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Dynamic carbon emission management

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Abstract

The control of carbon emissions by policymakers poses the corporate challenge of developing an optimal carbon management policy. We provide a unified model that characterizes how firms should optimally manage emissions through production, green investment, and the trading of carbon credits. We show that carbon pricing reduces firms' emissions but also induces firms to tilt towards more immediate yet transient types of green investment—such as abatement as opposed to innovation—as it becomes costlier to comply. Green innovation subsidies mitigate this effect and complement carbon pricing in ensuring innovation-driven sustainability. Perhaps surprisingly, we show that carbon regulation need not reduce firm value.

Keywords: Carbon pricing, Carbon Emissions, Carbon Abatement, Green Innovation, Sustainability.

JEL Classification Numbers: G30; G31; G12; D62; O33

Non-technical summary

To limit global warming, several counties around the world have adopted carbon pricing mechanisms—being carbon taxes or emissions trading systems. A common feature of carbon pricing mechanisms is that they impose additional costs on businesses: Every tonne of carbon dioxide produced through industrial processes needs to be paid for, either by surrendering carbon credits (which are costly) or by paying a tax on it. In response to carbon pricing, firms may pursue several strategies to reduce their emissions; for instance, they can adjust their scale of production, engage in green investment of various horizon and cost profiles, and manage their carbon credits. A relevant question in the climate debate is which mix of strategies firms will follow. This paper tackles this question from a theoretical angle.

As a novel distinction, our theoretical framework acknowledges that green investment projects can be of different types. At one extreme, abatement projects aim at offsetting some of the emissions coming from firms' production processes. While reducing their carbon footprint in the short term, abatement does not transform firms' production technologies. At the other extreme, green innovation stirs the transition to novel, cleaner technologies. While having a long-term impact on sustainability, green innovation is costlier than abatement, has a long gestation period, and has an uncertain outcome.

Our analysis illustrates that carbon pricing effectively leads firms to decrease their emissions, both by reducing the scale of production and by engaging in green investment. Yet, as it becomes costlier to comply, polluting firms tilt their green investment mix towards more immediate yet short-lived options—such as solely reducing emissions (abatement) instead of investing in green innovation. The reason is that abatement immediately reduces a firm's expected cost of carbon pricing, whereas green innovation has an uncertain and delayed outcome. Shifting to abatement, however, can slow down the introduction of greener technologies. Regulators could take this into account by complementing carbon pricing with subsidies to green innovation. Such subsidies not only stir a greater engagement in green investment, but also tilt the mix in favor of green innovation.

When specifically looking at emissions trading systems, the analysis shows that firms become less committed to curbing their emissions when holding large balances of carbon credits. The reason is that firms seek to minimize their need to buy credits by managing their corporate policies in a precautionary fashion. As their balance of carbon credits shrinks, firms scale down production and increase their green investment to reduce their emissions and, thus, the need of buying credits. This result suggests that limiting the distribution of free carbon credits can make firms more committed to becoming green.

Overall, the analysis suggests that carbon regulation is not necessarily value-decreasing for businesses. Despite the long-standing perception of a conflict between the interests of businesses and of environmental regulators, empirical research documents that the effect of climate regulation is heterogeneous across firms. Our analysis provides theoretical arguments supporting this evidence. Indeed, the sale of carbon credit, subsidies to green firms, and innovation can each increase the value of firms that are sufficiently committed to becoming sustainable.

1 Introduction

Keeping carbon emissions under control is one of the greatest challenges of our time. In this context, economic research started addressing the questions concerning the macroeconomic implications of climate change as well as the welfare consequences of different regulatory regimes.[1] Perhaps surprisingly, while streamlining the firm's problem to solve for aggregate implications, the existing literature has not yet addressed a fundamental question that naturally surfaces from the *corporate* perspective: Considering all the tools available to businesses, how should firms best manage their carbon emissions? The urgency in the corporate community to understand carbon trade-offs and explicitly address this question is palpable: Looking ahead, firms are increasingly implementing carbon accounting and green investments to either fulfill or comply with ambitious climate goals.[2]

Starting from the canonical premise in finance that corporate decisions should maximize shareholder value, we thus study the trade-offs posed by carbon regulation on firms. We develop a novel theoretical framework that characterizes firms' optimal carbon management policy under alternative regulatory systems as well as the ensuing implications for asset prices. Corporate carbon management is dynamic in nature, as virtually all forms of carbon regulation track the stock of a firm's carbon emissions over time. Accordingly, we propose a continuous-time dynamic model in which the firm best handles carbon emissions by balancing the scale of production, the engagement in immediate abatement measures to offset emissions, and the investment in innovative solutions to permanently switch to more sustainable technologies. Importantly, unlike previous studies, firms are not exogenously polluters or cleansers: In our model, the firms' carbon footprint is endogenous and may switch sign over time as management maximizes shareholder value.

A notable feature of our framework is being broad enough to characterize optimal carbon management policies under the three most prevalent regulatory frameworks: laissez-faire, a carbon tax system, and a carbon credit trading system (henceforth, carbon trading system). Under the carbon tax system, a central authority sets a predetermined price that emitters pay for each tonne of emissions. Under the carbon trading system, costly carbon credits give firms the right to deploy a fixed volume of carbon emissions into the atmosphere and, thus, produce.[3]

[1]See, for instance, the seminal models by Acemoglu, Aghion, Bursztyn, and Hemous (2012) and Acemoglu, Akcigit, Hanley, and Kerr (2016), or the policy considerations by Pindyck (2021) and Stavins (2022).

[2]See the reports by McKinsey & Company (2021, 2022) and The World Bank (2022).

[3]In different jurisdictions, this scheme has been labeled as "emissions trading scheme" (ETS) or "cap-and-

As carbon credits are tradable, firms in need of credits can buy them in the carbon markets, whereas firms with excess of credits may instead choose to sell them. A unifying feature across carbon pricing systems is then that they impose additional costs on firms, incentivizing them to reduce their carbon footprint. Firms continuously evaluate their optimal policies to curtail emissions by managing production, investing in long-term green innovation, engaging in more immediate abatement projects, or trading carbon credits.

We start by investigating the optimal carbon emission management under the carbon tax system. Compared to laissez-faire, we prove that such system is effective at reducing carbon emissions. A twofold strategy supports this result: (1) The firm produces less compared to laissez-faire, which naturally curbs its gross emissions, and (2) the firm engages in green investment projects, to clean up at least part of its emissions. Yet, the carbon tax system also affects the firm's optimal *mix* of green investment. As the carbon tax increases, we show that the firm puts more emphasis on abatement—which, while immediately reducing carbon emissions, is transient as it does not transform the production technology—and less emphasis on green innovation—which, while ensuring more sustainable production in the long run, has a longer gestation period and an uncertain outcome. Because abatement is more effective than innovation in reducing net emissions in the short-term—and, thus, the firm's carbon tax liability—the firm shifts to abatement and away from innovation as it becomes costlier to comply.

We next investigate the optimal carbon emission management under the carbon trading system. The firm optimally sells credits on carbon markets when its balance exceeds an endogenous threshold, and buys credits when running out of them. Importantly, the firm manages production and green investment as a function of carbon credits in a precautionary fashion: The firm aims at preserving its stock of credits to avoid resorting to the costly carbon credit market.[4] When the firm has a low credit balance, the firm optimally cuts on production to reduce its consumption of credits and, additionally, increases its investment in abatement and green innovation. Conversely, when the firm has a large carbon credit balance, the firm increases production—and, thus, its emissions—and reduces its engagement in green investment. The model then reveals that having a large balance of carbon credits actually *reduces* the firm's commitment to curb emissions. Consistent with De Jonghe, Mulier, and Schepens (2020), a firm

trade." The most established ETS is the European Union's one (EU ETS). Other major ETS are those in China, the UK, Switzerland, and South Korea. In North America, there are subregional schemes in California, Quebec, and in Northeastern United States (so called "Regional Greenhouse Gas Initiative")

[4] This result aligns with previous findings in dynamic inventory models, such as Bolton, Chen, and Wang (2011) and Décamps, Mariotti, Rochet, and Villeneuve (2011).

with a large stock of carbon credits is not "greener," as it produces (and, thus, pollutes) more and engages less in green investment. Our findings are also consistent with Bolton, Kacperczyk, and Wiedemann (2023), who document that firms exhibiting the larger carbon emissions are those investing in innovation the least.

Considering jointly our findings on the carbon trading and the carbon tax systems, a number of important implications follow. First, compared to laissez-faire, carbon pricing systems are effective at reducing carbon emissions, consistent with the evidence in Fowlie, Holland, and Mansur (2012), Martin, Muuls, and Wagner (2016), or Metcalf and Stock (2023). As just described, this result stems from both a reduction in production and a pickup in green investment in our model: Firms subject to carbon pricing internalize that production generates emissions which, by imposing additional operating costs, affect firm value. Interestingly, whereas advocates of the carbon trading system typically emphasize that the ability to buy credits leaves production uncapped—as it allows firms to carry on producing beyond their maximum emission allowance by buying credits on the market—we show instead that firms under-produce in this system compared to the laissez-faire benchmark. A notable feature of our model is then that a firm can dynamically be a net polluter and exhibit positive net emissions—i.e., not fully offseting its emissions—or instead be a net cleanser and exhibit negative net emissions.

More importantly, our analysis sheds light on an unexplored effect of carbon pricing on green investment. Either under the carbon trading or the carbon tax systems, we show that firms shift away from green innovation and engage more in abatement as it becomes costlier to comply, for instance, due to an increase in the price of carbon credits or in the carbon tax rate.[5] That is, to minimize the cost of carbon emissions and irrespective of the carbon pricing system, firms focus on immediate though short-lived types of green investment, which nonetheless do not transform firms' technologies by making them more sustainable. Yet, as emphasized by Acemoglu, Aghion, Barrage, and Hemous (2019) and Aghion, Boneva, Breckenfelder, Laeven, Olovsson, Popov, and Rancoita (2022) among others, the reduction in carbon emissions necessary to limit global warming requires the application of innovative technologies.[6] We thus investigate whether the provision of a subsidy to green innovation can milden this effect. Irrespective of the carbon

[5]Under the carbon trading system, the firm also puts more emphasis on abatement projects and less emphasis on green innovation as its balance of carbon credits approaches zero, to avoid resorting to the costly carbon credit market.

[6]In the context of the shale gas revolution in the United States, Acemoglu et al. (2019) show that providing a cheaper way to firms to limit their carbon emissions displaces green innovation and, thus, can trap the economy to continue using fossil fuels by postponing the switch towards green innovation.

pricing system, we confirm that a subsidy indeed boosts a firm's investment in green innovation, increasing a firm's engagement in green investment.

We assess the robustness of our core predictions by allowing a regulator to optimize over carbon regulation (both carbon pricing and an innovation subsidy), then taking into account the general equilibrium effect of firm-level policies. Importantly, and consistent with our analyses in partial equilibrium, we confirm that carbon pricing discourages innovation and encourages abatement of polluting firms, though a subsidy to innovation can mitigate this shift. The general equilibrium setting further reveals that carbon pricing and the innovation subsidy target distinct goals: Optimal carbon pricing guarantees that emissions are reduced in the short term, whereas the optimal innovation subsidy ensures a faster, innovation-driven greening path for the economy. Relatedly, we show that environmental regulators find it optimal to set carbon pricing below the social cost of carbon. This result challenges the Pigouvian view and yet confirms that carbon pricing may lead firms to focus too little on long-term innovation to reduce their cost of carbon regulation—highlighting the tension between short-term reduction in emissions and ensuring a long-run transition to more sustainable technologies.

Lastly, our analysis reveals that carbon regulation need not reduce firm value unconditionally, in spite of the perceived conflict between the interests of environmental regulators and those of businesses. This prediction is consistent with recent evidence showing the heterogeneous effects on firms of climate regulation (among others, Martin, de Preux, and Wagner, 2014; Bolton, Lam, and Muuls, 2023) or climate considerations (Sautner, van Lent, Vilkov, and Zhang, 2023). In particular, for firms with positive net emissions (i.e., net polluters), carbon regulation is largely a cost, which lowers firm value and reduces investment in green innovation compared to laissez-faire. In turn, for firms with negative net emissions or net cleansers, carbon regulation can yield gains from selling credits or rewards on negative emissions. Hence, for these firms, carbon regulation can raise firm value as well as the engagement in innovation compared to the laissez-faire benchmark.

Related literature Our model relates to the growing climate finance literature. In particular, our model relates to theoretical studies investigating how firms can be incentivized to internalize the social cost of emissions. Pioneering this literature, Heinkel, Kraus, and Zechner (2001) assess how exclusionary ethical investing affects corporate behavior and firm's cost of capital. Broccardo, Hart, and Zingales (2022) and Oehmke and Opp (2021) study the conditions under which socially-responsible investors affect firm behavior. Landier and Lovo (2020) study how socially-responsible funds can induce firms to reduce their toxic emissions. Hong, Wang, and Yang (2023) study a model featuring the gradual accumulation of decarbonization capital by firms that, while nonproductive, reduces their cost of capital. Gans (2012) questions the ability of carbon regulation to effectively spur green innovation. Ramadorai and Zeni (2022) study the impact of firms' beliefs on emission abatement, whereas Dangl, Halling, Yu, and Zechner (2023) study the impact of stochastic social preferences on corporate green investment decisions. Our contribution is to study optimal carbon management policies in a dynamic setting, by taking a comprehensive look at the tools that firms can apply to curb emissions and transition to a cleaner economy, and by also investigating the ensuing optimal policy of environmental regulators.

