correlated jumps occur in one single day and the jump measures are based on daily data. Moreover, we focus on price

jumps or discontinuities, which are narrower than the aforementioned studies. Jumps seem to be an extreme case of crisis-type propagation. de Bandt and Hartmann (2010) provide a good survey on systemic risk including theoretical models and empirical evidence.

Jumps that affect broad markets are also headaches for policy makers such as finance ministers and central bankers. This is all the more true if jumps are significantly correlated internationally, for policy makers will then find it necessary, albeit difficult, to coordinate their reactions across countries.

We present evidence about the international co-movement of jumps across 82 countries. Our general finding is that jumps are less correlated across countries than raw returns. In other words, jumps are less systematic than the smoother (non-jump) component of country price indexes. Except for a few pairs of countries (which we identify) jumps are relatively idiosyncratic. This suggests that jumps are mainly induced by country-specific events such as political events or natural disasters.⁴ They are not often induced by shocks to global factors such as energy or investor confidence. This is good news for international investors – diversification provides reasonably satisfactory insurance against jumps. Policy makers should not be complacent from our results because the future crisis might be broad and be associated with contagion.

Little has been previously documented about the international nature of jumps. To this end, we compare their prevalence and severity across countries. We also tabulate calendar periods that had the most influence on jump correlations and compare them with the most influential periods for return correlations. This provides an intuitive depiction of the frequency and importance of jumps.

2 Data and Summary Statistics for Returns

2.1 Data

⁴ This conclusion is in full agreement with the recent paper by Lee (2012), who reports that U.S. jumps are mostly attributable to events such as Federal Reserve announcements or initial jobless claims (which are mainly idiosyncratic from a global perspective) or else are due to clearly idiosyncratic firm-specific events such as earnings reports.

Daily data are extracted for 82 countries from DataStream, a division of Thomson Financial. The data consist of broad country indexes converted into a common currency (the US dollar). Appendix A lists the countries, identifies the indexes, reports the time span of daily data availability, and provides the DataStream mnemonic indicator (which could help in any replication.) If the mnemonic contains the symbol “RI”, the index includes reinvested dividends; otherwise, the index an average daily price.

Daily data availability extends back to the 1960s for a few countries but most joined the database at a later time. The latest available date, when all the data were downloaded, is October 26, 2009 for all countries except Zimbabwe, (which closed its stock market after October 2006.)

Daily returns are calculated as log index relatives from valid index observations. An index observation is not used if it exactly matches the previous reported day’s index. When an index is not available for a given trading day, DataStream inserts the previous day’s value. This happens whenever a trading day is a holiday in a country and also, particularly for smaller countries, when the market is closed or the data are simply not available. Our daily returns are thus filtered to eliminate such invalid observations.

Using the daily data for valid observations, calendar month and semiannual returns are computed by adding together the (log) daily returns. The subsequent analysis uses these longer-term returns, which also helps alleviate the effect of invalid daily observations. In order to be included in the computations, a country must have at least ten valid monthly observation or 30 valid observations within a semester.

2.2 Summary statistics for return correlations

Simple product moment correlations are computed for each pair of countries. Summary statistics for the correlations are reported in Table 1, Panel A for monthly correlations and Panel B for semiannual. The number of observations depends on data availability. The maximum number of months is 538, (e.g., Germany and the United Kingdom), and the minimum is eight, (e.g., Greece and Zimbabwe.) Most pairs of countries have at least 100 concurrent monthly observations and quite a few have several hundred. For semiannual periods, the maximum number is 90 and the minimum is eight. Greece and Zimbabwe do not have enough concurrent semiannual observations to compute a correlation.

As the table reveals, correlations are somewhat higher with semiannual than with monthly returns; both the mean and median are higher by about 0.12. Cross-country-pair variation is only slightly higher for semiannual returns as indicated by the standard deviation and the mean absolute deviation while the number of highly significant correlations is lower; this is probably attributable to the lower sample sizes for semiannual data. There is no evidence of skewness or kurtosis.

Table 2 provides a list of the single most influential observation for the return correlation between each pair of countries. To obtain these results, we simply computed the de-measured product of returns that was the algebraically largest over all the available observations. The table lists each influential period, the number of country pairs with data available for that period, and the fraction of country pairs for which that particular period was the most influential. Periods are omitted if their influential observations amounted to less than one percent of the available correlations.

Perhaps the most striking aspect of Table 2 is the pronounced dominance of October 2008 for monthly data and the second semester of 2008 for semiannual data. For 3,240 monthly correlation coefficients among the 82 countries, October 2008 was the single most influential observation in 2,457, more than 75% of the cases. The second semester of 2008 was the most influential in 87.1% of the 3,240 semiannual correlations. No other periods even come close. The next most influential monthly observation is October 1987, with 16.9% of the 378 correlations available then. The next most influential semester was the second half of 1993, a paltry 4.86% of the 1,378 available correlations.

3 International jump correlations

Our approach consists of two steps. First, we compute the Barndorff-Nielsen and Shephard (2006, hereafter BNS) jump statistic “G” over a sequence of fixed-length calendar periods within each country.⁵ Second, for each pair of countries, we correlate the resulting BNS G jump statistics across all available periods. The intuition is simple: if jumps are contemporaneous and more intense simultaneously, the BNS jump statistics will be positively

⁵ The BNS G statistic is based on the difference between “bipower” variation and squared variation; (See Appendix B.) BNS also derive an H statistic based on the ratio of bipower to squared variation. The G and H statistics provide very similar inferences in all cases. Full details are available upon request.

correlated across time. Such jump correlations can conceivably have a very different pattern than ordinary return correlations.

3.1 The Barndorff-Nielsen and Shephard (2006) statistic

For each country and each period k , either a calendar month or a semester, the BNS G statistic is computed from the daily return observations during the period. The full tabulation of results is available upon request.

The BNS G statistic is asymptotically unit normal under the null hypothesis of no jumps. The alternative hypothesis, that one or more jumps has occurred, tends to make the BNS G negative. Our results reveal that the average value of G is negative for every one of the 82 countries and all of the T -statistics for the sample mean G indicate significance, most being highly significant. If the underlying returns are independently distributed across time, Barndorff-Nielsen and Shephard show that their jump statistics are also time-series independent, so the T -statistics should be fairly reliable.

Table 3 provides summary statistics for the BNS G measure computed over both monthly and semiannual periods.⁶ For example, the mean over 82 countries of the country mean BNS G is -6.799 and the mean country standard deviation is 15.19. If there had been no jumps, the mean and standard deviation should have been approximately zero and 1.0 on average. The country average T -statistic is -5.232.

Similarly, the average skewness and kurtosis, (which would be approximately zero if there were no jumps) are -5.177 and 47.160, both indicating dramatic departure from the asymptotic normality that would arise under the null hypothesis of no jumps. Skewness is negative for every country, which shows that some months during the sample have dramatically smaller values of the jump measure than could be expected under the null; (recall that negative values of G indicate jumps within the month.) The uniformly large values of kurtosis reveal extreme value of G in some months.

⁶In these averages, measures that exceed 1,000 in absolute value are expunged because they are probably due to data errors. For example, the January 1999 monthly G measure for Ghana is -202,343. In the original data, the Ghanaian price index changed only in the seventh significant digit every day in January until the last (typical successive values are 426.8350, 426.8352, and so on, up and down.) Then, on the last day of January, the index shot up to 452.95. In February, the index remained around 452.95 until the last day as well. It seems likely that no trades occurred on most days in these months and the index changed only because of rounding error.

The individual monthly and semiannual maxima and minima also indicate the strongly negative character of empirical BNS G measures. Very few individual jump measures are positive and the maximum is less than one for both monthly and semiannual periods. The minimum, in contrast, is orders of magnitude larger in absolute value.

BNS G measures based on semiannual observations are less significant because the sample sizes are smaller. But all indications agree that a null hypothesis of no jumps should be rejected for all countries. Evidently, jumps are ubiquitous.

Since Table 3 show clearly that jumps are happening all over the globe, the next step is to ascertain how correlated they are across countries. To this end, using the calculated BNS G computed for both months and semesters within individual countries, we compute two international correlation matrices. Table 4 provides summary statistics from these two different estimates of international jump correlations.

The international correlations of jump measures reported in Table 4 stand in stark contrast with the return correlations reported earlier in Table 1. The jump measures are simply not that correlated. The mean correlation coefficients are only around 0.01 to 0.03. Although the means are supposedly statistically significant based on the *T*-statistic for the mean, only a modest number of individual correlations have individual *T*'s greater than 2.0, between six and seven percent of them. This differs dramatically from individual correlations among returns, which Table 1 reports have *T*'s exceeding 2.0 in 60% to 80% of the cases.

This conclusion is further supported by Table 5, which gives influential months and semesters for the correlations among jump measures. Unlike the influential periods for returns (Table 2), there are no grossly dominant periods. The first semester of 1973 has the largest percentage of influential observations, but only 21.9%, in contrast with the 87.1% of influential observations exhibited by the second semester of 2008 for return correlations. Moreover, there were many more available pairs during the second semester of 2008, 3,240, versus only 105 in the first semester of 1973, so the dominance of 2008 is all the more impressive.

For monthly jump measures, Table 5 shows that no month reaches even a ten percent level as being most influential. Notice also that the two most dominant months for returns, October 2008 and October 1987, do not even appear in Table 5. Similarly, the second half of 2008, the main time of the recent financial “meltdown,” does not appear as significantly contributing to semiannual jump correlations.

