State-dependent pricing and cost-push inflation in a production network economy *

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Abstract

This paper analyzes the link between state-dependent pricing and cost-push inflation in a multi-sector new-Keynesian model with input-output linkages and state-dependent price rigidity. Empirically, I estimate sector-specific price flexibility and the degree of its state dependence by fitting the model to sectoral price and wage series for the US. I find a significant degree of state dependence in most sectors of the US economy. Theoretically, I show that state-dependent pricing can change the size and reverse the sign of cost-push inflation compared to the non-state-dependent pricing model. Based on the empirical estimates of sector-specific state dependence, I evaluate the quantitative importance of state-dependent pricing for the cost-push inflation in the US over time. State dependence substantially affects model-implied cost-push inflation during particular historical episodes - after the Great Recession and during and after the Covid crises.

Keywords: production networks, state-dependent pricing, cost-push inflation

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1 Introduction

The observed inflation rate consists of the underlying demand and cost-push factors, and discerning their relative contributions is vital for effective monetary policy. The multi-sector New Keynesian literature establishes that cost-push inflation may result from sectoral shocks and depends on the input-output structure of the economy and the price rigidity distribution across sectors (Erceg et al., 2000; Aoki, 2001; La'O and Tahbaz-Salehi, 2020; Rubbo, 2020). This literature, however, pays limited attention to the importance of the price rigidity framework as it relies on non-state-dependent pricing approximation (e.g., Calvo). The limitation of non-state-dependent pricing is that the degree of price rigidity in each sector is fixed. Therefore, state-dependent pricing presents a more realistic approximation of pricing behavior as it allows price rigidity to change with shock size. If state dependence is a quantitatively important feature of pricing, a model with a fixed degree of price rigidity could yield an incorrect assessment of the size and sign of the cost-push effect.

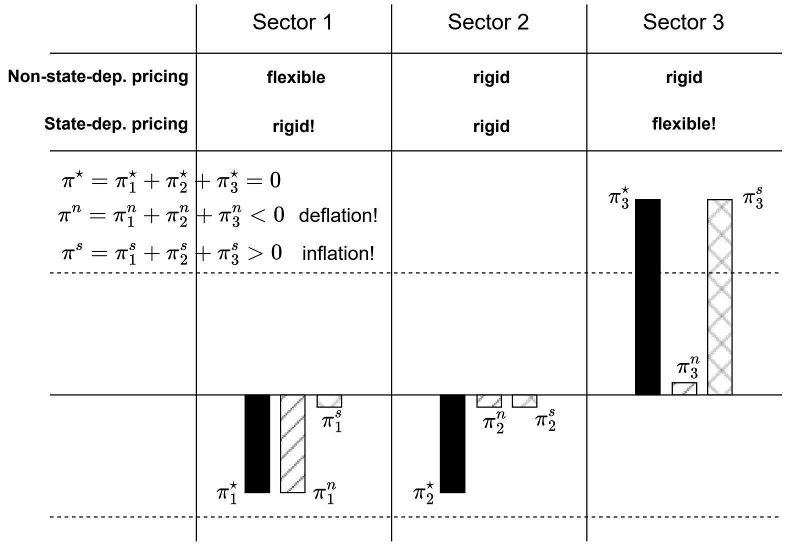
This project analyzes how sector-level state-dependent pricing shapes cost-push inflation in a multi-sector New-Keynesian model. Specifically, it shows that the empirically plausible degree of sector-specific state dependence can change the size and the sign of cost-push inflation compared to the case of non-state-dependent pricing.

Figure 1 provides a three-sector example illustrating the possible implications of state-dependent pricing for inflation in a multi-sector economy. Solid bars represent the desirable price adjustment in each sector such that the aggregate desired inflation is zero. If pricing is non-state-dependent, the degree of price flexibility in each sector is fixed in advance: Sector 1 has fully flexible prices, while Sectors 2 and 3 have fully rigid prices. In this case, Sector 1 is the only sector adjusting its price, and the aggregate inflation is negative. In contrast, if pricing is state-dependent, the degree of price flexibility depends on the size of the desired price change. In this case, only Sector 3 adjusts since its desired price change is sufficiently large, and the resulting aggregate inflation is positive. In this example, non-state-dependent pricing yields *deflation* driven by Sector 1 while state-dependent pricing yields *inflation* driven by Sector 3.

The analysis of this paper relies on the New-Keynesian Input-Output framework (La'O and Tahbaz-Salehi, 2020; Rubbo, 2020), which I extend to include state-dependent price rigidity at the sectoral level. This contrasts the existing literature, which also considers the I-O structure but relies on non-state-dependent price rigidity. State-dependent price rigidity is both intuitive and empirically plausible framework[1], but the conventional state-dependent pricing models, such as the menu cost model, do not allow the analytical solution. To overcome this problem, I model state dependence as a combination of a sticky information model (Mankiw and Reis, 2002) with a heterogeneous inattention framework. This approach

[1] See Nakamura and Steinsson (2008); Eichenbaum et al. (2011); Campbell and Eden (2014); Cavallo and Rigobon (2016); Carvalho and Kryvtsov (2021).

Figure 1: Price adjustment with non-state-dependent and state-dependent pricing



Hashed bars and π^n cross-hashed bars and π^s show price adjustment under state-dependent pricing; aggregate inflation is a sum of price changes in each sector.

yields the degree of price flexibility that corresponds to information flexibility, which in turn depends on the sector-specific state of the economy.