Our paper provides new testable implications and theoretical grounds to the growing empirical literature on how climate considerations affect firms' decisions and outcomes. Consistent with our predictions, Fowlie, Holland, and Mansur (2012) and Martin, Muuls, and Wagner (2016) conclude that carbon trading systems are effective at curbing emissions—while rationalizing this finding, we analyze the channels through which this happens. Moreover, Bushnell, Chong, and Mansur (2013) and Martin, de Preux, and Wagner (2014) show that carbon regulation in the European Union has an ambiguous effect on firm performance, consistent with our result that carbon regulation need not reduce valuations. Derrien, Krueger, Landier, and Yao (2022) show that negative environmental, social, and governance news trigger significant downgrades in earnings forecasts at all horizons, suggesting that consumers value sustainability, a feature that we feed into firms' product demand in our model.

Our model also provides guidance to future empirical work studying firms' engagement to becoming more sustainable. This literature is developing as new data on firm's green investment becomes available. Some predictions of our model have been recently documented—such as Bolton, Kacperczyk, and Wiedemann (2023), who find a negative relation between emissions and green innovation in the cross section of firms. Several other findings are novel and remain to be explicitly tested—such as the result that carbon regulation affects a firm's green investment mix. Nonetheless, consistent with this idea, Brown, Martinsson, and Thomann (2022) find that carbon regulation leads polluting firms to adopt existing technologies, instead of developing new clean ones.[7] Borghesi, Cainelli, and Mazzanti (2015) document that the stringency of regulation

[7] Specifically, they show that firms do not increase their patenting despite a greater engagement in R&D activities, which they interpret as polluting firms not developing new clean products or technologies but rather expanding their capacity to adopt and assimilate external knowledge and technical knowhow.

is negatively associated with innovation. Last, from a banking angle, Fuchs, Nguyen, Nguyen, and Schaeck (2023) find that polluting borrowers of banks subject to climate stress tests increase efforts to become greener through short-term measures.[8]

Methodologically, our analysis of corporate policies in the carbon credit system relates to the dynamic inventory models in corporate finance. Previous contributions in this strand have focused on dynamic cash management, spurred by the increase in corporate cash holdings since the Eighties. Notable contributions include Bolton, Chen, and Wang (2011, 2013), Décamps et al. (2011), Hugonnier, Malamud, and Morellec (2015), Décamps, Gryglewicz, Morellec, and Villeneuve (2017), Malamud and Zucchi (2019), Della Seta, Morellec, and Zucchi (2020), and Zucchi (2023). Our paper characterizes instead a novel type of corporate inventory management: The dynamic management of carbon credits, which can be accumulated, bought, and sold.

The paper is organized as follows. Section 2 describes the model. Section 3 analyzes optimal corporate policies in a laissez-faire benchmark. Section 4 studies optimal corporate policies under the carbon tax system, whereas Section 5 focuses on the carbon trading scheme. Section 6 develops further implications across carbon pricing systems. Section 7 investigates the role of a subsidy to green innovation. In Section 8, we endogenize carbon pricing and the innovation subsidy in general equilibrium. Section 9 concludes. Technical developments and proofs are gathered in the Appendix.

2 The model

We design a dynamic model for a firm that manages its carbon emissions. The model is flexible enough to acknowledge that firms may operate under different carbon pricing regimes: a carbon tax system or a carbon trading system. We define the firm's "greenness" g_t as the degree of technological sustainability of the firm. The firm can increase its greenness by investing in green innovation.

Time is continuous, and the economy admits a constant risk free rate denoted by r.

[8] In turn, Kaenzig (2023) documents that, mostly focusing on the years before the 2017 reform, the EU ETS had a quantitatively small effect on patenting.

2.1 The firm

Production and carbon emissions Through its production process, the firm generates carbon emissions. Denoting by Y_t the firm's endogenous scale of production, the ensuing flow of carbon emissions is described by the following dynamics:

$$dE_t = (\nu Y_t - \xi g_t - s_t g_t)dt + \sigma Y_t dW_t. \tag{1}$$

This specification implies that emissions are greater and more volatile if the firm's scale of production is larger.[9] In this equation, W_t is a standard Brownian motion, and σ is a positive constant representing the volatility of the firm's emissions per unit of production. Moreover, ν represents an industry-specific parameter associated with emission intensity per unit of production. The term ξg_t is instead firm-specific, and implies that the higher the firm's greenness—which the firm can improve through green innovation breakthroughs—the lower the firm's net emissions for a given scale of production Y_t. The term s_t represents the firm's engagement in emission abatement (on which we elaborate more below). Importantly, we do not impose any restrictions on the sign of the drift $(\nu Y_t - \xi g_t - s_t g_t)$, meaning that the firm endogenously decides whether to be a polluter or a cleanser. If the drift is positive, the firm is a net polluter, and does not fully offset its emissions. If negative, the firm is a net cleanser, and more than offsets its emissions.

The firm's choice of production affects output prices. In particular, we assume that the firm faces the following inverse demand function:

$$p(Y_t) = a - b\frac{Y_t}{g_t} \quad a > 0\,, b > 0. \tag{2}$$

Notably, the sensitivity of prices to Y_t is scaled by greenness g_t to capture that the greener the firm is, the greater the amount of product demanded by consumers for a given price level. The assumption is consistent with the growing evidence that consumers reward greener firms (see, e.g., Derrien et al. (2022)).[10] For simplicity, we consider that the firm is a monopolist, and we

[9] In a similar vein, Hong, Wang, and Yang (2023) assume that emissions are proportional to capital.

[10] Derrien et al. (2022) show that analysts significantly downgrade earnings forecasts on a firm following negative ESG news on such firm. They show that the negative revision of earnings forecasts reflects expectation of lower sales rather than higher future costs. See also Choi, Gao, and Jiang (2020). More broadly, our demand function is consistent with the argument that the marginal utility of households depends on the quality of the environment, already present in Acemoglu et al. (2012).

normalize the cost of production to zero.

Abatement and green innovation The specification in equation (1) implies that the firm can limit its net carbon emissions by investing in abatement. Examples of abatement are, for instance, investment in (international) carbon offset projects, afforestation or reforestation projects, or purchasing electricity from renewable sources. That is, abatement does not affect the firm's production technology—as it does not affect the firm's greenness g_t—but simply offsets the firm's gross emissions. Denoting the firm's engagement in carbon abatement by s_t, we assume that the cost associated with such projects is given by the quadratic specification $\frac{s_t^2}{2}\theta g_t$, where θ is a positive parameter.[11]

On top of investing in abatement, the firm also invests in green innovation. Differently from abatement, green innovation permanently improves the firm's degree of sustainability g_t. Our modeling of innovation builds on the endogenous growth literature (see Aghion, Akcigit, and Howitt (2014) for a survey). Namely, if the firm spends the quadratic cost $\frac{z_t^2}{2}\zeta g_t$, then a green breakthrough arrives at Poisson rate ϕz_t. The innovation rate z_t is an endogenous choice of the firm—a greater z_t entails greater cost, but increases the likelihood of attaining a green breakthrough. When a breakthrough happens, the firm's greenness g_t increases by a factor $\lambda > 1$. Furthermore, if the firm has already had many breakthroughs (i.e., g_t is higher), we assume that the cost of innovation increases. A higher level of greenness g_t brings along long-term benefits to the firm. First, for a given scale of production, the firm exhibits lower net emissions—and, thus, it is less polluting—regardless of the firm's engagement in abatement. Second, it leads to an increase in the demand for the firm's product (see equation (2)).

Whereas both abatement and green innovation aim at making the firm more sustainable, the key difference between the two types of green investment is the horizon of their impact. Abatement aims at reducing the expected flow of emissions in the present—basically, it cleans up (some of) the firm's gross emissions. Yet, it represents a short-lived effort as it does not change the firm's technology. Conversely, green innovation has long-lasting effects as it leads to a permanent increase in the sustainability of the firm's technology, which in turn makes the firm permanently less polluting for a given scale of production.

[11] We assume a quadratic formulation for the abatement cost, similar to the study by Hong, Wang, and Yang (2023) on the dynamics of decarbonization capital.

2.2 Carbon pricing

Carbon tax system A number of countries around the world have implemented carbon taxes—i.e., taxes levied on a firm's carbon footprint. Under carbon tax systems, a central authority sets a predetermined price that firms have to pay for a given volume of emissions. We denote such price—the carbon tax rate—as κ, which is levied on the firm's net emissions. If the firm's expected net emission flow is positive, then the firm is effectively charged the carbon tax. Whereas carbon taxes are designed to reduce the externalities produced by the industrial processes of polluting firms, regulators may reward firms that exhibit negative emissions—also referred to as "net cleansers," which contribute negatively to pollution. To capture this aspect, we assume that firms with negative net emissions enjoy a reward at rate κ on their negative emissions.[12]

Carbon trading system Under the carbon trading system (or emission trading systems), carbon credits are akin to permission slips that enable the firm to release carbon emissions into the atmosphere. Once the firm emits a fixed quantity of carbon emissions (typically, a tonne of carbon dioxide), one credit is retired. Accumulating credits entails a benefit, as carbon credits guarantee the firm the ability to carry on producing. At the same time, carbon credits entail a maintenance/storage cost χ, which is proportional to the firm's stock of credits.[13]

The credit system is nested into a trading scheme. Namely, if the firm exhausts its credits, it either has to stop emitting pollutants into the atmosphere, or it can buy credits from other firms willing to sell them. In turn, if the firm finds itself with an excess of credits, it can sell them to firms willing to buy them. Consistent with the trend observed in recent years, we assume that trading is centralized and the firm buys its credits on a carbon credit platform at the price $\gamma > 0$. In turn, the firms can sell credits at the cost $\gamma(1 - \psi)$, with $\psi > 0$ representing the compensation of the platform.[14]

The dynamics of the firm's carbon credits then satisfy:

$$dC_t = -dE_t + dP_t - dO_t + dI_t. \tag{3}$$

[12] For simplicity, we assume that the tax on positive net emissions and the reward on negative net emissions have the same magnitude. Our focus is mostly on the carbon management of polluting firms.

[13] In several jurisdictions, carbon credits require maintenance because they need to be certified.

[14] Effectively, platforms ease the needs of firms with an excess of credits (sellers) and firms with a shortfall of credits (buyers). The assumption that the fee is charged to sellers is consistent with the functioning of major carbon credit platforms, see, e.g., https://ctxglobal.com/the-exchange/.

The above equation shows that the firm's emission flow (defined in equation (1)) depletes the firm's stock of carbon credits. The process P_t represents the purchased credits, which increase the firm's credit stock. Conversely, O_t captures the credits that the firm sells to other firms, which deplete the firm's credit stock. Finally, the process I_t is the inflow of credits when a firm attains a breakthrough stemming from its investment in green innovation. Namely, because green innovation effectively improves the sustainability of its technology, the firm is awarded a lumpy amount of credits at no charge, which replenishes the firm's credit balance.[15]

2.3 Optimality

The firm maximizes its value by managing production Y_t, its engagement in abatement s_t, and its green innovation rate z_t. In addition, under the carbon trading system, the firm manages its stock of carbon credits C_t, by choosing both the upper bound at which the firm starts selling credits (which we denote by C^*), as well as the optimal buy/sell strategy (P_t and O_t).

To better single out the effect of the carbon trading system on firm's choices, we start by investigating the laissez-faire benchmark case in the next section, in which the firm does not hold nor manage carbon credits. We next solve for optimal policies and value under carbon pricing, first looking at the carbon tax case (section 4), and then at the carbon trading system (section 5).

3 Optimal policies under laissez-faire

As a benchmark, we start by considering a laissez-faire environment with no carbon regulation. In this case, firm value is solely a function of greenness, i.e. $V^B(g_t)$. Following standard arguments, firm value satisfies the following equation:

$$rV^B = \max_{Y^B, z^B, s^B} \phi z^B \left(V^B(\lambda g) - V^B(g)\right) + \left(Y^B p(Y^B) - \frac{(z^B)^2}{2} g\zeta - \frac{(s^B)^2}{2} g\theta\right). \tag{4}$$

The left-hand side of this equation is the return required by the investors. The right-hand side is the change in firm value on each time interval. In particular, the first term is the effect of a

[15]This assumption is consistent with the idea that innovative firms are rewarded some credits for free. For instance, in the EU ETS, manufacturing industries receive a share of their emission allowances for free, based on benchmarks that reward most efficient installations in each sector (so called "free allocation to industrial installations"). Technically, we assume that upon a breakthrough the firm can replenish its balance all the way to an endogenous upper bound, at which the firm starts selling credits.

breakthrough stemming from green innovation, which increases the firm's greenness. The last term is the expected net cash flow to shareholders on each time interval.

To obtain the firm's optimal policies and value, we conjecture and verify that firm value scales with greenness, i.e., $V^B(g_t) = g_t v^B$, where we denote by v^B the scaled firm value in the laissez-faire benchmark. We also denote by $y^B = Y_t^B/g_t$ the scaled production quantity. Substituting into equation (4) and maximizing with respect to y gives the optimal production quantity:

$$y^B = \frac{a}{2b}. \tag{5}$$

This expression implies that, under laissez-faire, the optimal production rate is independent of the firm's emissions. Moreover, the firm has no incentives to invest in abatement—as illustrated by equation (4), abatement projects entail a cost but do not bring any upside in the laissez-faire environment. The optimal abatement strategy is thus zero in this case ($s^B = 0$). In turn, the firm invests in green innovation, and its optimal innovation rate is:

$$z^B = \frac{\phi}{\zeta} v^B (\lambda - 1). \tag{6}$$

The firm invests more in green innovation if the Poisson coefficient ϕ (affecting the likelihood of a breakthrough) is higher, if the associated increase in greenness λ is greater, or if the cost of innovation ζ is smaller. Notably, the firm invests in innovation under laissez-faire as higher greenness increases the firm's demand for its product (see equation (2)).