Combining the results in Tables 3, 4, and 5, one can only conclude that jumps are occurring in all countries but not usually at the same time. This is good news for investors because it seems to suggest that diversification can be effective in protecting against extreme movements in prices even though the smooth component of return variation is quite correlated internationally. Evidently, jumps are much more idiosyncratic than normal variation.

Despite the weak international correlation among jumps, it could still be useful to examine special cases of countries that exhibit somewhat more jump co-movement. Table 6 presents a list of country pairs whose jump correlations have T -statistics exceeding 3.0 for the BNS G measure. Many of these seem intuitively plausible since they are close neighbors and trading partners; indeed, quite a few pairs are countries within the European community.

There are some, however, that seem a bit odd, particularly for the jump measures computed with semiannual data. Examples are Argentina, partnered with both Bangladesh and Kuwait, or China partnered with Jordan, or Brazil with Lithuania. Perhaps some of these oddities are simply attributable to randomness that is the inevitable companion of large-scale data comparisons

Other cases might very well be worthy of a more in-depth investigation. For example, are semiannual jumps correlated between Indonesia and Morocco because their religious faith subjects them to occasional common shocks? Are Israel and Switzerland paired through technology? What is the relation between Kuwait and Romania, South Korea and Sweden, or Ecuador and the Philippines? It would be interesting to know the underlying reasons for such connections, if indeed there are any.

Most countries provide good diversification protection against extreme movements in prices. But there are a few exceptions such as those listed in Table 6.

3.2 Other jump statistics

In addition to the BNS jump statistic discussed in the previous section whose empirical results are reported in Tables 3 to 6, we also investigated three other competing methods of jump detection. These approaches were developed by Lee and Mykland (2008), Jiang and Oomen (2008), and Jacod and Todorov (2009). All three are detailed in Appendix B, but since this is a paper about finance and not about statistics and because of limited space, the associated

empirical results are not reported in detail but are described briefly below. All results are available upon request.

The Lee and Mykland (hereafter LM) statistic indicates slightly fewer jumps than the Barndorff-Nielsen and Shephard (BNS) statistic but it agrees that jumps are occurring in every one of our 82 countries. LM also indicates that a few countries have correlated jumps. In 11.50% of the bi-country comparisons, LM reveals significant jump correlation with a p-value of 0.05. This exceeds, though only modestly, what one would expect under the null hypothesis of no jump dependence between any two countries. A majority of these significantly correlated pairs involve countries in Europe. A total of 54 countries had their largest LM statistic in a calendar month that was not shared by any other country. This suggests again that the most extreme jumps are relatively isolated and idiosyncratic events.

The Jiang and Oomen (hereafter JO), statistic contrasts to some extent with BNS and LM. Jump correlations based on JO are a bit larger on average, 0.134, and more statistically significant. They are not as significant as correlations between returns but they are closer to returns than the jump correlations for BNS and LM.

JO picks out a few of the same influential months as BNS; e.g., November 1978, and January 1991 and 1994. But it also identifies October 1987 as the most influential jump month of all and October 2008 as next most; these are months having the largest influence on return correlations. It thus seems that the JO measure of jumps portrays them as more systematic, though not to the same extent as returns, and less idiosyncratic as compared to the BNS and LM measures. According the JO measure of jumps, extreme international correlations do not happen for developing countries. Also, many significant country pairs are European, as they are for the LM measure of extreme jump co-movements.

In agreement with the other statistics above, the Jacod and Todorov (hereafter JT) tests suggest that international jumps are frequent. They are strictly idiosyncratic in more than half the country pairs but they do occur jointly on occasion. There is also essential agreement with respect to both the most influential months in the sample and on the pairs of countries that exhibit the largest average values. No month stands out as being overwhelmingly influential; the single most prominent month is September 2008, but it was largest for only 197 out of 3281 pairs of countries. There are 45 pairs of countries with significant jump co-movements at the five

percent level of significance. The majority of these (28) are European. Greece alone figures in 18 pairs.

3.3 Other tests we do not employ

While JT tests for cojumps in a pair of returns based on higher order power variation, Gobbi and Mancini (2006, 2008) propose a strategy to separate the covariation between the diffusive and jump components in a pair of returns. Using a related method, Bollerslev, Law, and Tauchen (2008) do not test for cojumps between a particular pair of returns, but rather in the cojumps embodied in a large ensemble of returns.

Aït-Sahalia and Jacod (2009) and Tauchen and Zhou (2010) propose nonparametric tests for presence of price jumps based on high-frequency data. Also, more recently, Aït-Sahalia, Cacho-Diaz, and Laeven (2010) model asset return dynamics with a drift component, a volatility component, and mutually exciting jumps known as Hawkes processes. They use this approach to capture adverse mutual shocks to stock markets, with a jump in one region of the world propagating a different jump in another region of the world.

Bollerslev and Todorov (2011a) also focus on high-frequency data and use a threshold approach to distinguish jumps from ordinary variation. In a related paper, Bollerslev and Todorov (2011b) estimate risk premia that depend on the existence of jumps in both volatility and prices, but they do not derive a separate estimator for jump detection within a sample period.

Of course, this paper would be unacceptably lengthy if every existing jump test were thoroughly examined. Hence, we selected a single test (BNS) that seemed promising and is relatively easy to implement. Most importantly, in the next section we employ simulations that check the test power of BNS, and verify that it seems more than adequate for our application.

4 The Efficacy of Jump Measures for Detecting Correlated Jumps

Given the fact that jump statistics have not heretofore been used to assess the international correlation of jumps, it is absolutely imperative that we develop some insight about test power. Hence, this section represents an extremely important understructure for the overall empirical approach. Here, we report simulations for which the true nature of correlated jumps is known. We generate artificial data that has a smooth Gaussian variation, including non-zero smooth correlation between the two bivariate return series, appended by artificial jumps of

various sizes, frequencies, and co-movements. Using these artificial data, we study the efficacy of the BNS jump measure for detecting correlated jumps. We also describe briefly the relative efficacies of the other three jump statistics, LM, JO, and JT and we contrast their test power graphically.

Without loss of generality, our simulated bivariate smooth Gaussian process is specified to have mean zero and unit variance for both series plus a pre-specified correlation. Since the average correlation in the monthly international return data is 0.314 (see Table 1, Panel A), we take this as an upper bound because it is also influenced by jumps and not just by smooth variation. In the simulations, we use a value in this general neighborhood, 0.30, and also two smaller values, 0.15 and zero.

The simulated jumps are also Gaussian with mean zero but their strength is modeled by specifying their standard deviation as a multiple (such as 5 or 15) of the underlying smooth series, whose standard deviations are both 1.0. Also, jumps arrive randomly with particular but rather small frequencies. For example, with a daily frequency of 0.02 and 21 trading days per month, the probability of a jump occurring on some day during the month is 0.42. The jump frequencies are studied over a range from very unlikely to very likely during each month. These frequencies are applied independently to both simulated return series.

Conditional on a jump arriving in either series on a given day, there is also a specified co-probability that the same jump will be transmitted to the other series. This co-probability is a key parameter, because it specifies jump co-movement, the object of our study. In the simulations, we allow it to vary from zero (no common jumps) to 0.999 (almost completely common jumps.) Note that the two simulated series can also have common jumps during the same month simply because of random arrivals, even though the jumps are not really common. The co-probability simply increases their natural commonality.

In summary, there are four parameters that vary across simulations: (1) smooth correlation, (2) jump strength, (3) jump frequency, and (4) jump co-probability. Other parameters are held constant: the mean and volatility of the bivariate smooth returns, the type I error (5%), and the number of replications for each parameter combination (1,000). We experimented with different replication numbers but they all deliver essentially the same results.

Each simulation produces an entire probability distribution of the test statistic for correlated jumps, but these numbers are too voluminous to report in their entirety. Instead, we

report only a single indication of effectiveness, test power. When the jump co-probability is positive in the simulated returns, (and hence there are genuinely correlated jumps), the test power is the fraction of replications that reject the false null hypothesis of no jump co-movement. In the special case when the co-probability is actually zero, and hence jumps are only randomly common in the two simulated return series, the test power is the fraction of replications that falsely reject the true null hypothesis of no jump co-movement.

As a base case, we first look at the computed test power when the jump frequency is zero for both simulated return series. Since jumps cannot occur, they cannot be common across the two series. Nonetheless, we compute test power in this case, which is essentially the probability of falsely rejecting the true null hypothesis that there are no correlated jumps. The results are plotted in Figure 1 for BNS, LM and JO.⁷ When the smooth variation correlation is zero, the BNS test provides appropriate results: i.e., at a 5% type I rejection level, it rejects (wrongly) in the vicinity of five percent of the time.

As the smooth correlation increases, going from zero in the left panel to 0.15 in the center panel and then to 0.30 in the right panel, the BNS test increases the incorrect rejection frequency only slightly; i.e., it is behaving well.

With true co-movements in jumps, Table 7 reports some representative simulation results. The table includes two values of the smooth variation correlation (zero and 0.15), two values of jump strength, (5 and 15), two values of jump frequency (0.01 and 0.03), and three values of the co-probability of jumps, (0.30, 0.60, and 0.90.) We actually produced simulation results for a variety of other parameter values, but those in Table 7 provide a reasonable depiction of the overall results.⁸

First notice that BNS seems to provide reasonably reliable results overall. Its test power is higher with more intense jumps and with a higher level of jump co-movement between the two simulated series. This is what one would hope to obtain in a test procedure. It is interesting though, that test power seems to be lower when jumps are more frequent. At first, this might seem surprising but on further reflection, it seems sensible for the following reason: really frequent jumps are more or less akin to smooth variation but simply with a higher volatility. The daily jump frequencies in Table 7 are 0.01 and 0.03, which imply monthly jump probabilities of

⁷ JT is discussed for this case below.