Plugging equations (5) and (6) back into the (scaled) HJB equation gives a closed-form expression for firm value:

$$v^B = \zeta \frac{r - \sqrt{r^2 - \frac{a^2}{b}\frac{\phi^2}{2\zeta}(\lambda - 1)^2}}{\phi^2 (\lambda - 1)^2}. \tag{7}$$

This expression shows that firm value does not depend on its carbon emissions under laissez-faire. As we show next, this is not the case when the firm is suject to carbon pricing.

4 Optimal policies in a carbon tax system

We next turn to investigate optimal policies under the carbon tax system. Under this system, we denote firm value by $V^\tau(g)$, which continues to be a function of greenness. Using standard arguments, firm value satisfies:

$$rV^{\tau}(g) = \max_{Y^{\tau}, z^{\tau}, s^{\tau}} \ \phi z^{\tau} \left[V^{\tau}(\lambda g) - V^{\tau}(g)\right] + \left[Y^{\tau} p(Y^{\tau}) - \frac{(z^{\tau})^2}{2} g\zeta - \frac{(s^{\tau})^2}{2} g\theta - \kappa(Y^{\tau}\nu - \xi g - s^{\tau} g)\right]. \tag{8}$$

This equation differs from equation (4) in the last term on the right-hand side, as the firm is taxed on its net emissions. To solve for the firm's optimal policies, we again use the scaling property (see Appendix A.2 for details). The optimal production quantity then satisfies

$$y^{\tau} = \frac{a - \kappa\nu}{2b}. \tag{9}$$

This expression implies that the larger the carbon tax rate, the lower the firm's production rate, and even more so if the emission intensity ν is larger. That is, by making emissions costly, the carbon tax makes firms internalize the externality generated by its production processes. Moreover, the firm's optimal abatement satisfies

$$s^{\tau} = \frac{\kappa}{\theta}, \tag{10}$$

which implies that the firm invests more in abatement if the carbon tax rate is larger. Lastly, the firms' optimal innovation rate satisfies:

$$z^{\tau} = \frac{\phi}{\zeta}(\lambda - 1)v^{\tau}(\kappa), \tag{11}$$

The next proposition follows.

Proposition 1 *In the presence of the carbon tax:*
(1) The firm always under-produces compared to laissez-faire (i.e. $y^{\tau} < y^{B} = \frac{a}{2b}$) and exhibits smaller revenues, even more so if κ is larger;
(2) The firm engages in abatement $s^{\tau} > s^{B} = 0$, and more so if κ is higher;
(3) The firm invests less in innovation than under laissez-faire if its net emissions are positive, and less so if κ is higher.

Because the firm's scale of production directly impacts the firm's gross emissions—and, thus, affects how much the firm is levied—Proposition 1 shows that y^{τ} is consistently below the laissez-faire (unconstrained) level y^B, and more so for higher levels of κ. As a result, the firm exhibits lower revenues under the carbon tax. Proposition 1 also illustrates that the firm invests in abatement under the carbon tax, whereas it does not under laissez-faire. Notably, the firm's engagement in abatement increases with κ: Abatement leads to an immediate reduction in the firm's net emissions which, in turn, lead to a lower carbon tax liability. Proposition 1 also indicates that firms that exhibit positive emissions—and, thus, subject to the carbon tax liability—reduce their engagement in green innovation under the carbon tax, and more so if the carbon tax rate κ increases.

The following corollary stems directly from Proposition 1.

Corollary 2 *Expected net emissions are lower under the carbon tax system compared to laissez-faire, and more so as κ is greater. Yet, under the carbon tax system, polluting firms (i.e., with positive net emissions) tilt their green investment mix towards abatement and away from innovation, and more so as it becomes costlier to comply (i.e., as κ increases).*

Corollary 2 is a direct implication of the optimal choices described in Proposition 1. Namely, because firms cut production and engage in green investment when subject to the carbon tax, they reduce their emissions compared to laissez-faire, and more so if κ is higher. Yet, as the carbon tax imposes a cost on polluting firms (i.e., firms exhibiting positive net emissions), such firms shift their green investment mix towards abatement and away from green innovation. As it becomes costlier to comply (as κ increases), this shift strengthens. Thus, the carbon tax tilts the green investment mix towards short-term, immediate types. The reason is that short-term abatement leads to an immediate reduction in the expected cost of carbon regulation—as it reduces the firm's net emissions—whereas green innovation has a delayed and uncertain outcome.

5 Optimal policies under the carbon trading system

We now analyze the optimal policies under the carbon trading system. Given that carbon credits are costly, the firm does not buy them until it runs out of them. In turn, the firm does not sell credits as long as the marginal value of credits inside the firm is greater than the marginal gain from selling them. As a result, we conjecture that there is a region $[0, C^*]$ in which the firm accumulates credits and does not trade them. At $C = 0$, the firm buys credits. Conversely, at the upper bound C^*, the firm sells credits.

In the region $[0, C^*]$, the dynamics of firm value satisfy the following Hamilton-Jacobi-Bellman (HJB) equation:

$$rV(C,g) = \max_{z,Y,s} \; -(Y\nu - \xi g - sg)V_C + \frac{\sigma^2}{2}Y^2 V_{CC} + \phi z \left(V(C^*, \lambda g) - V(C,g)\right) + \left(Yp(Y) - \frac{z^2}{2}g\zeta - \frac{s^2}{2}g\theta - \chi C\right). \quad (12)$$

The left-hand side of this equation is the return required by the investors. The right-hand side is the expected change in firm value on each time interval. The first term is the effect of a change in the credit balance on firm value. If $Y\nu - \xi g - sg > 0$, the firm depletes its credit balance in expectation, and faster if it produces more or if it invests less in abatement. The higher the firm's investment in abatement or the greater the firm's greenness, the lower the firm's expected net emissions. The second term on the right hand side is the effect of carbon emission volatility on firm value. The third term is the effect of a green innovation breakthrough, in which case the firm's greenness as well as its credit balance increase. The last term is the expected net cash flow to shareholders on each time interval. Similar to the carbon tax system (and differently from laissez-faire), equation (12) illustrates that, under the carbon trading scheme, emissions impact the dynamics of firm value.

In this system too, we define quantities scaled by greenness. Namely, we define

$$V(C,g) = g\, v\left(\frac{C}{g}\right) \equiv gv(c) \quad (13)$$

where $v(c)$ denotes firm value scaled by greenness and c denotes scaled credits so that $C = gc$. The scaled upper bound for the carbon credit balance is denoted by $c^* = C^*/g$ and is endogenously determined. We also define scaled production by $y = Y/g$. Substituting into equation (12) gives the scaled HJB (see equation (30) in Appendix A.3), which we differentiate with respect to y to get the optimal production quantity:

$$y(c) = \frac{a - \nu v'(c)}{2b - \sigma^2 v''(c)}. \quad (14)$$

Differently from the laissez-faire setup but similar to the carbon tax, the optimal scale of production under the carbon trading scheme internalizes the pollution externality (i.e., production becomes a function of the firm's emission intensity ν). Equation (14) implies that the greater the marginal value of a carbon credit (v') or the greater the firm's emission intensity (ν), the lower the firm's optimal scale of production.

Consider next the optimal abatement policy, obtained by differentiating the scaled HJB equation with respect to s:

$$s(c) = \frac{v'(c)}{\theta}. \tag{15}$$

This expression implies that the greater the marginal value of carbon credits, the more the firm will engage in abatement to reduce its net emissions, and slow down its consumption of carbon credits.

Finally, maximizing the scaled HJB equation with respect to z gives the optimal green innovation rate:

$$z(c) = \frac{\phi}{\zeta}(\lambda v(c^*) - v(c)). \tag{16}$$

This equation shows that the firm invests more in innovation if the surplus associated with a breakthrough ($\lambda v(c^*) - v(c)$) is higher, if the cost of green innovation ζ is smaller, or if the likelihood of a green breakthrough is greater thanks to a higher Poisson coefficient ϕ.

Substituting equations (14), (15), (16) into the HJB equation gives an ordinary differential equation (ODE), which we solve subject to three boundary conditions. First, when the firm runs out of credits at $c = 0$, it resorts to the credit market. Recall that buying credits entails a proportional cost γ. Thus, the following boundary condition holds:

$$v'(0) = \gamma, \tag{17}$$

which implies that, when the firm buys credits, the marginal value of credits inside the firm equals their marginal cost.[16] Next, the firm sells credits exceeding the endogenous upper bound c^*. Hence, the following boundary condition holds at c^*:

$$v'(c^*) = \gamma(1 - \psi). \tag{18}$$

That is, at c^*, the marginal value of credits inside the firm is equal to the gain associated with

[16]Realistically, we assume that buying credits does not put at stake the viability of the firm—i.e., it does not push the firm below its liquidation value. I.e., the following inequality: $v(0) > \ell$ holds, where we denote by ℓ the firm's liquidation value.

selling them.[17] Last, the threshold c^* is pinned down by the super-contact condition:

$$v''(c^*) = 0, \tag{19}$$

which guarantees that the threshold is optimally chosen (Dumas, 1991).

Optimal policies with carbon credit management We start by investigating firm value as a function of the stock of carbon credits c.

Proposition 3 *Under the carbon trading scheme, firm value is increasing and concave in its stock of carbon credits c.*

Proposition 3 shows that, under the carbon trading scheme, firm value is concave in the stock of carbon credits. Because carbon credits are costly, the firm optimally accumulates carbon credits in a precautionary perspective, to ensure continuity of production and avoid having to resort to the costly carbon credit market too often. Indeed, keeping a non-negative carbon credit balance acts as an operating constraint that enables the firm to produce.[18] As we show in the next proposition, the stock of accumulated credits—and, thus, the precautionary behavior just described—explains the firm's optimal corporate decisions.

Proposition 4 *Under the carbon trading scheme:*

(1) Production $y(c)$ increases with the stock of credits c and so do revenues $y(c)\,(a - by(c))$;

(2) Abatement $s(c)$ and green innovation $z(c)$ decrease with the stock of credits c;

(3) The firm's expected net emissions $(\nu y(c) - \xi - s(c))$ increase with the stock of credits c.

Proposition 4 shows that the larger the firm's stock of carbon credits, the greater its production rate. That is, when the stock of credits is low, the firm decreases production so to curtail its emissions and reduce its consumption of carbon credits, to avoid having to resort to the costly carbon credit market. By contrast, the firm increases production as its stock of carbon credits is larger, so revenues also increase with c.

Proposition 4 also illustrates that the firm's abatement and innovation policies are complementary to its production choices in keeping emissions in check. Namely, when c is low, the firm

[17] For any $c > c^*$, firm value is linear, as the firm sells all the credits exceeding such threshold.

[18] Despite our stock variable is carbon credits (rather than cash), this result is similar to the finding of previous inventory models (see Bolton, Chen, and Wang, 2011; Décamps et al., 2011; Hugonnier, Malamud, and Morellec, 2015), in which financially constrained firms find it optimal to accumulate cash to buffer cash flow shocks and mitigate exposure to costly or uncertain external funding.

actively reduces its emissions not only by scaling down production but also by investing more in abatement and green innovation, to avoid having to resort to the costly carbon credit market. In turn, as c increases, the firm's engagement in abatement or in green innovation decreases. Overall, the proposition suggests that the firm manages production, abatement, and innovation policies to steer its carbon credit balance away from the $c = 0$ boundary, at which point it has to buy costly credits in order to sustain production.

As a direct implication of the monotonicity of s and y with respect to the stock of credits, Proposition 4 also highlights that the firm's expected net emissions increase as c increases. That is, a firm with a large stock of credits is not "cleaner" from an environmental perspective. By constrast, because a larger stock of credits gives the right to the firm to emit more pollutants into the atmosphere, the firm takes advantage of it by increasing production and, hence, polluting more. Hence, having a large carbon credit balance relaxes the firm's commitment to curb pollution. This result is consistent with De Jonghe, Mulier, and Schepens (2020), who document that, within the EU ETS, polluting firms have weaker incentives to become greener if they have more emission allowances. These policies are then consistent with the precautionary behavior suggested by Proposition 3.

Carbon pricing versus laissez-faire We proceed to compare policies in the carbon trading system and in the laissez-faire benchmark, while drawing parallels with the carbon tax system. The next proposition summarizes our results.

Proposition 5 *Compared to the laissez-faire benchmark, the carbon trading scheme:*
(1) Leads the firm to produce below the laissez-faire level ($y(c) < y^B$) and to earn lower revenues for any $c \leq c^*$;
(2) Incentivizes the firm to invest in abatement, i.e., $s(c) > s^B = 0$;
(3) Leads to a decrease in green innovation so that $z(c^*) < z^B$ is if net emissions are positive.

Recall from Proposition 3 that, under the carbon trading scheme, the firm is effectively risk averse for any $c < c^*$ (i.e., $v'' < 0$). Moreover, for any $c < c^*$, the marginal value of credits inside the firm is always greater than the marginal gain associated with selling them (i.e., $v'(c) > \gamma(1 - \psi)$). Consequently, equation (14) implies that the firm's scale of production always falls below that associated with the laissez-faire benchmark, which is consistent with the evidence in Martin, Muuls, and Wagner (2016) for the EU ETS. To avoid having to resort to buying costly credits, the firm reduces its need for credits by lowering its production and, thus, its emissions. Hence, while the proponents of the carbon trading system typically emphasize that the ability to buy credits allows the firm to carry on producing even when it reaches its maximum emission allowance (i.e., when its credit balance is depleted), our model shows that this system gives rise to a precautionary behavior: The firm always produces below its laissez-faire benchmark y^B for any level c of its credit balance. Comparing this result to Proposition 1 then points to the prediction that carbon pricing—irrespective of being a carbon tax or a carbon trading system—unambiguously leads firms to decrease their scale of production.

Proposition 5 also shows that, in contrast with the laissez-faire benchmark but similar to the carbon tax case, the firm *does* invest in abatement under the carbon trading system. Indeed, the firm seeks to reduce its net emissions and, hence, its need to resort to the costly carbon credit market. Proposition 5 also shows that the carbon trading system leads to a drop in the firm's engagement in green innovation for firms exhibiting positive emissions (i.e., net polluters) at least at c^*, similar to the carbon tax case.[19] The next result stems from Proposition 5.

Corollary 6 *The firm exhibits lower expected net emissions under the carbon trading system than under the laissez-faire benchmark, i.e.,* $y\nu - \xi - s < y^B\nu - \xi - s^B$*. Nonetheless, for polluting firms (i.e., exhibiting positive emissions), the carbon trading system spurs investment in abatement and reduces the firm's engagement in green innovation at* c^**.*

Corollary 6 resonates Corollary 2 in the carbon tax case. As shown in Proposition 5, the firm exhibits a higher scale of production in the laissez-faire environment and, at the same time, it does not invest in abatement. Therefore, the firm pollutes more in expectation in that environment than under the carbon trading system, a result similar to our conclusion in the carbon tax system. We conclude that the carbon trading system is indeed effective at curbing emissions, as firms internalize the environmental impact of their production activity. The reduction in emissions triggered by carbon trading systems has been empirically documented, e.g., by Fowlie, Holland, and Mansur (2012) or Martin, Muuls, and Wagner (2016). Moreover, polluting firms under the carbon credit system shift their green investment mix from green innovation towards abatement,[20] as abatement reduces the consumption of carbon credits immediately by leading to a reduction in firm's net emissions. Hence, our analysis shows that carbon pricing leads to

[19]In the numerical analysis in Section 6, we verify this result holds for any c.

[20]Similar to Proposition 5 (on which the result in this corollary is based), we verify this result holds for any c in the numerical analysis in Section 6.

a similar firm's response across its different policies, irrespective of whether carbon pricing is implemented as a carbon tax or as a carbon trading system.

The model also provides interesting insights on firm value. Because carbon pricing is a cost for polluting firms exhibiting negative emissions, it naturally decreases their value. Nonetheless, for firms exhibiting negative net emissions, the effect of carbon pricing on valuations is ambiguous, as illustrated by the next proposition.

Proposition 7 *If the firm's expected net emissions under the carbon trading system are sufficiently negative, firm value in the carbon trading system can exceed its counterpart in the laissez-faire benchmark. If $v(c^*) > v^B$, the rate of green innovation is greater under the carbon trading system than under the laissez-faire benchmark.*

In contrast with the long-standing perception of a fundamental conflict between the interests of environmental regulators and businesses, our analysis shows that carbon pricing is not necessarily value-decreasing for corporations. That is, the credit system can represent both a gain and a cost for firms: Firm value can be higher or lower than in the laissez-faire benchmark. This prediction is consistent with recent evidence showing that the effects of climate considerations and regulations on firms are heterogeneous, see e.g. Bolton, Lam, and Muuls (2023), Martin, de Preux, and Wagner (2014), or Sautner et al. (2023). In particular, the carbon trading system represents an opportunity for profits for a firm whose net emissions are sufficiently negative—so that the firm is effectively a net cleanser. In expectation, a net cleanser accumulates (as opposed to consumes) credits, which are eventually sold when the credit balance reaches its upper bound c^*. The present value of such gain thus increases firm value beyond its laissez-faire benchmark if negative net emissions are sufficiently large. A similar result holds in the carbon tax system if net cleansers are rewarded for their negative net emissions (as described in Section 2).

6 Further implications across carbon pricing systems

Calibration Table 1 reports our baseline parameterization. We assume that the risk free rate is equal to 0.02. We normalize ϕ to 1 as in other innovation models—hence, z effectively represents the arrival rate of a green breakthrough.[21] We assume that λ is equal to 1.065, which is consistent with the innovation step size estimated by Acemoglu et al. (2016). We assume that

[21] E.g., Akcigit, Hanley, and Serrano-Velarde (2021), Akcigit and Kerr (2018), or Bustamante and Zucchi (2023).

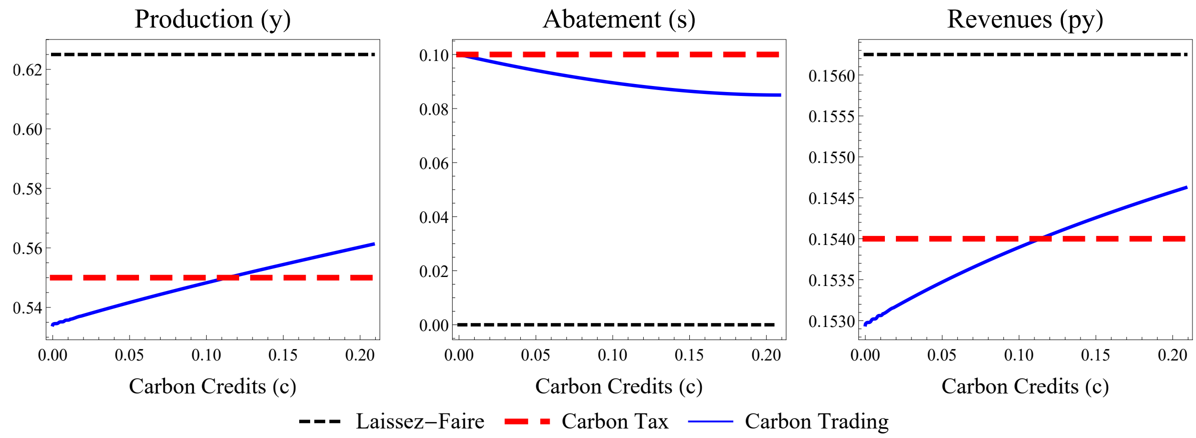

Figure 1: PRODUCTION, ABATEMENT, AND REVENUES. The figure shows the firm's optimal scale of production y, abatement policy s, and revenues $(p(y)y)$ as a function of credits c and under different environments, namely in the carbon credit system (solid blue line), the carbon tax system (large-dashed red line), and the laissez-faire case (tiny-dashed black line).

the cost ζ is four times larger than α, meaning that innovation is much costlier than abatement. We assume that the emission intensity ν is equal to 0.3, so to match the average emissions around the world per purchasing power parity of GDP as reported by the World Bank. Moreover, we assume that the volatility coefficient σ is 0.3, which implies that the volatility of emissions is about 15% in line with the findings in Ramadorai and Zeni (2022). For consistency, we assume that both the carbon tax rate κ as well as the price of carbon credits γ are equal to 0.2, so that the expected net emissions' flow of our baseline firm is positive under both carbon pricing systems.[22] Moreover, we assume that the maintenance cost χ is 0.0125, and the transaction fee ψ is 15%, consistent with the information on trading fees reported by carbon exchanges.[23] We set ξ to 0.001 and explore our results for higher values in our comparative statics. We set the parameters of the inverse demand function to $a = 0.5$ and $b = 0.4$, which give a markup of around 30% at c^* (consistent with Hall, 2018) and a return on assets consistent with that of dividend-paying firms in Farre-Mensa and Ljungqvist (2016).

The dynamics of optimal choices and outcomes Figures 1 and 2 jointly illustrate the firm's optimal choices and outcomes in the carbon pricing systems (carbon tax and carbon trading systems) compared to the laissez-faire benchmark. Confirming our result in Propositions

[22]The price of carbon varies widely within and across jurisdictions. As a result, we assess the robustness of our results extensively through comparative statics.

[23]The fees reported by exchanges live in a broad range, up to 20%. In general, smaller exchanges charge larger fees. Moreover, tax treatment of the proceeds from selling credits effectively increases the value of ψ.

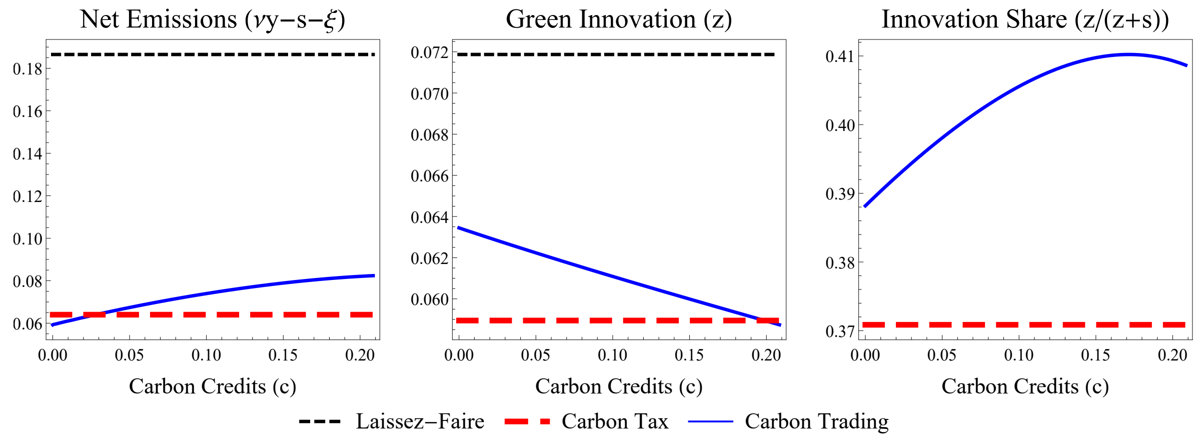

Figure 2: Carbon emissions, green innovation, and the green innovation mix. The figure shows net emissions, the optimal innovation policy, and the innovation share (i.e., the weight of green innovation in the firm's green investment mix) as a function of credits c in the carbon credit system (solid blue line), in the carbon tax system (large-dashed red line), and in the laissez-faire benchmark (tiny-dashed black line).

1 and 5, the left panel of Figure 1 shows that production is consistently lower under carbon pricing than under laissez-faire. The lower scale of production under carbon pricing systems implies that firms exhibit lower revenues than under laissez-faire, as illustrated by the right panel. Furthermore, and again consistent with the aforementioned propositions, the middle panel shows that the firm engages in abatement when subject to carbon pricing, whereas it does not in the laissez-faire benchmark.

The left panel of Figure 2 illustrates that, under our baseline parameterization, firms have positive net emissions and, thus, are net polluters. Regardless of the carbon pricing system, the figure shows that expected net emissions are always lower than under laissez-faire, consistent with Corollaries 2 and 6. That is, by attributing financial value to emissions, both the carbon tax and the carbon trading systems are effective in leading polluting firm to reduce their carbon footprint.

In turn, the middle panel of Figure 2 shows that firms with positive net emissions exhibit lower engagement in innovation compared to laissez-faire. Therefore, the findings in Figures 1 and 2 are consistent with Propositions 1 and 5, predicting that polluting firms reduce their investment in green innovation and increase that in abatement when subject to carbon pricing. Moreover, the right panel of 2 illustrates the ratio $z/(s+z)$, which captures the weight of innovation over the sum of the two types of green investment, denoted hereafter as the "innovation share." Regardless of the carbon pricing system, the innovation share is strictly lower than one

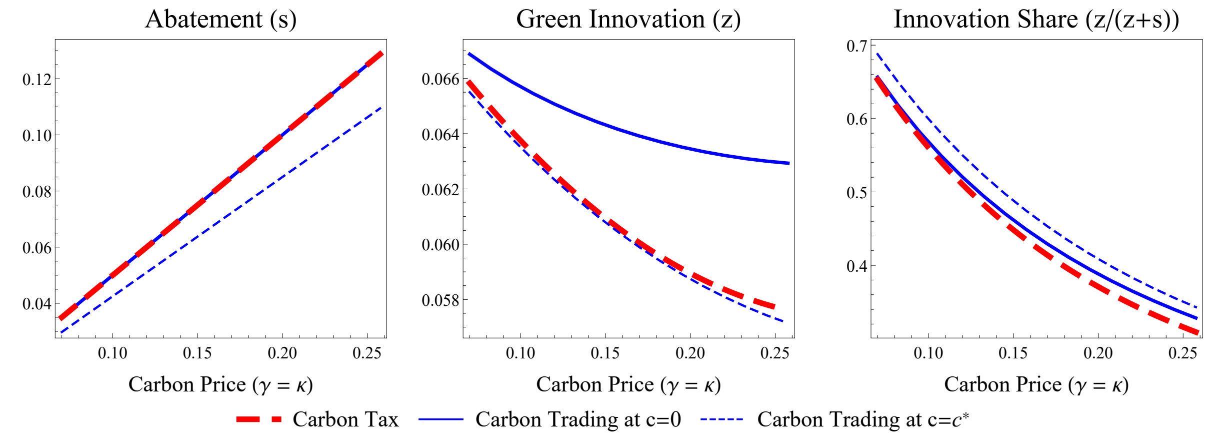

Figure 3: POLLUTING FIRMS, GREEN INVESTMENT, AND CARBON PRICES. This figure focuses on net polluters (i.e., firms exhibiting positive net emissions) and shows the optimal abatement policy, green innovation policy, and the innovation share (i.e., the weight of innovation in the firm's green investment mix) as a function of the the carbon price both in the carbon tax system (κ, red line) and carbon credit system (γ, blue lines, plotting the values of these quantities at $c = 0$ and at $c = c^*$).

and, thus, strictly below the innovation share in the laissez-faire benchmark.[24]

The right panel of Figure 2 additionally shows that, under our baseline parametrization, the innovation share of the firm in the carbon trading system is non-monotonic in the credit balance c. As c approaches zero, the innovation share decreases, as the likelihood that the firm resorts to buying credits increases. Hence, the firm focuses on more immediate though transient measures to curb pollution to immediately reduce the cost of carbon pricing and steer away from the boundary at which the firm has to buy carbon credits. In turn, as c approaches the upper bound $c^\star$, we observe that the innovation share is decreasing in c, as the expected gain from an innovation breakthrough also becomes smaller.