⁸ The complete set of results for all parameter values will be provided to interested readers.

at least 0.21 and 0.63, respectively. With a monthly probability of around 0.60, it is highly likely that at least one of the two simulated return series will have a jump in a given month and this is transferred to the other series with the specified co-probability. Evidently, the commonality that is easiest to detect, at least by the BNS method, involves rather rare jumps.

In comparison to BNS, the LM test provides relatively weaker test power. Nonetheless, the LM approach seems to have the appropriate pattern; it simply requires very strong and highly correlated jumps to have much power.

The JO test has more power than the LM test at all levels of intensity, frequency, and co-probability. However, it has less power than BNS throughout. Moreover, unlike BNS and LM, it tends to detect jumps that do not exist. When there are no jumps, it incorrectly rejects the null hypothesis (no jumps) about 40% of the time for the mid-range smooth correlation of 0.15 and almost 90% of the time at the high end, a correlation of 0.30 (See Figure 1.) Jiang and Oomen (JO) assert in their paper that their test is very sensitive to even small jumps. Evidently, even a small amount of smooth correlation leads to an incorrect inference that there are common jumps.

The JT test never rejects the null hypothesis (no jumps) wrongly, even five percent of the time; hence, it actually has too few rejections, the opposite of JO. However, for high jump strength (15) and the high co-probability of jump transmission (0.90), the JT measure achieves 100% power. It is perfect. It does not perform as well when jump strength is lower; at a jump strength of 5, its power is negligible unless the co-probability is very high. It does better when the jump frequency is higher, *ceteris paribus*.

These results and comparisons are further illustrated in Figures 2 to 4. Figure 2 shows test power for the four jump measures and high jump intensity across three levels of smooth correlation. BNS has the highest power overall. The test powers of BNS, LM and JO do not change much when the smooth correlation goes from zero to 0.30; (the latter value is in the same general vicinity as the average smooth correlation in the international index returns.) However, JT's power increases dramatically, from around 10% to over 70%. In simulations, Jacod and Todorov (2009, Section 6) also find that power is affected by the level of smooth correlation, though the effect appears to be less dramatic than in our application here.

Figure 3 depicts the influence of jump strength. Again, BNS has good power throughout. Its power exceeds 60% even at low levels of intensity (5) and it grows to 80% at an intensity of 10. Both LM and JO exhibit strongly increasing power with growing intensity and JO has the

higher of these two at all levels but neither reaches the power of BNS. JT's power is outstanding and the best of all measures at higher jump intensities (10 and 15) but has only about 10% power at an intensity of 5.

Finally, Figure 4 plots the power for each of the four jump measures against jump frequency and jump co-movement probability. BNS, LM and JO have the pattern one would expect, very low probability of incorrectly rejecting a true null hypothesis (when the co-movement probability is zero) and increasing power against a false null hypothesis as the co-movement probability increases from 0.30 through 0.999. However, when there is truly some jump co-movement, BNS has higher power than LM and JO throughout; (the latter are similar.) Notice too that power is generally better for rare jumps, when the frequency is lower, for BNS, LM and JO. The pattern for JT is quite different. It has virtually no power until the co-movement probability reaches 0.60 but it has the best power of all when this probability is 0.90 and above. Another contrast is that JT's power is (slightly) better for higher jump frequencies. The bottom line from these simulations turns out to be fairly clear-cut. BNS G, the Barndorff-Nielsen and Shephard difference jump measure, seems preferable overall for the explicit purpose we have here, estimating the co-movement of jumps across international markets. It performs well when there are no correlated jumps and it has acceptable power when there are many such jumps. Although the LM and JO measures display a similar pattern, they have weaker power when there are actually jumps. Moreover, JO (but not LM) incorrectly indicates the presence of correlated jumps when there are actually none. JT has outstanding power at very high levels of jump co-movement but performs poorly at lower levels.

5 A Simple Validity Check

To this point, our basic inference from the empirical results is that jumps, though common in all countries, are mostly idiosyncratic and not very related across countries. This suggests that any well-diversified portfolio should exhibit fewer jumps than any single country considered alone.⁹ This can be readily checked by constructing a globally diversified portfolio and estimating the prevalence of jumps by using one of the measures studied above. Previously, Bollerslev, Law, and Tauchen (2008) using the BNS measure, and Lee and Mykland (2008)

⁹ We are grateful to Hanno Lustig for suggesting this idea.

document more frequent and larger sized jumps for the individual stocks as compared to an index.

We take the simplest possible approach by first constructing an equal-weighted global portfolio from the available daily returns of the 82 countries listed in Table 1. Thus, the constructed index is a simple average of the countries already investigated and covers the same time period. Since the previous section's simulations suggested that the BNS jump measure has relatively sound properties, we adopt it for this validity check.

Table 8 presents the results. The first panel is copied from Table 3 and simply provides summary statistics for individual countries. The second panel reports on the BNS G jump measure for the global equal-weighted portfolio. The difference is indeed striking and completely supports the notion that jumps are largely diversifiable. Notice that the mean value of individual country BNS G measures is -6.799 while the equal-weighted index' mean measure is only -0.276. (Recall that large negative values of the BNS G measure reject the null hypothesis of no jumps.)

Other comparisons in Table 8 also support the same inference. For example, the index has much smaller standard deviation across months, only 0.787 versus 15.190 for countries on average. The minimum monthly value for the index is -9.527 as compared to -102.100 for countries.

Although the index displays much smaller jump measures, the average jump measure is still significantly negative. The T-value for the sample mean is even larger than for individual countries, -8.127 versus -5.232. This can be attributed to the index having more available observations than countries having on average and also to the much smaller variance of the index' jump measure across months. The bottom line here is that jumps are largely diversified away but not completely. Evidently, country jumps are mostly, but not entirely, idiosyncratic.

6 Conclusions

The extent of international correlation is very important for diversifying investors and government officials attempting to coordinate policies across borders. In this paper, we examine daily data for broad equity indexes from 82 countries and adopt several competing jump measures suggested in previous papers.

Returns are quite correlated internationally. Almost all the monthly return correlations are positive and almost 80% are statistically significant at the 1% level; this is for 3,321 individual correlation coefficients computed with returns from 82 countries. But jumps are less correlated. For some of the jump measures, the correlation is very weak and is statistically significant in only a few pairs of countries. This is based on the Barndorff-Nielsen and Shephard (BNS) (2006) jump measure. Simulations in Section 4 show that BNS performs very well; it does not indicate the presence of correlated jumps when there are actually none and it has good power to reject a false null hypothesis of no correlated jumps.

We also document two other interesting features of jumps: first, we display particular calendar periods that contribute the most to international jump correlations. Perhaps surprisingly, these are not usually the same months that are most influential for return correlations, though again, there are some differences among the jump measures. Second, we provide information on particular pairs of countries that are most influenced by extreme jumps.

Another surprise is that the most jump-correlated countries are larger and more developed and are mainly in Europe. Because jumps are more correlated among European neighbors, international diversification is less effective in that region. In contrast, jump co-movement is uncommon among developing countries or in non-European developed countries. The rarity of international correlation among jumps suggests they are mostly caused by local influences such as political events and not by common global factors such as energy prices.

Jumps estimated in our paper are jumps in equity returns, not real economic output or returns of other financial assets. Second, jumps in our paper are different from true crises. Although we find most jumps are not globally systematic, jumps are mostly found in Europe. A jump is an event of sharp increase or decrease in equity returns whereas true crises or contagion of downfall returns are a broader event. Our correlated jumps occur in a single day whereas contagion captures a spread of downfall over time. Furthermore, our jump includes both positive and negative jumps. We did not exclude positive jumps from our experiment. Future research should exclude them and thus the findings will be applied only to true crises.

Moreover, our approach can be readily adapted to ascertain whether jumps are entirely contemporaneous or whether they have a lead/lag relation on occasion. This interesting issue is left for future research. The bottom line is a bit of good news for investors. Although jumps are

frequent in all countries and are probably hard to predict, they are not as correlated internationally as returns themselves.

Returns seem to be more driven by global systematic influences while jumps are somewhat more idiosyncratic.

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Table 1

Cross-country return correlations

Product moment correlation coefficients are computed from dollar-denominated monthly and semiannual returns for all pairs of 82 countries. There are 3,321 pairs. For monthly observations, 3,321 coefficients are computed but the Greece/Zimbabwe correlation is missing from the semiannual calculations. The summary statistics below are computed across all the available coefficients. Sigma is the cross-coefficient standard deviation. T is the T -statistic assuming cross-coefficient independence (and hence may not be reliable.) MAD is the mean absolute deviation. The last two columns give the percentage of all correlation coefficients whose individual T -statistic exceeds 2.0 and 3.0, respectively.¹⁰ The data are extracted from DataStream, a division of Thomson Financial.

Mean	Median	Sigma	T	MAD	Skewness	Kurtosis	Maximum	Minimum	$T > 2$	$T > 3$
Panel A. Monthly returns, 3,321 correlation coefficients										
0.314	0.313	0.191	94.7	0.153	0.302	0.006	0.935	-0.238	78.0%	63.9%
Panel B. Semiannual returns, 3,320 correlation coefficients										
0.436	0.439	0.233	108.0	0.188	-0.166	-0.157	0.989	-0.420	61.5%	27.8%

¹⁰ The individual correlation coefficient is assumed to have a standard error equal to $1/(\text{Sample Size})^{1/2}$.

Table 2

The most influential periods for inter-country return correlations

An influential observation is defined here as the single calendar period that contributes the most to return correlations among each pair of countries. Periods with less than one percent of the most influential observations are omitted for reasons of space. The raw data are extracted from DataStream, a division of Thomson Financial.