Sensitivity to carbon pricing We next investigate the role of the price of carbon under both the carbon trading and the carbon tax systems (γ and κ, respectively). Figure 3 illustrates how firms with positive net emissions adapt their green investment mix depending on carbon pricing (i.e., on the cost of carbon emissions). Naturally, carbon pricing imposes additional operating costs on polluting firms. As a result, if carbon pricing increases, firms increase their investment in abatement (left panel) and reduce their investment in innovation (middle panel) which, then, results in a drop in the share of innovation as a fraction of total green investment (right panel).

[24]Because the firm does not invest in abatement in the laissez-faire benchmark, then the innovation share is always equal to 1.

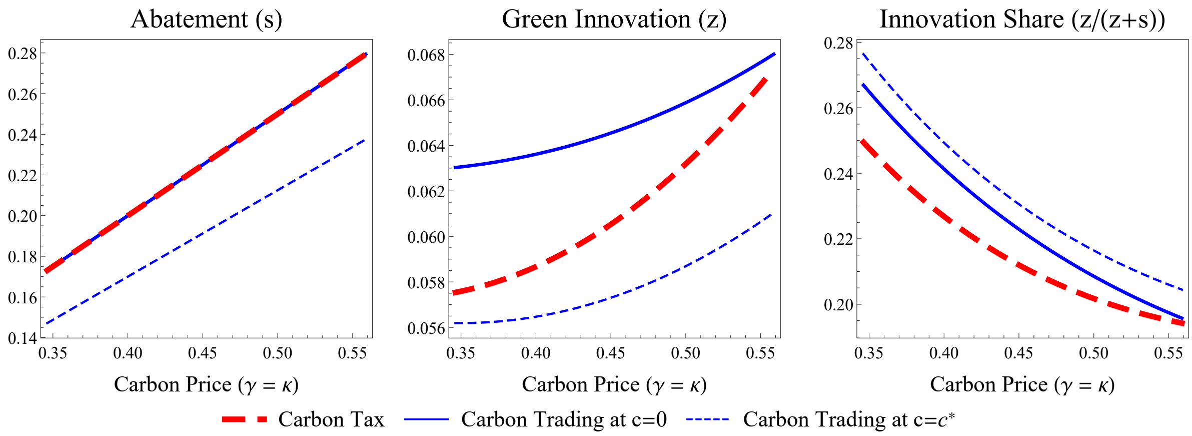

Figure 4: Net cleansers, green investment, and carbon prices. This figure focuses on net cleansers (i.e., firms exhibiting negative net emissions) and shows the optimal abatement policy, green innovation policy, and the innovation share (i.e., the weight of innovation in the firm's green investment mix) as a function of the the carbon price both in the carbon tax system (κ, red line) and carbon credit system (γ, blue lines, plotting the values of these quantities at $c = 0$ and at $c = c^*$).

The charts confirm the results in Proposition 1 and confirms Proposition 5 for any level of c. That is, while carbon pricing systems effectively lead the firm to reduce its net emissions, the firm reacts by undertaking short-term measures that nonetheless cannot address the transition to more sustainable technologies.[25]

Figure 4 complements our analysis by focusing on firms that exhibits negative emissions—i.e., the net cleansers. Under the trading system, a net cleanser expects to accumulate credits—and, thus, sell them—in expectation. In this case, an increase in γ has a positive impact on firm value, as described in Proposition 7. Consequently, as shown in the middle panel of Figure 4, innovation can increase in γ for at least some c if net emissions are sufficiently negative. By contrast, the left panel shows that the firm's abatement continues to be increasing in γ.[26] Driven by the greater sensitivity of abatement to γ, the innovation share continues to decrease with γ as in the case of net polluters. The same result holds under the carbon tax system, if net cleansers earn a reward for their negative net emissions (as described in Section 2).

The cross-section of green and brown firms Recent years have seen a rapid expansion of empirical work devoted to studying brown versus green firms, in particular how they differ

[25]See, Acemoglu et al. (2019), Aghion et al. (2022), or De Haas and Popov (2023), among others.

[26]Recall that abatement increases with the marginal value of carbon credits ($v'(c)$), as per equation (15). Because the marginal value of carbon credits increases with γ, so does the firm's optimal engagement in abatement. Similarly, under the carbon tax system, abatement increases with κ.

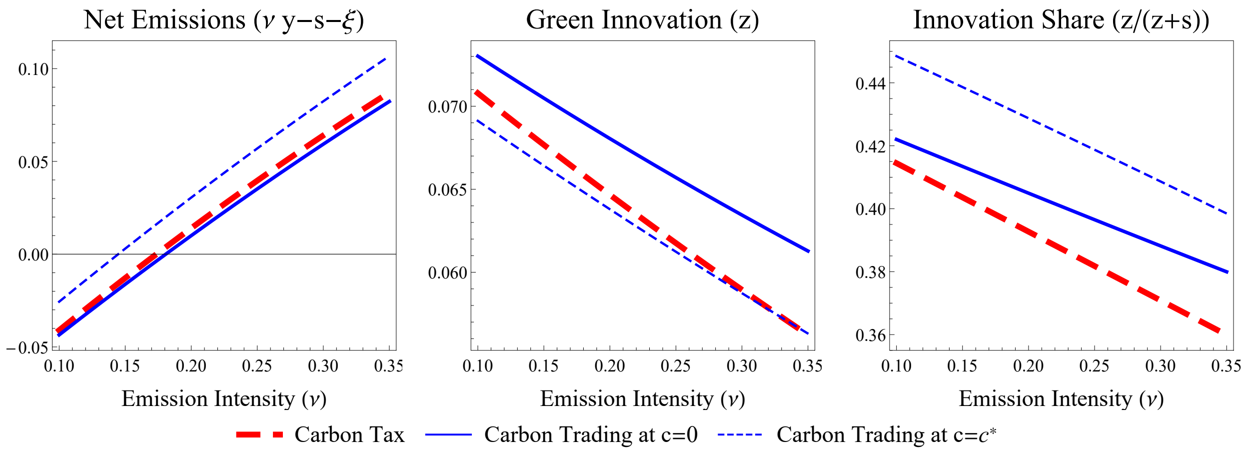

Figure 5: EMISSION INTENSITY AND INNOVATION. The figure shows the net carbon emissions, the optimal innovation rate, and the innovation share (i.e., the weight of innovation in the firm's green investment mix) as a function the degree of industry-specific emission intensity ν and both in the carbon tax system (κ, red line) and carbon credit system (γ, blue lines, plotting the values of these quantities at $c = 0$ and at $c = c^*$).

in their policies as well as their valuations. To this end, empiricists have relied on different strategies to identify green versus brown firms. Some works in the literature rely on an industry-level classification, as some industries are structurally more or less polluting than others (see, e.g., Brown, Martinsson, and Thomann, 2022). Other papers sort firms instead as green or brown based on their firm-level carbon emissions, controlling for fixed-effects (see, e.g., Bolton, Kacperczyk, and Wiedemann, 2023).

Our theoretical approach is rich enough to derive implications that can inspire empirical testing adopting either of the two approaches highlighted above. In particular, the industry-specific component of firm's carbon footprint is captured by sorting on ν.[27] When firms are subject to carbon pricing—being carbon tax or carbon trading systems—they downsize their production, and more so if they operate in a dirtier industry with a higher ν. The middle and right panels of Figure 5 illustrate that, all else equal, firms with a higher emission intensity reduce their engagement in green innovation. Because these firms intrinsically face a higher expected cost associated with carbon pricing, they reduce their engagement in long-term albeit uncertain measures to combat pollution (such as innovation), then increasing the relative weight of immediate though transient measures (such as abatement).

[27] As explained, for a given level of production and for a given emission intensity (i.e., ν), a firm can become less polluting by engaging in green investment. Hence, our model nests an industry-specific component as well as a firm-specific one, which stems from the firm's efforts in becoming green.

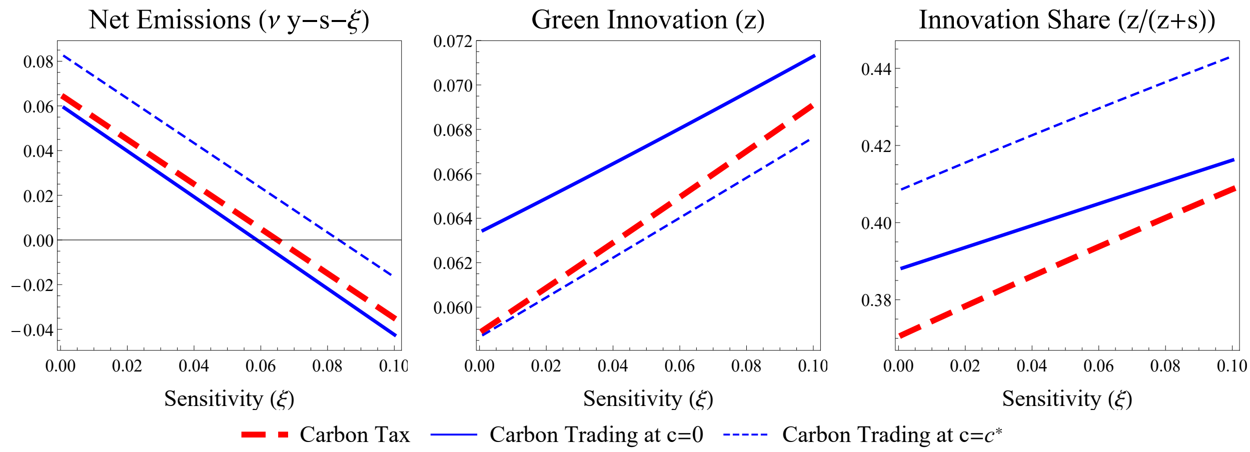

Figure 6: SUSTAINABILITY, INNOVATION, AND THE INNOVATION SHARE. The figure shows the net carbon emissions, the optimal innovation rate, and the innovation share as a function of parameter ξ, both in the carbon tax system (κ, red line) and carbon credit system (γ, blue lines, plotting the values of these quantities at $c = 0$ and at $c = c^*$).

Our discussion in the previous paragraph suggests that firms in industries with higher ν innovate less unconditionally. Consider now the sensitivity of optimal choices with respect to ξ, a parameter that captures the extent to which green breakthroughs reduce a firm's emissions for a given scale of production. Importantly, ν describes the emission intensity of the industry in which the firm operates (e.g., an oil company will structurally have a higher ν than a service company), whereas ξ represents how much a firm can improve its own specific degree of sustainability through green innovation. Figure 6 shows that the greater ξ, the more the firm invests in innovation, both in absolute terms and relative to total green investment. Thus, Figure 5 and Figure 6 jointly imply that ξ and ν have an opposite effect on green innovation—which effect dominates is an empirical question.

Consider now the empirical approach which sorts firms according to their carbon emissions. Our model provides guidance to empirical works adopting this approach too. Importantly, carbon emissions are endogenous in the model. For instance, in the carbon trading system, a firm can dynamically be a net polluter or cleanser depending on its credit balance, consistent with our result that firms undertake precautionary policies when they expect to run out of credits. In particular, a firm might be a net cleanser if its credit balance is low, in the attempt to avert having to resort to the costly carbon credit market, while being a net polluter if the credit balance is sufficiently high. This result is illustrated in the left panel of Figure 7. Similarly, as we vary other parameters of the model—such as the emission intensity ν or the extent to

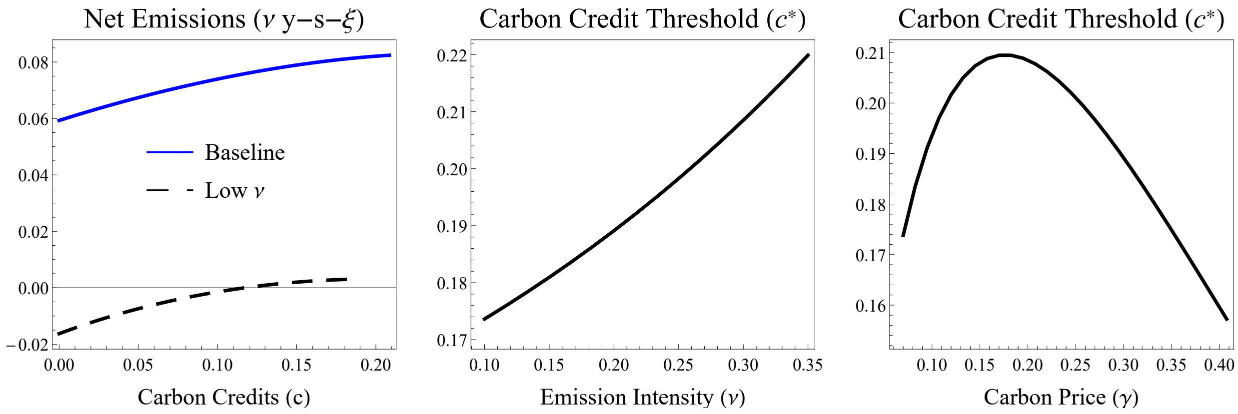

Figure 7: GREEN AND BROWN FIRMS IN THE CARBON CREDIT SYSTEM. The left panel shows how net emissions vary with respect to carbon credits under the baseline calibration (solid blue line) and considering a lower level of the industry-specific emission intensity $\nu = 0.15$ (dashed black line). The middle and right panels respectively illustrate how the endogenous upper bound of carbon credits c^*—at which the firm starts selling credits—varies with respect to ν and γ.

which innovation breakthroughs reduce emissions ξ—the left panels in Figures 5 and 6 illustrate that firms can be either net polluters or net cleansers in both the carbon tax and the carbon trading systems.