	Number of Influential Observations	Number of Available Country Pairs	Percentage of Influential Observations
January/1975	8	136	5.88%
October/1987	64	378	16.9%
December/1993	44	1431	3.07%
January/1994	33	1485	2.22%
August/1998	239	2628	9.09%
January/2006	40	3240	1.23%
September/2008	62	3240	1.91%
October/2008	2457	3240	75.8%
February/2009	34	3240	1.05%

	Monthly Returns		
Month/Year	Number of Influential Observations	Number of Available Country Pairs	Percentage of Influential Observations
January/1975	8	136	5.88%
October/1987	64	378	16.9%
December/1993	44	1431	3.07%
January/1994	33	1485	2.22%
August/1998	239	2628	9.09%
January/2006	40	3240	1.23%
September/2008	62	3240	1.91%
October/2008	2457	3240	75.8%
February/2009	34	3240	1.05%

	Semiannual Returns		
Semester/Year	Number of Influential Observations	Number of Available Country Pairs	Percentage of Influential Observations
2/1985	3	253	1.19%
1/1986	3	276	1.08%
2/1993	67	1378	4.86%
1/1994	23	1485	1.55%
2/1997	76	2415	3.14%
1/1998	36	2628	1.37%
2/2006	38	3321	1.14%
2/2008	2822	3240	87.1%

Table 3

Summary Statistics for country averages of the Barndorff-Nielsen/Shephard (2006) jump measures

The BNS G jump measure described in Appendix B is computed from daily observations within available calendar months and semiannual periods for each of 82 countries. Summary statistics are computed from the resulting country time series of jump measures and are then averaged over countries. N is the average sample size in months. Sigma is the country average time-series standard deviation. T is the average T-statistic assuming time-series independence. MAD is the average mean absolute deviation. The maximum and minimum values are over individual months or semiannual periods. Observations with absolute values greater than 1,000 are deleted. Daily stock index data are extracted from DataStream, a division of Thomson Financial.

N	Mean	Median	Sigma	T	MAD	Skewness	Kurtosis	Maximum	Minimum
G Measure (Difference), Monthly									
252.1	-6.799	-0.994	15.190	-5.232	8.781	-5.177	47.160	0.364	-102.10
G Measure (Difference), Semiannual									
42.9	-6.093	-2.518	10.976	-3.398	6.879	-2.261	7.512	0.110	-44.31

Table 4

Cross-country correlations of BNS jump measures

Product moment correlation coefficients are computed across countries for the Barndorff-Nielsen and Shephard (2006) (BNS) G jump measures based on squared variation versus bipower variation differences. G is calculated both monthly and semiannually. There are 3,321 pairs of countries. For monthly observations, 3,321 coefficients are computed but the Greece/Zimbabwe correlation is missing from the semiannual calculations. The summary statistics below are computed across all the available correlation coefficients. Sigma is the cross-coefficient standard deviation. T is the T -statistic assuming cross-coefficient independence (and hence may not be reliable.) MAD is the mean absolute deviation. The last column gives the percentage of all correlation coefficients whose individual T -statistic exceeds 2.0.¹¹ The data are extracted from DataStream, a division of Thomson Financial.

Mean	Median	Sigma	T	MAD	Skewness	Kurtosis	Maximum	Minimum	$T > 2$
G Measure (Difference), Monthly									
0.0126	0.0009	0.0926	7.85	0.0681	0.996	3.455	0.598	-0.358	6.38%
G Measure (Difference), Semiannual									
0.0258	0.0049	0.2211	6.73	0.1693	0.534	0.979	0.884	-0.843	6.72%

¹¹ The individual correlation coefficient is assumed to have a standard error equal to $1/(\text{Sample Size})^{1/2}$.

Table 5

Influential periods for inter-country correlations of jumps using the BNS G measure

An influential observation is defined here as the single calendar period that contributes the most to the correlation of jumps between countries. The Barndorff-Nielsen and Shephard (2006) measures are calculated for each period and then correlated over time for all available pairs of countries. For each listed period, the table contains the percentage of country pairs for which that period was the single most influential contributor to the estimated jump correlation. To save space, periods are excluded if there are fewer than 100 available pairs of countries or have less than two percent of the most influential observations. The raw data are extracted from DataStream, a division of Thomson Financial.

Month/Year	Most Influential %
October/1973	3.810%
December/1974	2.500%
April/1975	2.941%
November/1978	8.824%
May/1980	2.632%
February/1983	4.211%
November/1983	6.667%
January/1991	6.554%
January/1994	2.155%
March/2009	2.161%
Semester/Year	Most Influential %
1/1973	21.91%
1/1974	7.500%
1/1988	7.308%
1/1991	11.11%
1/1994	6.061%
2/2000	6.524%
1/2002	7.359%
1/2006	7.377%

Table 6

Country pairs with large jump correlations according to the BNS G measure

The Barndorff-Nielsen and Shephard (2006) G measure is calculated for each period and then correlated over time for all available pairs of countries. The pairs of countries listed here exhibit jump measure correlations with T -statistics of at least 3.0. The raw data are extracted from DataStream, a division of Thomson Financial.

Monthly Jumps		Semiannual Jumps	
Belgium	France	Argentina	Bangladesh
Belgium	Ireland	Argentina	Kuwait
Belgium	Netherlands	Austria	Spain
Belgium	Switzerland	Bangladesh	Kuwait
Brazil	Lithuania	Belgium	Netherlands
Canada	Sweden	Belgium	Switzerland
Estonia	Israel	Canada	Sweden
Finland	Romania	Chile	India
France	Germany	China	Czech Republic
France	Hungary	China	Jordan
France	Italy	Czech Republic	Jordan
France	Netherlands	Denmark	Nigeria
France	United Kingdom	Denmark	Sweden
Germany	Hungary	Ecuador	Philippines
Germany	Italy	Finland	Ukraine
Germany	Netherlands	France	Portugal
Hong Kong	Norway	Germany	Netherlands
Hungary	Norway	Germany	Switzerland
Israel	Switzerland	Ghana	Luxembourg
Kenya	Oman	Ghana	Mauritius
Netherlands	Poland	Hungary	Poland
Netherlands	Switzerland	Hungary	Spain
Netherlands	United Kingdom	Indonesia	Morocco
Portugal	Switzerland	Kenya	Oman
Romania	Sweden	Kuwait	Oman
Slovenia	Tunisia	Kuwait	Romania
South Korea	Sweden	Kuwait	Sweden
		Malta	Nigeria
		Netherlands	Switzerland

Table 7

**Simulations to check the power of the BNS G test for
detecting correlated jumps**

The G jump measure derived by Barndorff-Nielsen and Shephard (2006) is described in Appendix B. Simulated bivariate returns have two components, a smooth Gaussian variation with unit variance (for both bivariate returns) and a specified smooth correlation plus a Gaussian jump component with specified frequency, intensity (strength), and co-movement probability, “Co-Prob.” Jump intensity is in multiple units of the smooth variation volatility.

Smooth correlation = 0.00				Smooth Correlation = 0.15			
Jump Strength	Jump Frequency	Jump Co-Prob	Test Power	Jump Strength	Jump Frequency	Jump Co-Prob	Test Power
5	0.01	0.30	27.90	5	0.01	0.30	29.60
5	0.03	0.30	26.60	5	0.03	0.30	26.10
5	0.01	0.60	52.40	5	0.01	0.60	55.10
5	0.03	0.60	43.50	5	0.03	0.60	42.80
5	0.01	0.90	65.90	5	0.01	0.90	69.50
5	0.03	0.90	57.20	5	0.03	0.90	52.80
15	0.01	0.30	44.90	15	0.01	0.30	43.90
15	0.03	0.30	29.10	15	0.03	0.30	29.70
15	0.01	0.60	76.10	15	0.01	0.60	77.90
15	0.03	0.60	49.70	15	0.03	0.60	47.80
15	0.01	0.90	89.90	15	0.01	0.90	90.00
15	0.03	0.90	74.10	15	0.03	0.90	73.20

Table 8
Barndorff-Nielsen/Shephard (2006) G jump measures, Country Averages vs. Equal-Weighted Global Index

The BNS G jump measures described in Appendix B is computed from daily observations within available calendar month for each of 82 countries and also for an equal-weighted index of all countries. Summary statistics are computed from the resulting time series of jump measures. N is the average sample size in months for individual countries and the number of months for the equal-weighted index. Sigma is the average time-series standard deviation. T is the average T -statistic assuming time-series independence. MAD is the average mean absolute deviation. Observations with absolute values greater than 1,000 are deleted. Daily stock index data are extracted from DataStream, a division of Thomson Financial.

N	Mean	Median	Sigma	T	MAD	Skewness	Kurtosis	Maximum	Minimum
BNS G, Monthly, Individual Countries (from Table 3)									
252.1	-6.799	-0.994	15.190	-5.232	8.781	-5.177	47.160	0.364	-102.100
BNS G, Monthly, Equal-Weighted Global Index									
538	-0.276	-0.0895	0.787	-8.127	0.416	-5.815	49.820	0.411	-9.527

Figure 1

The probability of rejecting a true null hypothesis that there are no jumps in either of two simulated return series. The two return series both have a smooth unit Gaussian variation and a specified level of correlation. The underlying jump measures are those derived by Barndorff-Nielsen and Shephard [2006] (BNS), Lee and Mykland [2008] (LM), Jiang and Oomen [2008] (JO), and Jacod and Todorov [2009] (JT). The type I rejection level is 5%. Simulations have 1,000 replications.

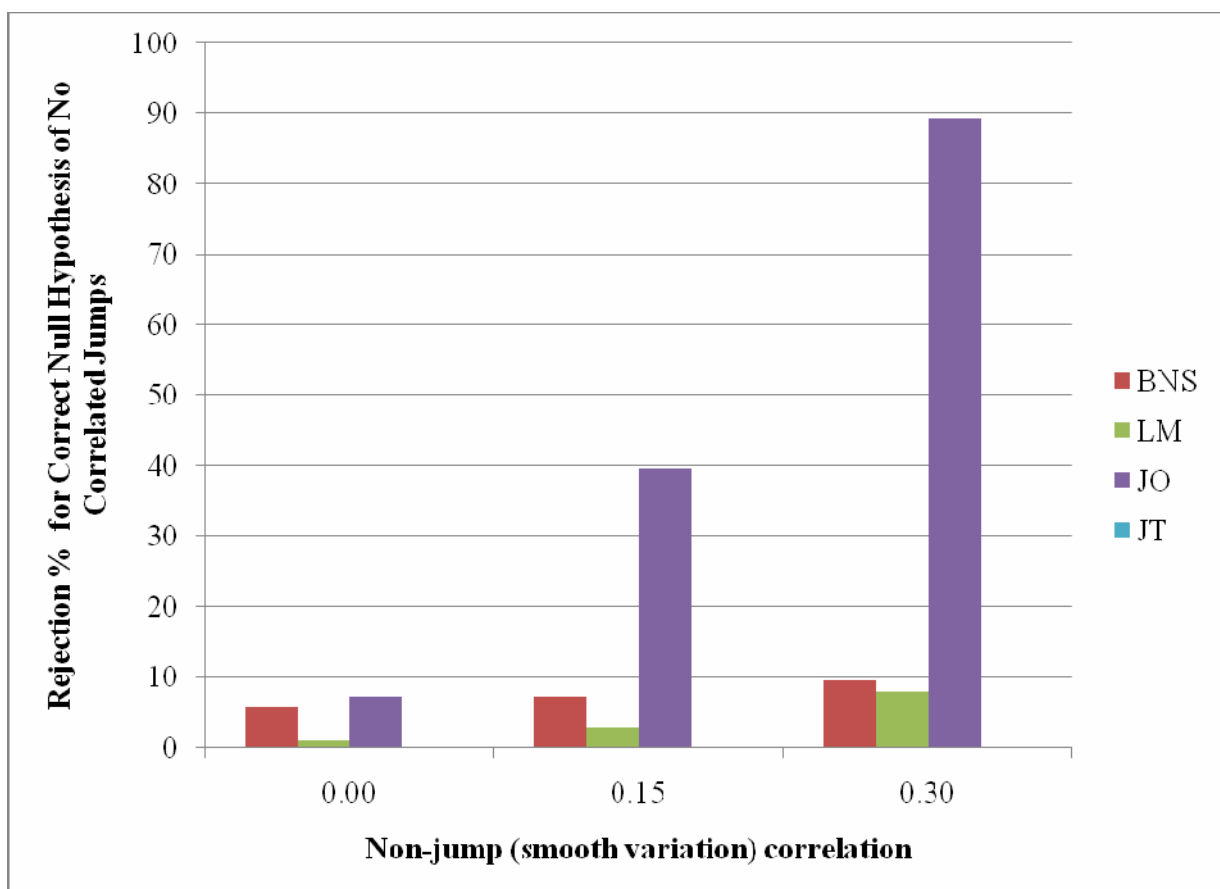


Figure 2

Smooth Correlation and Test Power Against a False Null Hypothesis of No Jump Co-Movement for Jump Intensity = 15, Jump Frequency = 0.02, and Jump Co-Movement Probability = 0.90. The two return series both have a smooth unit Gaussian variation and a specified level of correlation. The underlying jump measures are those derived by Barndorff-Nielsen and Shephard [2006] (BNS), Lee and Mykland [2008] (LM), Jiang and Oomen [2008] (JO), and Jacod and Todorov [2009] (JT). The type I rejection level is 5%. Simulations have 1,000 replications.

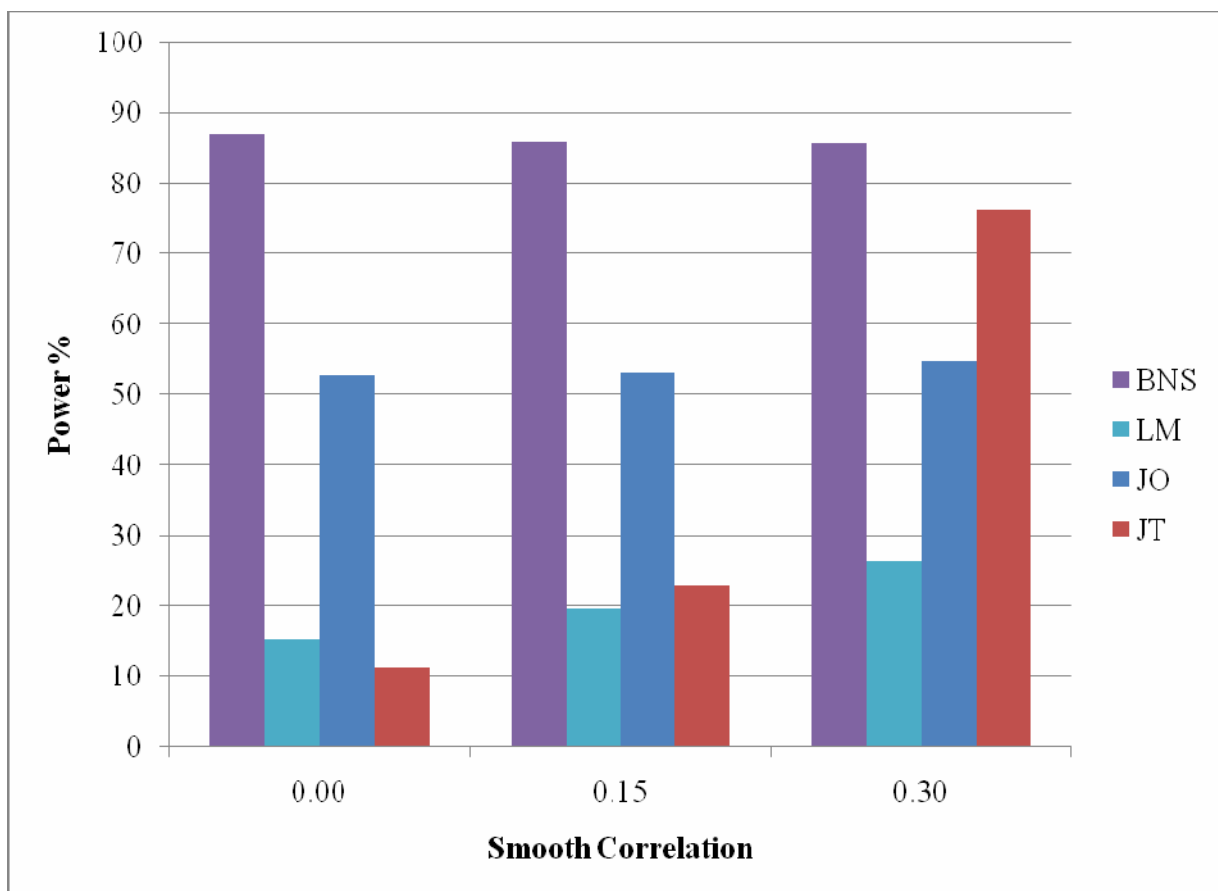


Figure 3

Jump Intensity and Test Power Against a False Null Hypothesis of No Jump Co-Movement for Smooth Correlation = 0.15, Jump Frequency = 0.02, and Jump Co-Movement Probability = 0.90. The two return series both have a smooth unit Gaussian variation and the specified level of correlation (0.15). The underlying jump measures are those derived by Barndorff-Nielsen and Shephard [2006] (BNS), Lee and Mykland [2008] (LM), Jiang and Oomen [2008] (JO), and Jacod and Todorov [2009] (JT). The type I rejection level is 5%. Simulations have 1,000 replications.