Notably, our model is capable of reproducing the negative relation between emissions and green innovation documented by Bolton, Kacperczyk, and Wiedemann (2023). When we vary firms' stock of carbon credits, results (2) and (3) of Proposition 4 jointly imply that firms exhibiting the higher emissions are less engaged in green innovation, controlling for any other source of technological differences across firms. Similarly, the left and middle panels in Figures 5 and 6 jointly imply that, regardless of the carbon pricing system, firms exhibiting larger emissions invest less in innovation. as we vary either the emission intensity ν or the sensitivity ξ.

As a last step, we assess the additional implications of the model on the optimal carbon credit trading policy of both brown and green firms. The middle panel of Figure 7 captures how industries with dirtier technologies (and hence greater ν) have greater incentives to store credits, so that c^* is increasing in ν. All else equal, as ν increases and net emissions are higher, so is the need to accumulate credits. Relatedly, the carbon credit price γ is either a cost or a gain depending of whether the firm is either a net polluter or a net cleanser, and it thus typically in the market to either buy or sell credits. The right panel of Figure 7 shows that this dual role of γ is reflected in the upper bound c^* at which the firm optimally starts selling credits, as c^* is hump-shaped in γ. A greater γ implies that the cost of buying credits is larger—which should lead the firm to keep a larger credit balance—but also implies that the gain from selling credits is greater too—then leading the firm to keep fewer credits.

7 Subsidy to green innovation

A key result of our analysis is that carbon regulation—being a carbon trading system or a carbon tax system—leads firms to tilt towards more immediate albeit transient policies to reduce their carbon emissions. A natural question then arises as to whether there are complementary tools that regulators can harness to partly offset this effect and encourage the investment in long-term, green innovation—whose outcome can lead to persistent improvements in firms' greenness.

In this section, we focus on one such tool: A subsidy to green innovation. To this end, we assume that the subsidy covers a portion ι of the firm's cost of innovation, which then decreases to $(1-\iota)\frac{z^2}{2}g_t\zeta$. Therefore, ι represents the magnitude of the subsidy, decreasing the effective cost of innovation. While the expression of the optimal abatement and production policies remain the same in this case, the optimal green innovation policy is

$$z^{\tau} = \frac{\phi}{\zeta(1-\iota)} v^{\tau}(\iota)\,(\lambda - 1) \tag{20}$$

in the carbon tax system, whereas it is

$$z(c) = \frac{\phi}{\zeta(1-\iota)} (\lambda v(c^*, \iota) - v(c, \iota))$$

in the carbon trading system. These equations suggest that because the innovation subsidy decreases the effective cost of green innovation, the firm's optimal innovation rate should increase. Moreover, the subsidy impacts firm value in both systems, so it should affect innovation expenditures through this channel too.

Figure 8 investigates quantitatively the impact of the innovation subsidy under both the carbon tax and the carbon trading systems. On the horizontal axis, we vary the magnitude of the subsidy ι. The figure shows that the subsidy effectively increases the firm's investment in innovation as well as the innovation share—and more so as the subsidy increases. That is, the subsidy counteracts the shift away from green innovation that is triggered by carbon pricing, and effectively tilts the green investment mix back to green innovation. Moreover, the subsidy

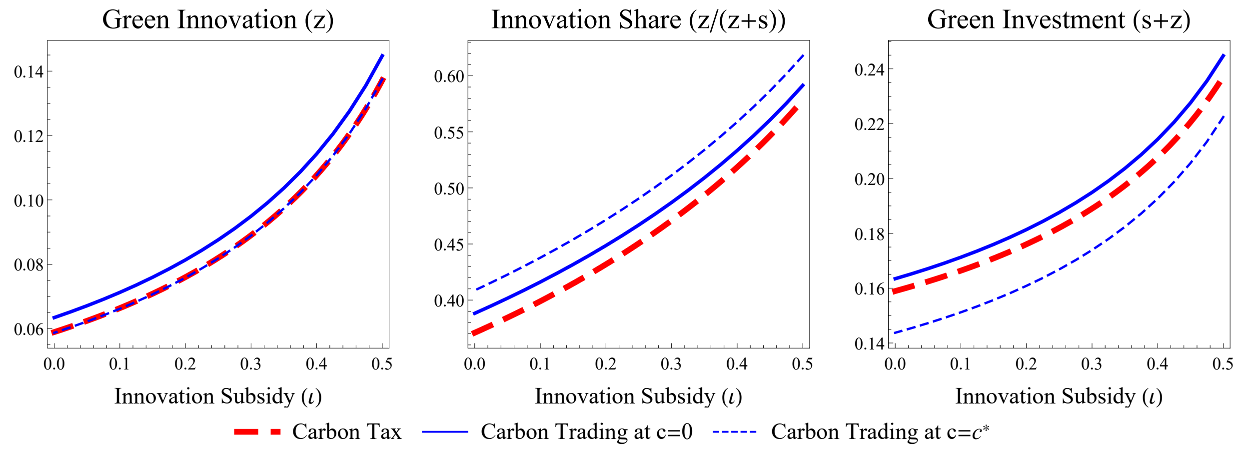

Figure 8: THE IMPACT OF THE INNOVATION SUBSIDY. The figure shows the optimal innovation rate, the innovation share, and total green investment (considering both abatement and innovation) as a function of the innovation subsidy ι and both in the carbon tax system (κ, red line) and carbon credit system (γ, blue lines, plotting the values of these quantities at $c = 0$ and at $c = c^*$)

spurs the total amount that firms spend on becoming green (abatement plus green innovation), as shown in the right panel.[28]

8 Endogenous carbon regulation

The main goal of our paper is to understand how firms respond to carbon pricing when allowing for a rich set of corporate policies, encompassing green investment projects of various horizon and risk profiles. So far, we have analyzed corporate policies taking carbon regulation as exogenous. In this section, we investigate whether our main results continue to hold if we allow a benevolent regulator to optimally choose both carbon pricing and the innovation subsidy.[29]

[28]Under the carbon trading system, abatement slightly decreases in the presence of the subsidy for interior levels of the cash reserves (whereas it does not at $c = 0$ or at $c = c^*$, where the value of credits is pinned down by γ and ψ). The reason is that the subsidy reduces the marginal value of carbon credits which, in turn, reduces the marginal benefit from investing in abatement (as illustrated by the numerator of equation (15)). Yet, such reduction in abatement is rather small and, overall, the increase in green innovation is the most notable firm outcome associated with the subsidy.

[29]In this section, we focus on the carbon tax case. Under the carbon trading system, the central authority sets the quantity of credits that are exchanged, whereas the market price of carbon credits is instead pinned down by demand and supply forces. Our partial equilibrium analysis in the carbon trading system (Section 5) collapses to the carbon tax case (Section 4) when there is no bid-ask spread ψ.

The economy We consider an economy that admits a representative agent with logarithmic preferences

$$E\left[\int_0^\infty e^{-\delta t}\ln \mathcal{C}_t\right], \tag{21}$$

where $\mathcal{C}_t$ denotes consumption and $\delta > 0$. Consumption is drained by the social cost of carbon, which we define below. The representative household is endowed with labor L, which is supplied inelastically and yields a competitive wage $\mathcal{W}_t$.

There is a final good sector that serves as the numeraire of the economy, whose output is denoted by $\mathcal{Y}_t$. This final good is produced competitively with a production function using labor and a continuum of inputs:

$$\mathcal{Y}_t = \int_0^1 (aY_{jt} + fg_{jt}L)\,dj, \tag{22}$$

where the last term captures that if the intermediate inputs are greener, labor becomes more productive (see, e.g., Acemoglu et al. (2012)). To nest our analysis so far in a general equilibrium setting, we consider that all intermediate goods are produced by polluting firms subject to carbon pricing—in particular, we assume intermediate firms are those analyzed so far in the paper, which not only produce but also invest in abatement and innovation. On top of the cost of inputs and of labor, and due to its sourcing of inputs from polluting firms, the final good sector bears a convex cost $\frac{b}{2g_{jt}}Y_{jt}^2$, which is greater the lower the degree of greenness (and, thus sustainability) of the input. When we solve the maximization problem of the final good sector, the resulting demand function for each intermediate product coincides with that assumed in equation (2)—indicating that our general equilibrium setting nests the partial equilibrium analysis from previous sections.[30]

We focus on a "balanced greening path" equilibrium, in which aggregate quantities become more sustainable at the constant, endogenous rate G, which aggregates the greening of all sectors of the economy. An equilibrium is an allocation such that: (i) intermediate firms set production, abatement, and green innovation to maximize value (as investigated so far); (ii) the final good sector maximizes profits; (iii) the representative household maximizes utility from consumption; (iv) a central authority sets carbon pricing and the innovation subsidy to maximize the utility of the representative agent; (v) all markets clear.

[30] See Appendix A.4 for the analytical details and calculations.

Endogenous aggregate quantities The representative agent's budget constraint implies:

$$\mathcal{C}_t = \mathcal{W}_t L + \mathcal{D}_t + \mathcal{T}_t - \Omega \int_0^1 \left(\nu Y_{jt} - s_{jt} g_{jt} - \xi g_{jt}\right) dj, \tag{23}$$

which spells out the sources of consumption of the representative agent. The first term represents labor income. The second term $\mathcal{D}_t$ represents dividends from the intermediate good sector, which are given by the flow of revenues of these firms net of abatement and innovation costs, and net of the cost imposed by carbon pricing on their emissions (see Appendix A.4 for details). The third term $\mathcal{T}_t$ represents the consumption flow coming from the regulator, which is given by the proceeds from carbon pricing net of the subsidy to innovation.[31] The last term of the equation is the social cost of carbon, which aggregates net emissions in the different sectors j. The parameter Ω captures the reduction in consumption due to a marginal increase in aggregate net emissions. Along the balanced greening path, all these quantities green at the endogenous rate G supported by green innovation in the intermediate sectors j with $e^{Gt} = \int_0^1 g_{jt} dj$.

Endogenous carbon regulation The regulator optimally sets carbon pricing and the innovation subsidy to maximize the inter-temporal utility of the representative agent. The corresponding first order conditions with respect to carbon pricing and the innovation subsidy are then given, respectively, by:

$$\frac{\partial y}{\partial \kappa}(a - \Omega\nu - by) + \frac{\partial s}{\partial \kappa}(\Omega - \kappa) + \left(\frac{\phi}{\delta}(\lambda - 1)\mathcal{C}_0 - \zeta z\right)\frac{\partial z}{\partial \kappa} = 0, \tag{24}$$

$$\left(\frac{\phi}{\delta}(\lambda - 1)\mathcal{C}_0 - \zeta z\right)\frac{\partial z}{\partial \iota} = 0. \tag{25}$$

The above first-order conditions characterize how optimal carbon regulation targets both the short-term goal of reducing emissions as well as the long-term goal of ensuring sustainable technological change. Equation (24) shows that optimal carbon pricing trades off reduced consumption due to lower profits—stemming from both a lower production rate and a higher engagement in abatement—against the upside of reducing aggregate carbon emissions, which overall reduces the social cost of carbon.[32] In turn, equation (25) reveals that the optimal innovation subsidy

[31] The proceeds from carbon pricing do not affect the representative agents' consumption directly: Carbon pricing reduces the dividend coming from the intermediate sector, and yet the consumer eventually receives the proceeds from the regulator. Similar arguments hold for the innovation subsidy, but flows run in reversed order.

[32] In fact, the last term in the first-order condition with respect to κ (equation (24)) is zero, as equation (25) needs to be zero too. If the central authority used carbon pricing only (i.e., no innovation subsidy), such term

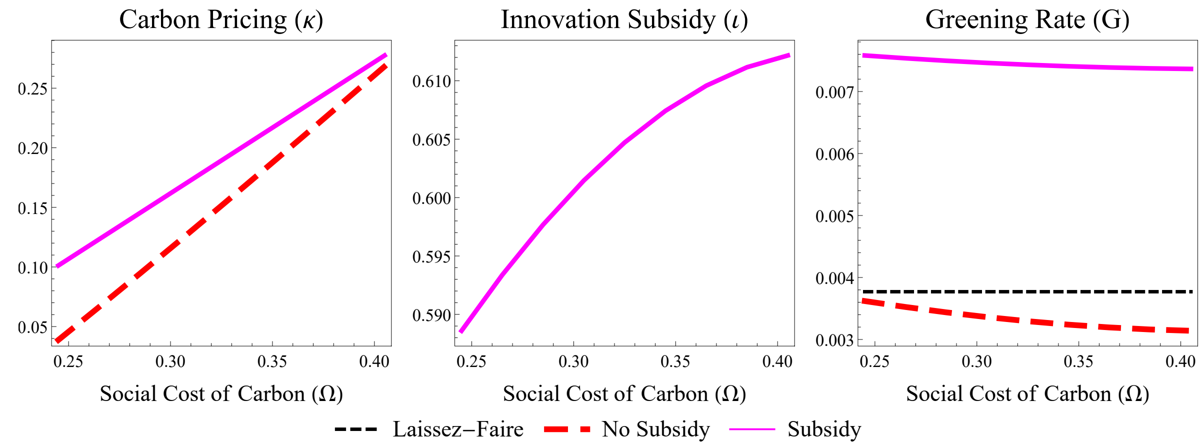

Figure 9: ENDOGENOUS CARBON REGULATION. The figure shows the optimal carbon price, the optimal innovation subsidy, and the ensuing greening rate of the economy. The short-dashed black line represents the laissez-faire benchmark, the red line represents the environment in which the regulator sets carbon pricing only, whereas the solid magenta line represents the environment in which the regulators pairs carbon pricing with the innovation subsidy.

balances the short-run reduction in consumption due to increased investment in innovation, with the long-term benefit of greening the economy at a faster pace.

Notably, when the regulator optimally uses both carbon pricing and the innovation subsidy, we obtain an explicit formula for the optimal price of carbon:

$$\kappa^* = \frac{\Omega(4b + 2\theta\nu^2) - a\theta\nu}{4b + \theta\nu^2}. \tag{26}$$

This expression predicts that the regulator optimally sets the carbon tax below the social cost of carbon Ω if the following inequality holds: $\nu\Omega < a$. The inequality requires that the social cost of an additional unit of production (the left-hand side) is lower than its value added (the right-hand side).[33]

Equilibrium implications. We provide further implications by solving the equilibrium numerically.[34] Figure 9 considers optimal carbon regulation in two cases: In the first, the regulator sets both carbon pricing and the innovation subsidy (solid magenta line); in the second, the reg-

need not to be zero, meaning that the regulator would need to address both short- and long-term goals with one instrument only. We analyze this case numerically in the next paragraph.

[33]Intermediate production y_j is positive if $\nu\Omega < a$ holds: When we evaluate y_j at κ^*, it holds that $y_j > 0$.

[34]In addition to our baseline calibration parameters, we consider $\delta = 0.02$, $f = 0.15$, and normalize L to 1. Our choice for f ensures the ratio of wages over value added in the final goods sector (i.e., the labor share) is in the range of 40% and 45% in our numerical analyses. We run comparative statics for the social cost of carbon.

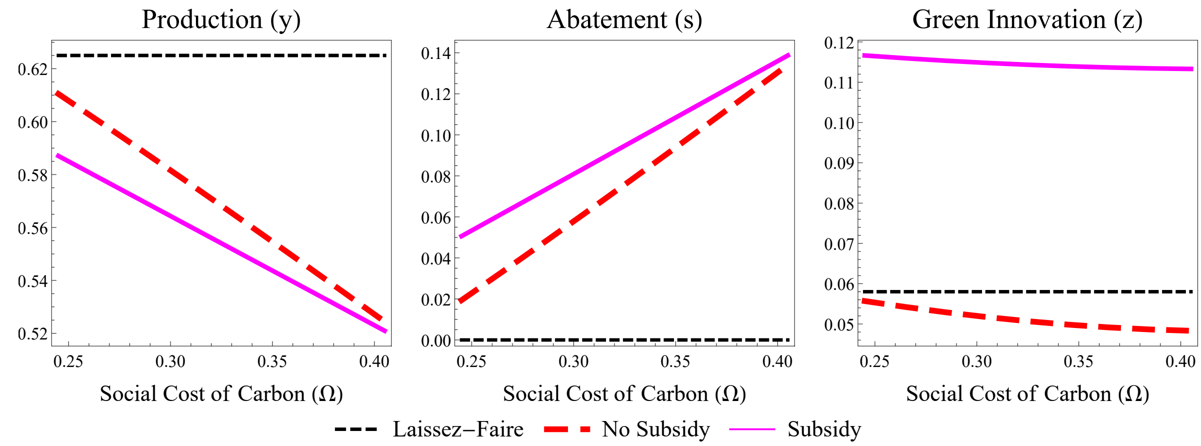

Figure 10: OPTIMAL CARBON REGULATION AND CORPORATE STRATEGIES. The figure shows the production, abatement, and innovation policies of the intermediate-good sector when carbon regulation is endogenous. The short-dashed black line represents the laissez-faire benchmark, the red line represents the environment in which the regulator sets carbon pricing only, whereas the solid magenta line represents the environment in which the regulators pairs carbon pricing with the innovation subsidy.

ulator uses carbon pricing only (dashed red line). The figure illustrates how optimal carbon regulation and the greening rate of the economy vary with the social cost Ω. Carbon pricing and the innovation subsidy both increase with the social cost of carbon. Interestingly, the optimal price of carbon is lower in the case with no subsidy. Moreover, the greening rate of the economy G is the lowest in this case, whereas it is the highest when carbon regulation contemplates an innovation subsidy on top of carbon pricing. As shown in Figure 10, this result is explained by the fact that, absent the innovation subsidy, carbon pricing leads polluting firms to invest less in innovation.[35]

Figure 10 illustrates the optimal corporate policies with endogenous carbon regulation as a function of the social cost of carbon Ω.[36] Importantly, and consistent with our previous analyses in partial equilibrium, we confirm that carbon pricing discourages innovation and encourages abatement for polluting firms. Yet, the innovation subsidy can offset this shift and help support sustainable technological change. Moreover, the figure confirms our result in partial equilibrium that carbon pricing reduces production below the laissez-faire benchmark.[37] Our results in

[35]Naturally, welfare is higher in the case in which carbon regulation embraces both carbon pricing and an innovation subsidy compared to either the laissez-faire benchmark or the case with carbon pricing only (and no subsidy).

[36]As a reminder, these firms in the intermediate good sector are the polluting firms on which we focused in our partial equilibrium analysis.

[37]Consistent with our partial equilibrium analysis, the quantities y, s, and z in Figure 10 are expressed at the firm level, and thus scaled by each firm's degree of greenness g_t.

Figure 10 further imply that carbon pricing reduces net emissions at the firm level.

Several implications follow from our analysis. First, when endogenizing carbon regulation, our analysis continues to indicate that a larger carbon price leads firms to shift from green innovation to abatement. Second, and relatedly, to stimulate the engagement in green innovation of polluting firms, environmental regulators may consider setting the carbon price below the social cost of carbon, then challenging the standard Pigouvian view. Third, complementing carbon pricing with a subsidy to green innovation stimulates a faster greening path (captured by a larger G). Moreover, when contemplating such a subsidy, the regulator can set the carbon price *higher*. Thus, our analysis reveals that carbon pricing and innovation subsidies target distinct goals—namely, carbon pricing guarantees that emissions are reduced in the short-term, whereas the innovation subsidy fosters a faster greening path.

9 Concluding remarks

The systematic control of firms' carbon emissions by regulators poses a new challenge in the corporate world, which involves maximizing shareholder wealth by developing an optimal carbon management policy. This paper provides a unified model to study, precisely, how firms should optimally manage carbon emissions through production, green investment (both abatement and green innovation), and the management of carbon credits. Our theoretical framework is broad enough to allow a comparison of optimal corporate policies under the three main regulatory schemes observed internationally—the carbon trading system, the carbon tax system, and a laissez-faire benchmark.

The main takeaway of our model is that carbon pricing leads firms to put more emphasis on more immediate albeit transient forms of green investment, such as abatement as opposed to green innovation. We show that subsidies to green innovation can partly undo this effect and, overall, boost the firm's engagement to becoming greener. Notably, these insights continue to hold when we endogenize carbon regulation.

Our analysis of firms' corporate policies further shows that, under a carbon trading scheme, firms adopt a precautionary behavior such as under-producing compared to the laissez-faire benchmark. This result challenges the conventional wisdom that the carbon trading scheme preserves a firm's ability to produce (and, thus, pollute) by making carbon credits tradable. We find that firms with a large stock of carbon credits are less committed to reducing emissions.

Last, we conclude that carbon regulation is not necessarily a cost for corporations, as firms can actually attain higher value than under laissez-faire if their net emissions are sufficiently negative.

In sum, our paper delivers new insights that speak to the long- and short-term effects of carbon regulation—a macroeconomic concern—but more importantly to the microeconomic and financial aspect of corporate carbon management. Our analysis can be extended in many ways—which we intend to examine in future research—to further uncover how the firms' new need to manage emissions can affect existing theories on corporate decision making.

A Appendix

A.1 Proof of the results in Section 3

Using the scaling property gives the scaled HJB equation:

$$rv^B = \max_{y^B, z^B, s^B} \quad \phi z^B v^B (\lambda - 1) + \left(y^B \left(a - b y^B \right) - \frac{(z^B)^2}{2} \zeta - \frac{(s^B)^2}{2} \theta \right).$$

Plugging equations (5) and (6) into this equation gives:

$$\frac{\phi^2}{2\zeta} (\lambda - 1)^2 (v^B)^2 - rv^B + \frac{a^2}{4b} = 0$$

which we solve with respect to v^B and obtain equation (7).

A.2 Proof of the results in Section 4

To solve for the firm's optimal policies in the carbon tax system, we again use the scaling property and get:

$$rv^\tau = \max_{y^\tau, z^\tau, s^\tau} \quad \phi z^\tau (\lambda - 1) v^\tau + \left(y^\tau (a - b y^\tau) - \frac{(z^\tau)^2}{2} \zeta - \frac{(s^\tau)^2}{2} \theta - \kappa (y^\tau \nu - s^\tau - \xi) \right). \tag{27}$$

Plugging the optimal policies into the above equation we have

$$\frac{\phi^2}{2\zeta} (\lambda - 1)^2 (v^\tau)^2 - rv^\tau + \frac{(a - \kappa\nu)^2}{4b} + \frac{\kappa^2}{2\theta} + \kappa\xi = 0.$$

so we solve for firm value

$$v^\tau = \frac{r - \sqrt{r^2 - \frac{\phi^2}{\zeta} (\lambda - 1)^2 \left[\frac{(a-\kappa\nu)^2}{2b} + \frac{\kappa^2}{\theta} + 2\kappa\xi \right]}}{\frac{\phi^2}{\zeta} (\lambda - 1)^2} \tag{28}$$

Proof of Proposition 1 Part (1) of the proposition is straightforward given that $\kappa > 0$ and $\nu > 0$. Revenues are given by $\frac{a^2}{4b}$ under laissez-faire whereas are given by $\frac{a^2 - \kappa^2 \nu^2}{4b}$.

Part (2) of the proposition is also straightforward given that $s^\tau > 0$ under the carbon tax system, whereas $s^B = 0$ under laissez.faire. Given the expression for s^τ, it is obvious to infer that investment in abatement increases with κ.

To show part (3) of the proposition, we start by noting that the expression for the optimal investment in innovation is the same in the laissez-faire and in the carbon tax environment but for the fact that firm value differs in the two environment. Thus, part (3) boils down to showing that polluting firms (i.e., firms exhibiting positive net emissions) are less valuable under the carbon tax system than under laissez-faire. Using the expressions for firm value, we have that $v^B > v^\tau$ if the following inequality holds:

$$\nu\frac{2a-\kappa\nu}{2b}-\frac{\kappa}{\theta}-2\xi>0. \tag{29}$$

A firm exhibits positive emissions under the carbon tax if the following inequality holds $y^\tau\nu - s^\tau - \xi \equiv \nu\frac{a-\kappa\nu}{2b}-\frac{\kappa}{\theta}-\xi>0$. In turn, a firm exhibits positive emissions under laissez-faire if $y^B\nu-\xi\equiv\frac{a\nu}{2b}-\xi>0$. Notably, condition (29) holds if the firm exhibits positive net emissions in these environments. Lastly, consider the first derivative of v^τ with respect to κ.

$$\frac{\partial v^\tau}{\partial\kappa}=\frac{\frac{\kappa}{\theta}+\xi-\nu\frac{a-\kappa\nu}{2b}}{\sqrt{r^2-\frac{\phi^2}{\zeta}\left(\lambda-1\right)^2\left[\frac{(a-\kappa\nu)^2}{2b}+\frac{\kappa^2}{\theta}+2\kappa\xi\right]}}$$

Notably, this derivative is negative if $\frac{\kappa}{\theta}+\xi-\nu\frac{a-\kappa\nu}{2b}<0$—in which case the firm exhibits positive emissions. The claim follows. ■

A.3 Proof of the results in Section 5

To solve the firm's problem when it accumulates credits, we use the homogeneity property explained in the main text. That is, we have:

$$V_C(C,g)=v'(c)\quad V_{CC}(C,g)=\frac{v''(c)}{g},$$

which we substitute into equation (12) and get the scaled HJB:

$$rv(c)=\max_{z,y,s}\;(s+\xi-y\nu)v'+\frac{\sigma^2}{2}y^2v''+\phi z\left(\lambda v(c^*)-v(c)\right)+\left(y(a-by)-\frac{z^2}{2}\zeta-\frac{s^2}{2}\theta-\chi c\right). \tag{30}$$

Differentiating equation (30) gives the optimal policies reported in equations (14), (15), and (16). Plugging these policies into (30) gives the following equation:

$$rv(c) = \frac{\phi^2}{2\zeta}(\lambda v(c^*) - v(c))^2 + \frac{(v'(c))^2}{2\theta} + \frac{(a-\nu v')^2}{2(2b-\sigma^2 v'')} - \chi c + \xi v'. \tag{31}$$

At $c = c^*$, this equation boils down to

$$\frac{\phi^2}{2\zeta}(\lambda-1)^2 v^2(c^*) - rv(c^*) + \frac{\gamma^2(1-\psi)^2}{2\theta} + \frac{(a-\nu\gamma(1-\psi))^2}{4b} - \chi c^* + \xi\gamma(1-\psi) = 0,$$

which we solve with respect to $v(c^*)$:

$$v(c^*) = \frac{\zeta}{\phi^2(\lambda-1)^2}\left[r - \sqrt{r^2 - 2\frac{\phi^2}{\zeta}(\lambda-1)^2\left[\frac{\gamma^2(1-\psi)^2}{2\theta} + \frac{(a-\nu\gamma(1-\psi))^2}{4b} - \chi c^* + \xi\gamma(1-\psi)\right]}\right]$$

Next we show Proposition 3.