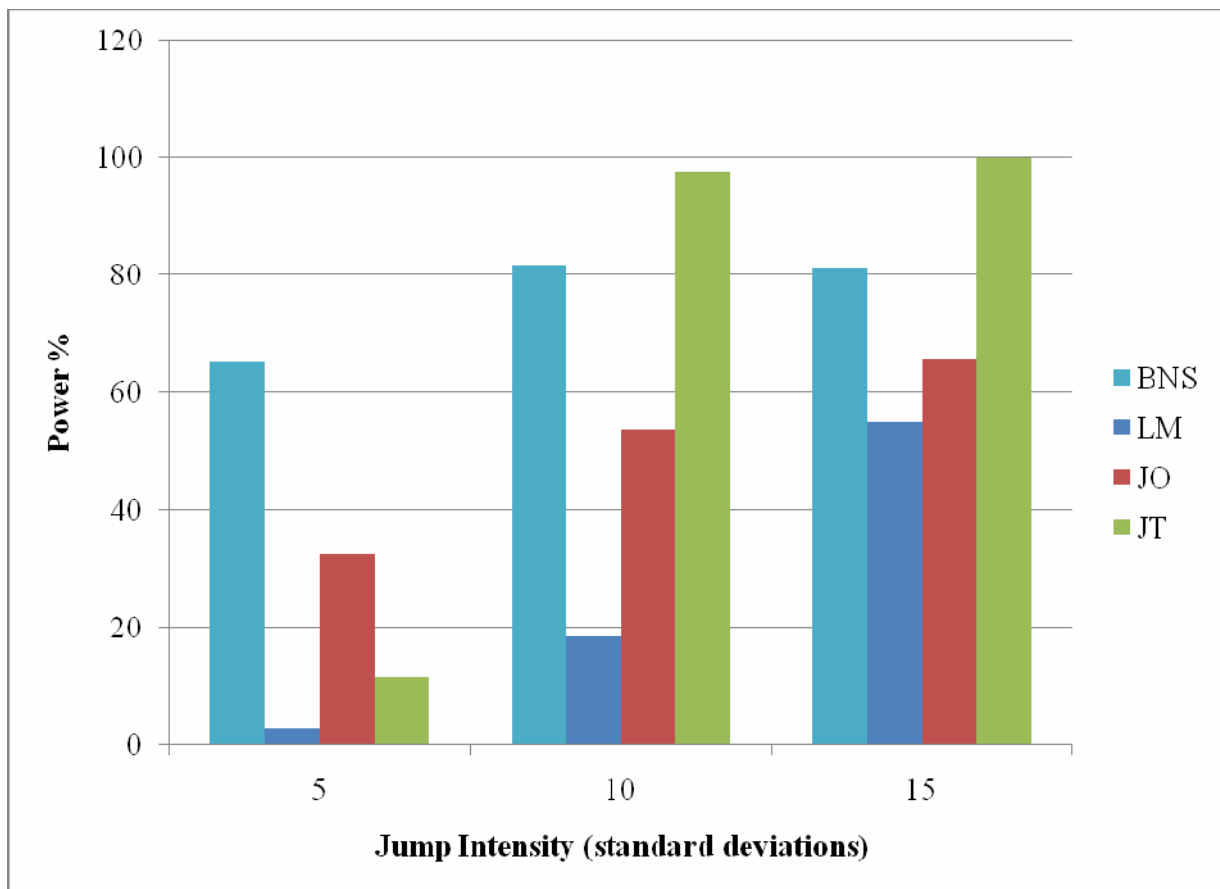
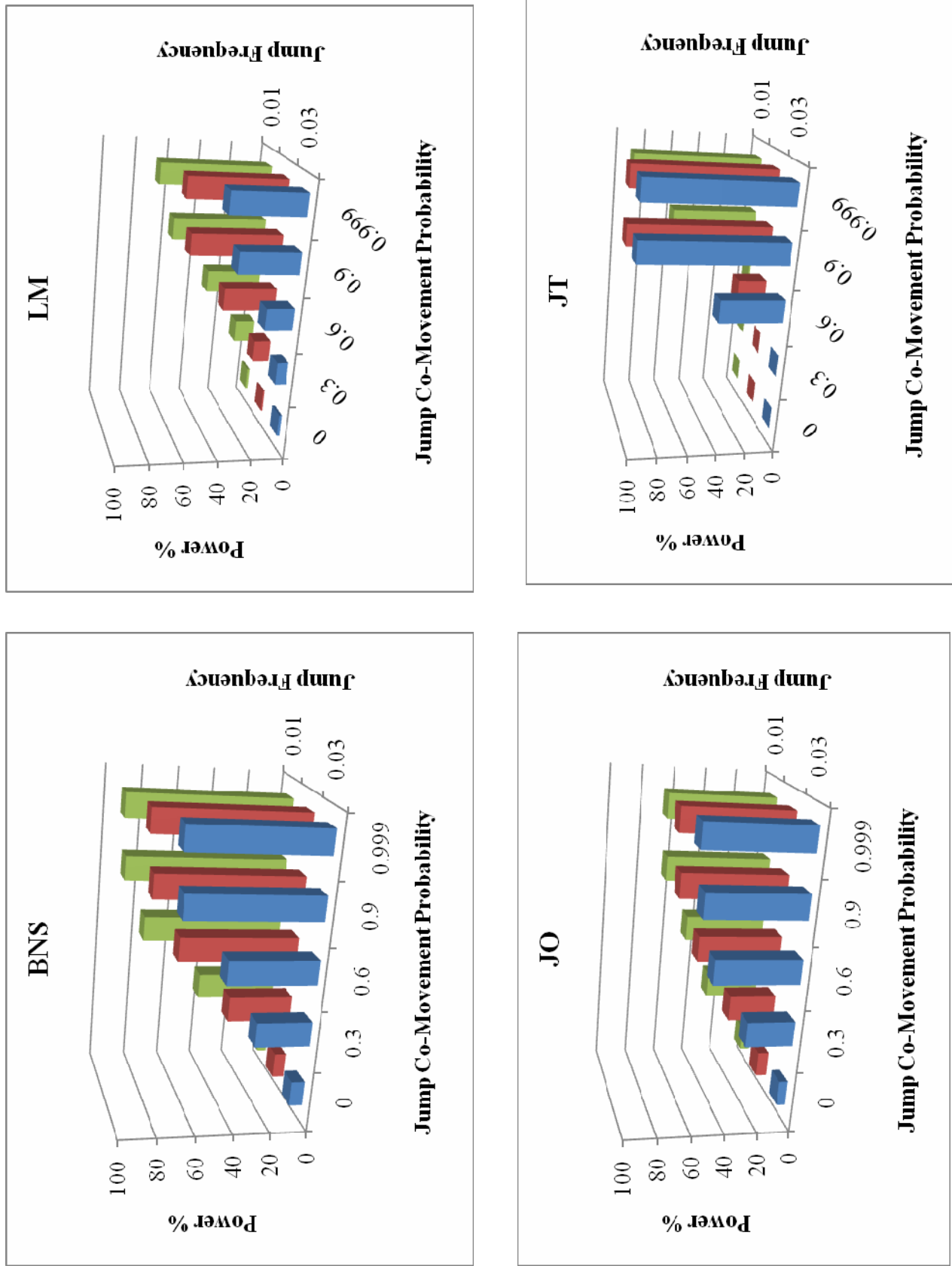


Figure 4

Jump Frequency, Co-Movement Probability and Test Power against a Null Hypothesis of No Jump Co-Movement for Smooth Correlation = 0.15 and Jump Intensity = 15. For a Co-Movement Probability of zero, the Null Hypothesis is true; otherwise, it is false. The two return series both have a smooth unit Gaussian variation and a specified level of correlation. The underlying jump measures are those derived by Barndorff-Nielsen and Shephard [2006] (BNS), Lee and Mykland [2008] (LM), Jiang and Oomen [2008] (JO), and Jacod and Todorov [2009] (JT). The type I rejection level is 5%. Simulations have 1,000 replications.



Appendix A

Country Index Sample Periods and Index Identification

Eighty-Two countries have index data availability from DataStream, a division of Thomson Financial. Some countries have several indexes and the index chosen has the longest period of data availability. All index values are converted into a common currency, the US dollar. An index with the designation “RI” is a total return index (with reinvested dividends.) The designation “PI” denotes a pure price index. When calculating log returns from the indexes, neither the beginning nor the ending index value can be identical to its immediately preceding index value; (this eliminates holidays, which vary across countries, and days with obviously stale prices.)

Country	DataStream Availability		Index Identification	DataStream Mnemonic
	Begins	Ends		
Argentina	2-Aug-93	26-Oct-09	ARGENTINA Merval	ARGMERV(PI)~US
Australia	1-Jan-73	26-Oct-09	AUSTRALIA-DS MARKET	TOTMAU\$(RI)
Austria	1-Jan-73	26-Oct-09	AUSTRIA-DS Market	TOTMKOE(RI)~US
Bahrain	31-Dec-99	26-Oct-09	DOW JONES BAHRAIN	DJBAHR\$(PI)
Bangladesh	1-Jan-90	26-Oct-09	BANGLADESH SE ALL SHARE	BDTALSH(PI)~US
Belgium	1-Jan-73	26-Oct-09	BELGIUM-DS Market	TOTMKBG(RI)~US
Botswana	29-Dec-95	26-Oct-09	S&P/IFCF M BOTSWA0.	IFFMBOL(PI)~US
Brazil	7-Apr-83	26-Oct-09	BRAZIL BOVESPA	BRBOVES(PI)~US
Bulgaria	20-Oct-00	26-Oct-09	BSE SOFIX	BSSOFIX(PI)~US
Canada	31-Dec-64	26-Oct-09	S&P/TSX COMPOSITE INDEX	TTOCOMP(RI)~US
Chile	2-Jan-87	26-Oct-09	CHILE GENERAL (IGPA)	IGPAGEN(PI)~US
China	3-Apr-91	26-Oct-09	SHENZHEN SE COMPOSITE	CHZCOMP(PI)~US
Colombia	10-Mar-92	26-Oct-09	COLOMBIA-DS Market	TOTMKCB(RI)~US
Côte d'Ivoire	29-Dec-95	26-Oct-09	S&P/IFCF M COTE D'IVOIRE	IFFMCIL(RI)~US
Croatia	2-Jan-97	26-Oct-09	CROATIA CROBEX	CTCROBE(PI)~US
Cyprus	3-Sep-04	26-Oct-09	CYPRUS GENERAL	CYPMAPM(PI)~US
Czech Republic	9-Nov-93	26-Oct-09	CZECH REP.-DS NON-FINICIAL	TOTLICZ(RI)~US
Denmark	31-Dec-69	26-Oct-09	MSCI DENMARK	MSDNNMKL(RI)~US
Ecuador	2-Aug-93	26-Oct-09	ECUADOR ECU (US)	ECUECUI(PI)
Egypt	2-Jan-95	26-Oct-09	EGYPT HERMES FINANCIAL	EGHFINC(PI)~US
Estonia	3-Jun-96	26-Oct-09	OMX TALLINN (OMXT)	ESTALSE(PI)~US
Finland	2-Jan-91	26-Oct-09	OMX HELSINKI (OMXH)	HEXINDX(RI)~US
France	1-Jan-73	26-Oct-09	FRANCE-DS Market	TOTMKFR(RI)~US
Germany	31-Dec-64	26-Oct-09	DAX 30 PERFORMANCE	DAXINDX(RI)~US