Proof of Proposition 3 $v''(c) \leq 0$ means that v' is monotonically decreasing for any c. Towards a contradiction, suppose that there are two credit levels $0 < c_1 < c_2 < c^*$ such that $v'(c_1) = v'(c_2) = \Gamma > \gamma(1-\psi)$. This means that there is a $\bar{c} \in [c_1, c_2]$ at which v' attains a local maximum—i.e., $v''(\bar{c}) = 0$, $v''(c_1) > 0$, and $v''(c_2) < 0$. At c_1, firm value satisfies:

$$rv(c_1) = \frac{\phi^2}{2\zeta}\left(\lambda v(c^*) - v(c_1)\right)^2 + \frac{(v'(c_1))^2}{2\theta} + \frac{(a-\nu v'(c_1))^2}{2(2b-\sigma^2 v''(c_1))} - \chi c_1 + \xi v'(c_1). \tag{32}$$

whereas at c_2:

$$rv(c_2) = \frac{\phi^2}{2\zeta}\left(\lambda v(c^*) - v(c_2)\right)^2 + \frac{(v'(c_2))^2}{2\theta} + \frac{(a-\nu v'(c_2))^2}{2(2b-\sigma^2 v''(c_2))} - \chi c_2 + \xi v'(c_2). \tag{33}$$

Subtracting (33) from (32) gives:

$$\begin{aligned} r\left[v(c_1) - v(c_2)\right] =& \frac{\phi^2}{2\zeta}\left[\left(\lambda v(c^*) - v(c_1)\right)^2 - \left(\lambda v(c^*) - v(c_2)\right)^2\right] \\ &+ \frac{(a-\nu v'(c_1))^2}{2}\left[\frac{1}{(2b-\sigma^2 v''(c_1))} - \frac{1}{(2b-\sigma^2 v''(c_2))}\right] - \chi(c_1 - c2). \end{aligned} \tag{34}$$

Because v' is positive over the interval $[c_1, c_2]$, then $v(c_1) < v(c_2)$. Therefore, the left-hand side of this equation is negative. In turn, the first term on the right-hand side is positive, because

$c_1 < c_2$ and $v(c)$ is increasing in c. Because $v''(c_1) > 0$ and $v''(c_2) < 0$, the second term is positive too. Finally $-\chi(c_1 - c_2) > 0$ too. Thus, equation (34) does not hold. It then means such two levels c_1 and c_2 do not exist. v' then monotonically decreases in c, and the claim holds.
■

We now turn to prove Proposition 4.

Proof of Proposition 4 We start by showing part (1) of the claim. Differentiating the optimal policy gives:

$$y'(c) = \frac{-v''(c)\nu + \sigma^2 y(c)v'''(c)}{2b - \sigma^2 v''} \tag{35}$$

As $v'' \leq 0$ (by Proposition 3) and $y \geq 0$, then to prove the claim we simply need to show that $v''' > 0$ for any c. Differentiating equation (31) gives:

$$rv'(c) = \quad -\frac{\phi^2}{\zeta}(\lambda v(c^*) - v(c))v'(c) + \frac{v'(c)v''(c)}{\theta} + \frac{a - \nu v'}{2b - \sigma^2 v''}\left(-\nu v''(c) + \frac{\sigma^2(a - \nu v')v'''}{2(2b - \sigma^2 v'')}\right) - \chi + \xi v''.$$

At $c = c^*$, the equation becomes:

$$rv'(c^*) = \quad -\frac{\phi^2}{\zeta}(\lambda v(c^*) - v(c^*))v'(c^*) + \frac{a - \nu v'(c^*)}{2b}\left(\frac{\sigma^2(a - \nu v'(c^*))v'''(c^*)}{2(2b)}\right) - \chi.$$

Because the left-hand side is positive (recall that $v'(c^*) = \gamma(1 - \psi)$) whereas the first and last terms on the right-hand side are negative, $v'''(c^*)$ must be positive for the equation to hold. Toward a contradiction, suppose this is not the case and there is a point c^{**} at which $v'''(c^{**}) = 0$. Hence, there are two levels $c_1 < c^{**} < c_2$ such that $v''(c_1) = v''(c_2)$ (and negative by Lemma 3), and $v'''(c_1) < 0$ and $v'''(c_2) > 0$. Thus, we have at c_1

$$\begin{aligned} rv'(c_1) = &-\frac{\phi^2(\lambda v(c^*) - v(c_1))v'(c_1)}{\zeta} + \frac{v'(c_1)v''(c_1)}{\theta} \\ &+ \frac{a - \nu v'(c_1)}{2b - \sigma^2 v''(c_1)}\left(-\nu v''(c_1) + \frac{\sigma^2(a - \nu v'(c_1))v'''(c_1)}{2(2b - \sigma^2 v''(c_1))}\right) - \chi + \xi v''(c_1) \end{aligned} \tag{36}$$

and at c_2

$$\begin{aligned} rv'(c_2) = &-\frac{\phi^2(\lambda v(c^*) - v(c_2))v'(c_2)}{\zeta} + \frac{v'(c_2)v''(c_1)}{\theta} \\ &+ \frac{a - \nu v'(c_2)}{2b - \sigma^2 v''(c_1)}\left(-\nu v''(c_1) + \frac{\sigma^2(a - \nu v'(c_2))v'''(c_2)}{2(2b - \sigma^2 v''(c_1))}\right) - \chi + \xi v''(c_2). \end{aligned} \tag{37}$$

Subtracting (36) from (37) gives

$$r(v'(c_2)-v'(c_1)) = \frac{\phi^2}{\zeta}\left[(\lambda v(c^*) - v(c_1))v'(c_1) - (\lambda v(c^*) - v(c_2))v'(c_2)\right] + \frac{(v'(c_2) - v'(c_1))v''(c_1)}{\theta} + \frac{\nu^2 v''(c_1)(v'(c_2) - v'(c_1))}{2b - \sigma^2 v''(c_1)} + \sigma^2 \frac{(a - \nu v'(c_2))^2 v'''(c_2) - (a - \nu v'(c_1))^2 v'''(c_1)}{2(2b - \sigma^2 v''(c_1))^2}.$$

By Proposition 3, the left-hand side is negative, whereas the first, second, and third terms on the right-hand side are positive. Moreover, under the conjecture, $v'''(c_2) > 0$ and $v'''(c_1) < 0$ hold, and the last term is positive too. Hence, the ODE would not hold so it must be that v''' does not switch sign, so $y(c)$ is monotonically increasing. The result about the monotonicity of revenues stems from the monotonicity of $y(c)$. In fact, revenues are $y(c)(a - by(c))$, whose derivative is $(a - 2by(c))$, which is indeed positive.

Part (2) of the claim stems from Proposition (3), as $s'(c) = v''(c)/\theta$. In turn, $z'(c) = -\frac{\phi}{\zeta}v'(c)$– as $v'(c) \geq \gamma(1 - \psi) > 0$ for any c.

Part (3) of the proof stems from the monotonicity of $y(c)$ and $s(c)$. The claim follows. ■

Proof of Proposition 5 Part (1) follows from Proposition 3. I.e., the numerator of $y(c)$ (see equation (14)) is smaller than a, as $v' \geq \gamma(1 - \psi)$. In turn, because $v'' < 0$, the denominator of $y(c)$ is equal or greater than $2b$ as $v'' \leq 0$. As a result, because the quantity $y(a - by)$ is non decreasing in y up to $y = a/(2b)$, then revenues are also uniformly smaller in the trading system vis-à-vis the laissez-faire environment.

Part (2) of the proof simply stems from the fact that the firm does not invest in abatement in the laissez-faire benchmark, whereas $s(c) \geq \frac{\gamma(1-\psi)}{\theta} > 0$ for any c.

Because of the definition of the optimal innovation rate, Part (3) relies on studying the relative magnitude of firm value in the carbon trading system versus laissez-faire. Namely, $z(c^*) < z^B$ if $v(c^*) < v^B$. Using the quantities previously derived, this boils down to

$$\nu\frac{2a - \nu\gamma(1 - \psi)}{2b} - \frac{\gamma(1 - \psi)}{\theta} - 2\xi + \frac{2\chi c^*}{\gamma(1 - \psi)} > 0$$

which holds if net emissions are positive under carbon trading and under the laissez faire system (plus, the last term on the left-hand side is non-negative). The result follows. ■

Proof of Proposition 7 Because $z(c)$ decreases with c, then $z(c) > z^B$ holds for any c if it is true at $c = c^*$. At $c = c^*$, this is guaranteed if $v(c^*) > v^B$. The inequality $v(c^*) > v^B$ boils down to:

$$a - \gamma(1-\psi)\left[\frac{\nu}{2} + \frac{b}{\theta\nu} + \frac{2\xi b}{\gamma(1-\psi)\nu} - 2\chi c^* \frac{b}{\gamma^2(1-\psi)^2\nu}\right] < 0. \quad (38)$$

Equivalently, this equation implies that $v(c^*) > v^B$ if a is sufficiently low.

Consider now the firm's net emissions. Suppose that these are positive for at least some values of c. Because they are monotonic increasing in c, it implies that they are at least positive at c^*.

$$\nu y(c^*) - s(c^*) - \xi = \frac{a\nu - \nu^2\gamma(1-\psi)}{2b} - \frac{\gamma(1-\psi)}{\theta} - \xi > 0$$

By calculations, this boils down to

$$a - \gamma(1-\psi)\left[\nu + \frac{2b}{\theta\nu} + \frac{2b\xi}{\nu\gamma(1-\psi)}\right] > 0$$

However, this inequality cannot coexist with equation (38). The claim follows. ■

A.4 Proof of the results in Section 8

The final good sector maximizes:

$$\mathcal{Y}_t - \mathcal{W}_t L - \int_0^1 p_{jt} Y_{jt} dj - \int_0^1 \frac{b}{2g_{jt}} Y_{jt}^2 dj$$

where the second and third terms are, respectively, the cost of intermediate inputs and the cost of labor. The above maximization gives the following demand function for the intermediate products:

$$p_{jt} = a - \frac{b}{g_{jt}} Y_{jt}$$

which is the demand function used in our partial equilibrium analysis. The only difference is that the price of carbon, the subsidy, and the interest rate are endogenous.

The expected increase in sustainability in the input j is given, per time interval dt, by

$$E_{t-}[dg_{jt}] = g_{jt-}(\lambda - 1)\phi z_{jt} dt.$$

Breakthroughs occur at independent Poisson times in different industries j. By the law of large numbers, the growth rate of sustainability gives

$$\int_0^1 g_{jt} dj = \bar{g}_t = e^{Gt} \tag{39}$$

as we assume a balanced greening path.[38]

Under the assumption of the balanced greening path, the output of the consumption good sector is given by

$$\mathcal{Y}_t^* = \int_0^1 \left(a\frac{a-\nu\kappa}{2b} g_{jt} + f g_{jt} L \right) dj = \left(a\frac{a-\nu\kappa}{2b} + fL \right) \int_0^1 g_{jt} dj$$

Moreover, because the final good sector is competitive, the ensuing equilibrium wage is

$$\mathcal{W}_t L = \mathcal{Y}_t^* - \int_0^1 \left(p_{jt} Y_{jt} - \frac{b}{2g_{jt}} Y_{jt}^2 \right) dj = \left(\frac{(a-\nu\kappa)^2}{8b} + fL \right) \int_0^1 g_{jt} dj$$

In turn, the aggregate dividends to the representative agent from the final good sector amounts to:

$$\mathcal{D}_t = \int_0^1 \left[Y_{jt} p_{jt} - \frac{(z_{jt})^2}{2} \zeta (1-\iota) g_{jt} - \frac{(s_{jt})^2}{2} g_{jt} \theta - \kappa (Y_{jt}\nu - s_{jt} g_{jt} - \xi g_{jt}) \right] dj.$$

Namely, the flow of dividends to the representative agent aggregates revenues net of abatement and innovation expenditures and net of carbon pricing in the different product lines. Finally, the net flow to the representative agent coming from the central authority is given by the proceeds from the carbon tax net of the subsidy to the intermediate good sector, which are given by

$$\mathcal{T}_t = \int_0^1 \kappa (Y_{jt}\nu - s_{jt} g_{jt} - \xi g_{jt}) dj - \iota \int_0^1 \frac{(z_{jt})^2}{2} \zeta g_{jt} dj.$$

Using the budget (23) and under the assumption of a balanced greening path, the initial consumption of the representative agent is given by:

$$\begin{aligned} \mathcal{C}_0 &= y(a - by) - \frac{z^2}{2}\zeta - \frac{s^2}{2}\theta - \Omega\left(\nu y - s - \xi\right) + \mathcal{W}_0 L \\ &= \frac{a-\nu\kappa}{2b} a - \frac{(a-\nu\kappa)^2}{8b} + fL - \frac{\phi^2(\lambda-1)^2}{2\zeta} (v^\tau)^2(\kappa,\iota) - \frac{\kappa^2}{2\theta} - \Omega\left(\nu\frac{a-\nu\kappa}{2b} - \frac{\kappa}{\theta} - \xi \right) \end{aligned}$$

[38] Initial greenness is normalized to one for any j.

The agent maximizes utility from consumption so that the equilibrium interest rate satisfies $r = \delta + G$. Moreover, the social planner maximizes the inter-temporal utility of the representative agent, which is given by:

$$E\left[\int_0^{\infty} e^{-\delta t} \ln \mathcal{C}_t\right] = \frac{1}{\delta}\left[\ln(\mathcal{C}_0) + \frac{G}{\delta}\right]$$

which is increasing in the expected level of greenness of the economy G, consistent with Acemoglu et al. (2012). Taking the first derivative of the above with respect to the carbon price κ and the innovation subsidy ι gives the first-order conditions reported in the main text (equations (24) and (25), respectively).

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Table 1: Baseline parameters.

Parameter	Description	Value
r	Risk-free rate	0.020
ν	Emission intensity	0.300
σ	Emission volatility coefficient	0.300
θ	Abatement cost coefficient	2.000
ζ	R&D cost coefficient	8.000
ϕ	Poisson coefficient of green breakthroughs	1.000
λ	Jump upon a green breakthrough	1.065
ξ	Effect of sustainability on emissions	0.001
γ	Price of carbon credits	0.200
κ	Carbon tax	0.200
ψ	Fee on carbon sales	0.150
χ	Maintenance cost	0.0125
a	Maximum clearing price	0.500
b	Slope of the inverse demand function	0.400

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