Ghana	29-Dec-95	26-Oct-09	S&P/IFCF M GHAA0.	IFFMGHL(PI)~US\$
Greece	26-Jan-06	26-Oct-09	ATHEX COMPOSITE	GRAGENL(RI)~US\$
Hong Kong	2-Jan-90	26-Oct-09	HANG SENG	HNGKNGI(RI)~US\$
Hungary	2-Jan-91	26-Oct-09	BUDAPEST (BUX)	BUXINDX(PI)~US\$
Iceland	31-Dec-92	26-Oct-09	OMX ICELAND ALLSHARE	ICEXALL(PI)~US\$
India	2-Jan-87	26-Oct-09	INDIA BSE (100) NATIONAL	IBOMBSE(PI)~US\$
Indonesia	2-Apr-90	26-Oct-09	INDONESIA-DS Market	TOTMKID(RI)~US\$
Ireland	1-Jan-73	26-Oct-09	IRELAND-DS MARKET	TOTMIR\$(RI)
Israel	23-Apr-87	26-Oct-09	ISRAEL TA 100	ISTA100(PI)~US\$
Italy	1-Jan-73	26-Oct-09	ITALY-DS MARKET	TOTMIT\$(RI)
Jamaica	29-Dec-95	26-Oct-09	S&P/IFCF M JAMAICA	IFFMJAL(PI)~US\$
Japan	1-Jan-73	26-Oct-09	TOPIX	TOKYOSE(RI)~US\$
Jordan	21-Nov-88	26-Oct-09	AMMAN SE FINANCIAL MARKET	AMMANFM(PI)~US\$
Kenya	11-Jan-90	26-Oct-09	KENYA NAIROBI SE	NSEINDX(PI)~US\$
Kuwait	28-Dec-94	26-Oct-09	KUWAIT KIC GENERAL	KWKICGN(PI)~US\$
Latvia	3-Jan-00	26-Oct-09	OMX RIGA (OMXR)	RIGSEIN(RI)~US\$
Lebanon	31-Jan-00	26-Oct-09	S&P/IFCF M LEBANON	IFFMLEL(PI)~US\$
Lithuania	31-Dec-99	26-Oct-09	OMX VILNIUS (OMXV)	LVILSE(RI)~US\$
Luxembourg	2-Jan-92	26-Oct-09	LUXEMBURG-DS MARKET	LXTOTMK(RI)~US\$
Malaysia	2-Jan-80	26-Oct-09	KLCI COMPOSITE	KLPCOMP(PI)~US\$
Malta	27-Dec-95	26-Oct-09	MALTA SE MSE -	MALTAIX(PI)~US\$
Mauritius	29-Dec-95	26-Oct-09	S&P/IFCF M MAURITIUS	IFFMMAL(PI)~US\$
Mexico	4-Jan-88	26-Oct-09	MEXICO IPC (BOLSA)	MXIPC35(PI)~US\$
Morocco	31-Dec-87	26-Oct-09	MOROCCO SE CFG25	MDCFG25(PI)~US\$
Namibia	31-Jan-00	26-Oct-09	S&P/IFCF M NAMIBIA	IFFMNAL(PI)~US\$
Netherlands	1-Jan-73	26-Oct-09	NETHERLAND-DS Market	TOTMKNL(RI)~US\$
New Zealand	4-Jan-88	26-Oct-09	NEW ZEALAND-DS MARKET	TOTMNZ\$(RI)
Nigeria	30-June-95	26-Oct-09	S&P/IFCG D NIGERIA	IFGDNGL(PI)~US\$
Norway	2-Jan-80	26-Oct-09	NORWAY-DS MARKET	TOTMNW\$(RI)
Oman	22-Oct-96	26-Oct-09	OMAN MUSCAT SECURITIES MKT.	OMANMSM(PI)~US\$
Pakistan	30-Dec-88	26-Oct-09	KARACHI SE 100	PKSE100(PI)~US\$
Peru	2-Jan-91	26-Oct-09	LIMA SE GENERAL(IGBL)	PEGENRL(PI)~US\$
Philippines	2-Jan-86	26-Oct-09	PHILIPPINE SE (PSEI)	PSECOMP(PI)~US\$
Poland	16-Apr-91	26-Oct-09	WARSAW GENERALINDEX	POLWIGI(PI)~US\$
Portugal	5-Jan-88	26-Oct-09	PORTUGAL PSI GENERAL	POPSIGN(PI)~US\$
Romania	19-Sep-97	26-Oct-09	ROMANIA BET (L)	RMBETRL(PI)~US\$
Russia	1-Sep-95	26-Oct-09	RUSSIA RTS INDEX	RSRTSIN(PI)~US\$
Saudi Arabia	31-Dec-97	26-Oct-09	S&P/IFCG D SAUDI ARABIA	IFGDSB\$(RI)

Singapore	1-Jan-73	26-Oct-09	SINGAPORE-DS MARKET EX TMT	TOTXTSG(RI)~US\$
Slovakia	14-Sep-93	26-Oct-09	SLOVAKIA SAX 16	SXSAX16(PI)~US\$
Slovenia	31-Dec-93	26-Oct-09	SLOVENIAN EXCH. STOCK (SBI)	SLOESBI(PI)~US\$
South Africa	1-Jan-73	26-Oct-09	SOUTH AFRICA-DS MARKET	TOTMSA\$(RI)
South Korea	31-Dec-74	26-Oct-09	KOREA SE COMPOSITE (KOSPI)	KORCOMP(PI)~US\$
Spain	2-Jan-74	26-Oct-09	MADRID SE GENERAL	MADRIDI(PI)~US\$
Sri Lanka	2-Jan-85	26-Oct-09	COLOMBO SE ALLSHARE	SRALLSH(PD)~US\$
Sweden	28-Dec-79	26-Oct-09	OMX STOCKHOLM (OMXS)	SWSEALI(PI)~US\$
Switzerland	1-Jan-73	26-Oct-09	SWITZ-DS Market	TOTMKS\$(RI)~US\$
Taiwan	31-Dec-84	26-Oct-09	TAIWAN SE WEIGHTED	TAIWGHT(PD)~US\$
Thailand	2-Jan-87	26-Oct-09	THAILAND-DS MARKET	TOTMTH\$(RI)
Trinidad	29-Dec-95	26-Oct-09	S&P/IFCF M TRINIDAD & TOBAGO	IFFMTTL(PD)~US\$
Tunisia	31-Dec-97	26-Oct-09	TUNISIA TUNINDEX	TUTUNIN(PD)~US\$
Turkey	4-Jan-88	26-Oct-09	ISE TIOL 100	TRKISTB(PD)~US\$
Ukraine	30-Jan-98	26-Oct-09	S&P/IFCF M UKRAINE	IFFMURL(PD)~US\$
Utd. Arab Emirates	1-Jun-05	26-Oct-09	MSCI UAE	MSUAEI\$
United Kingdom	1-Jan-65	26-Oct-09	UK-DS MARKET	TOTMUK\$(RI)
United States	4-Jan-68	26-Oct-09	S&P 500 COMPOSITE	S&PCOMP(RI)~US\$
Venezuela	2-Jan-90	26-Oct-09	VENEZUELA-DS MARKET	TOTMVE\$(RI)
Zimbabwe	6-Apr-88	6-Oct-06	ZIMBABWE INDUSTRIALS	ZIMINDS(PI)

Appendix B

Jump Measures

1. Barndorff-Nielson and Shephard (2006)

Barndorff-Nielson and Shephard (2006), hereafter BNS, develop a test statistic based on comparing bipower variation with squared variation. To understand their test, consider the following notation (that we will adopt throughout the paper.)

t , subscript for day

T_k , the number of days in subperiod k

K , the total number of available subperiods

$R_{i,t,k}$, the return (log price relative including dividends, if any)

for asset i on day t in subperiod k

The BNS bipower and squared variations are defined as follows:

$B_{i,k}$, bipower variation,

$$B_{i,k} = \frac{1}{T_k - 1} \sum_{t=2}^{T_k} |R_{i,t,k} \parallel R_{i,t-1,k} |$$

$S_{i,k}$, squared variation

$$S_{i,k} = \frac{1}{T_k} \sum_{t=1}^{T_k} (R_{i,t,k})^2.$$

BNS propose two variants of the quadratic versus bipower variation measure, a difference and a ratio. If the non-jump part of the process has constant drift and volatility, they show that $(\pi/2)B_{i,k}$ is asymptotically equal to the non-jump squared variation. Consequently, a test for the null hypothesis of no jumps can be based on $(\pi/2)B_{i,k} - S_{i,k}$, or $(\pi/2)B_{i,k}/S_{i,k} - 1$. Under the null hypothesis, the standard deviations of this difference and ratio depend on the ‘‘quarticity’’ of the process, which they show can be estimated by

$$Q_{i,k} = \frac{1}{T_k - 3} \sum_{t=4}^{T_k} |R_{i,t,k} \parallel R_{i,t-1,k} \parallel R_{i,t-2,k} \parallel R_{i,t-3,k} |.$$

Define the constant $\upsilon = (\pi^2/4) + \pi - 5$. Then the difference and ratio statistics,

$$G_{i,k} = \frac{(\pi/2)B_{i,k} - S_{i,k}}{\sqrt{\upsilon(\pi/2)^2 Q_{i,k}}}$$

$$H_{i,k} = \frac{(\pi/2)(B_{i,k}/S_{i,k})-1}{\sqrt{vQ_{i,k}/B_{i,k}^2}}$$

are both asymptotically unit normal.

These statistics have intuitive appeal because the squared variation ($S_{i,k}$) should be relatively small if there is smooth variation, as with the normal distribution. On the other hand, if the price jumps on some days, those jumps are magnified by squaring and the statistics above should be small. Small values of G and H relative to the unit normal reject the null hypothesis of no jumps.

From our perspective, these statistics also have the benefit that they can be computed sequentially over calendar periods of various lengths.¹² For example, beginning with daily observations, they can be computed monthly or semiannually for each asset. Subsequently, the resulting monthly or semiannual statistics can be correlated across assets to detect whether jumps are related. When the assets are broad country indexes, this provides the opportunity to test for internationally correlated jumps. For example, to check whether countries j and i exhibit correlated jumps, one can calculate the correlation over $k = 1, \dots, K$ between $G_{i,k}$ and $G_{j,k}$.

In previous papers, Huang and Tauchen (2005) and Andersen, Bollerslev, and Diebold (2007) adopt the BNS method and develop a Z statistic for jumps using tri-power quarticity. The latter paper also develops a “staggered” version of bi-power variation to tackle microstructure noise that induces autocorrelation in the high-frequency returns. Zhang, Zhou, and Zhu (2009) use the BNS method to identify jump risk of individual firms from high-frequency equity prices in order to explain credit default swap premiums.

2. Lee and Mykland (2008)

Like BNS, Lee and Mykland (2008), (hereafter LM), base their test on bipower variation, but it is employed differently. Bipower variation is used as an estimate of the instantaneous variance of the continuous (non-jump) component of prices. LM recommend its computation with data preceding a particular return observation being tested for a jump and the resulting test

¹² There is a caveat. BNS assume that the non-jump part of the process has constant mean and volatility, which rules out phenomena such as reductions in volatility with increasing prices, and vice versa. This should be only a minor annoyance, though, when the calendar period is fairly short.

statistic is $L = \frac{R_{t,t+1,k}}{\sqrt{B_{t,t,k}}}$. Under the null hypothesis of no jump at $t+1$, LM show that $\sqrt{L} \frac{L}{\sqrt{L}}$ converges to a unit normal.¹³ In addition, if there is a jump at $t+1$, $\sqrt{L} \frac{L}{\sqrt{L}}$ is equal to a unit normal plus the jump scaled by the standard deviation of the continuous portion of the process.

LM stress that high-frequency data minimizes the likelihood that a jump will be misclassified. A test might fail to detect an actual jump at $t+1$ or it might spuriously “detect” one at $t+1$ even though it has not occurred. Over a sequence of periods, tests might also fail to detect any jumps even when one or more have occurred or they may falsely indicate that one or more have occurred. LM provide explicit expressions for the probabilities of such misclassifications.

Unfortunately, we do not possess international stock index data at frequencies higher than daily, so we will have to live with possible misclassifications. But since our purpose is mainly to find evidence about the international correlation of jumps rather than the unambiguous identification of a jump at a particular time, occasional misclassification is less of an issue. We also finesse the problem to some extent by using a non-parametric enumeration of the test statistic.

Since the LM test statistic has the return in the numerator, it would not be appropriate to simply correlate it across countries. The resulting statistic would be polluted by the normal non-jump correlation of returns. Instead, we first identify periods when the statistic is significantly non-normal, thus indicating a likely jump. Using a simple contingency table test, we then ascertain whether these periods are related across each pair of countries.

3. Jiang and Oomen (2008)

Jiang and Oomen (2008) (hereafter JO) devise a test inspired by the variance swap, a contract whose payoff depends on the realized squared returns of an asset at a particular frequency and over a specified horizon. They cite Neuberger (1994) for the continuous replication strategy using a “log contract.” This leads to the idea of swap-based variation, defined during period k with our usual notation as

¹³ For short periods, the mean return is negligible and is ignored in the simplest version of the LM test.

$$SW_{i,k} = \frac{1}{T_k} \sum_{t=1}^{T_k} (R_{i,t,k}^{ar} - R_{i,t,k}^{ln})$$

where the new superscripts “ar” and “ln” denote, respectively, the arithmetic return (P_t/P_{t-1}) and the log return $\ln(P_t/P_{t-1})$ with P_t as the price (or index value) at time t . The squared variation, already earlier in the Appendix when introducing the BNS statistic, is compared with the swap variation in several proposed test statistics based on $SW_{i,k} - S_{i,k}$, or $\ln(SW_{i,k}) - \ln(S_{i,k})$, or a ratio test based on $1 - S_{i,k}/SW_{i,k}$.¹⁴

JO argue that these statistics are more sensitive to jumps than the BNS and LM statistics described above because they exploit the influence of jumps on the third and higher order moments rather than exclusively on the second moment. JO provide simulations that seem to demonstrate that their statistic performs comparatively well.

Their theorem 2.1, p. 354, states that any of the proposed test statistics are asymptotically normal with mean zero under the null hypothesis of no jumps during k . The variances of the tests are unknown but can be estimated by multi-power variations that are consistent and robust to jumps during the estimation period.

For our purpose of correlating jumps across international markets, we do not even need to estimate the variances of the JO tests provided that the variance is constant over time, (though different across countries.) Also, we use just the second of JO’s three proposed statistics, involving logs of SW and S, simply on the grounds that logs attenuate outliers.

4. Jacod and Todorov (2009)

The tests devised by Jacod and Todorov (2009), hereafter JT, seem to perfectly fit our goal here because they are explicitly intended to detect the common arrival of jumps in two time series. JT actually develop two statistics, one for the null hypothesis that jumps arrive at the same instant in both time series (“joint” jumps) and another for the null hypothesis that jumps arrive in both time series but not at the same instant (“disjoint” jumps.)

Within a finite subperiod k , the first JT test asks whether $R_{i,t,k}$ and $R_{j,t,k}$ ($i \neq j$) both experience a jump on the same date t , for at least one $t \in k$. Given a pair of countries, one can compute the first JT test for a sequence of subperiods, $k = 1, \dots, K$, and tabulate the frequency of

¹⁴ Because JO intend their estimator for very high frequency data, the means are ignored. De-meanded data can be used for lower frequency data.

common jumps. This provides a measure of jump co-movement frequency. One can also use the second test to measure the arrival frequency of disjoint jumps that arrive on different dates but both within the same subperiod k .

The JT tests require that at least one jump occurs in both countries i and j in at least one interval $k = 1, \dots, K$. So, the first step in implementing their procedure is to throw out countries that never experience a jump during the sample. The BNS statistics could be used for this purpose. In other words, one could first compute the $G_{i,k}$ and $G_{j,k}$ (or $H_{i,k}$ and $H_{j,k}$) according to the expressions described above in the Appendix, check whether the means of both G 's (or both H 's) fall below some pre-specified threshold, such as the .01 fractile of the unit normal, and retain for the JT test only those pairs of countries for which the threshold is breached. For monthly periods, this approach seems unnecessary because failure to reject both the “joint” and the “disjoint” jump null hypotheses is tantamount to accepting the hypothesis that the month contains no jump of any kind.

For month k , the monthly return is simply the sum of daily (log) returns,

$$R_{i,k} = \sum_{t=1}^{T_k} R_{i,t,k},$$

denote as for country i and month k which contains T_k daily returns. Inserting our return notation in JT's functional representation, we first define a functional sum as

$$V(f, \lambda) = \sum_{m=1}^{[K/\lambda]} f\left(\sum_{l=\lambda(m-1)+1}^{\lambda m} (R_{i,l})\right)$$

for integer $\lambda \geq 1$, where $[\cdot]$ denotes the integer part or the argument and the function $f(x)$ takes on two forms: a cross-product, $f_{ij} = (x_i x_j)^2$ and a quartic, $g_i = x_i^4$. For $\lambda = 1$, $V(f, 1)$ is simply the sum of the functions of individual monthly returns. For $\lambda > 1$, JT recommend the choices of $\lambda = 2$ or $\lambda = 3$; we will adopt the former and retain it throughout because this maximizes the number of terms in the sum, i.e., in $[K/\lambda]$. Consequently, in our application of the JT tests, the second sum in $V(f, 2)$ will involve bi-monthly returns.

The JT test statistic for simultaneous (“joint”) jumps is given by

$$\Phi_{i,j}^{(J)} = \frac{V(f_{i,j}, 2)}{V(f_{i,j}, 1)},$$

and for “disjoint” jumps (non-simultaneous ones), the statistic is

$$\Phi_{i,j}^{(D)} = \frac{V(f_{i,j}, I)}{\sqrt{V(g_i, I)V(g_j, I)}}.$$

JT derive asymptotic properties for both statistics. When there are joint jumps, $\Phi^{(J)}$ converges to a Gaussian with mean 1.0 and variance given by their equation 4.1, (p. 1800.) When there are only joint jumps, $\Phi^{(D)}$ also converges to 1.0, and it generally converges to a positive value when there are both joint and disjoint jumps. When there are uniquely disjoint jumps, $\Phi^{(D)}$ converges to zero and $\Phi^{(J)}$ converges to 2.0. If there are no jumps at all, $\Phi^{(D)}$ should also converge to zero, so a test of $\Phi^{(D)}$ against a null hypothesis of zero (and perhaps $\Phi^{(J)}$ against a null hypothesis of 2.0) should be rejected when jumps are joint and thus not idiosyncratic.