The inattention framework requires establishing a set of variables tracked by inattentive firms. Based on the equilibrium sectoral marginal cost equations, I assume that firms in each sector track only one variable, which I call the relevant productivity state. The relevant productivity state is a linear combination of sector-specific productivities that affect marginal cost in a given sector directly or through the input-output network. Changes in the relevant productivity state trigger a subset of firms to update their information about the economy, and those who update receive the full information.

The model consists of two blocks of equations. The first block gives equilibrium sectoral prices and quantities for a given set of sector-specific markups. This block does not depend on the price rigidity framework. The second block links sectoral markups to price changes through sectoral price rigidities. The novel contribution of the present work lies in exploiting this two-block structure to estimate sector-specific price flexibility and the sector-specific degree of state dependence. I assume that sectoral price flexibility consists of the average price flexibility component and the state-dependent component. Average flexibility differs across sectors but is fixed over time. This component corresponds to a non-state-dependent pricing framework with heterogeneous price flexibilities across sectors, a framework widely used in the literature. The state-dependent component captures how sector-specific price flexibility changes with the size of shocks affecting the sector directly or through the production network, which is summarized by the changes in the relevant productivity state.

Sector-specific average flexibility and the degree of state dependence are required for the realistic calibration of the state-dependent price rigidity in the model. The strategy behind estimating these two groups of parameters exploits the properties of model-implied contemporaneous prices/markups response to sectoral productivity shocks. Intuitively, sector-specific price flexibility is high if prices respond strongly to shocks. If the sensitivity of prices to shocks depends on the shock size - price flexibility is state-dependent. I start the estimation by calibrating the model block, which does not depend on price rigidity; in particular, I use the US input-output table to calibrate sectoral shares. The calibrated model consists of more than 360 sectors. I use monthly time series of sectoral prices and wages for the US to compute the relevant productivity state required for price flexibility and state-dependence estimation. The model with a calibrated I-O network allows the compute model-implied sector-specific productivities and markups at monthly frequency. The relatively high frequency of the data ensures that the contemporaneous price/markup response to productivity shock contains information about price rigidity as opposed to the policy reaction.

The average price flexibility and state-dependence estimates indicate that about 70% of sectors in the US waited by consumption share exhibit a statistically significant degree of state dependence in their price adjustment. The degree of state dependence differs strongly across sectors. The non-state-dependent component of price flexibility is also strongly heterogeneous across sectors, with commodity-related sectors having a higher degree of price flexibility on average.

The non-state-dependent component of price flexibility is positively correlated with the volatility of the relevant productivity state across sectors, meaning that the sectors with more volatile costs have more flexible prices on average. In contrast, the degree of state dependence negatively correlates with the relevant productivity state volatility. Sectors with the low volatility of relevant productivity state exhibit a higher degree of state dependence.

Theoretically, I show that the Phillips curve residual, which captures the cost-push effect in the model, can be decomposed into the main and input-output components. This decomposition helps analyze the effect of a state-dependent pricing framework compared to non-state-dependent pricing. The main effect captures the cost-push inflation in a counterfactual economy where reset prices equal the efficient prices. The I-O component captures the effect of "real rigidity" that arises in equilibrium due to the propagation of nominal rigidity through input-output network. The propagation of nominal rigidity makes reset prices differ from their efficient counterparts. While the I-O effect plays the amplifying role, it is the main component that largely shapes the cost-push effect. State-dependent pricing shapes the main component in significant ways. I show that state-dependent pricing may reverse the sign of the main component of the cost-push effect compared to non-state-dependent pricing.

From the model with a price rigidity framework calibrated according to the empirical estimates, I compute the cost-push effect for the US and the counterfactual effect without a state-dependent pricing component. Overall, the state-dependent pricing model produces a more volatile cost-push effect. State dependence plays a different role during different historical periods. In 2009, just after the Great Recession, the cost-push effect was positive in both state-dependent and non-state-dependent pricing models; state-dependence is an amplifying mechanism that strengthens the cost-push effect. In contrast, after the Covid crisis, which started in 2019, the state-dependent pricing model generated a cost-push effect, often having a different sign than the non-state-dependent pricing model. For instance, the state-dependent pricing model predicted a negative cost-push effect when the Covid crisis started and a positive cost-push effect when the Ukraine war broke out. At the same time, the non-state-dependent pricing model gives the opposite prediction. To validate the model-implied cost-push effect, I show that it has significant explanatory power when added to a standard Phillips curve regression and outperforms the oil prices and the non-state-dependent counterpart in explaining inflation fluctuations.

Finally, the analysis of sector-specific contribution reveals that while the 2009 cost-push effect was largely attributed to petroleum, the post-Covid cost-push episodes are attributed simultaneously to many groups of sectors, including healthcare, financial and insurance, and utilities.

2 Related literature

This paper relates to the literature on monetary policy trade-offs in multi-sector economies. Aoki (2001) study a two-sector horizontal economy and show that with one sticky and one flexible sector, cost-push inflation appears in response to sector shocks. Erceg et al. (2000) show that upstream rigidity (sticky wages) results in a monetary policy trade-off in a two-sector vertical economy. More recently, La'O and Tahbaz-Salehi (2020) and Rubbo (2020) showed that monetary policy trade-off arises in a more general production network economy under information-related price rigidity and Calvo-type price rigidity. The common feature of all these studies is the time-constant degree of price rigidity in each sector. However, Ball and Mankiw (1995) argue that what contributes to cost inflation is a combination of state-dependent price rigidity with asymmetric distribution of desired relative price changes. Building on Ball and Mankiw (1995) conceptual insight, I aim to understand the importance of state dependence for cost-push inflation in a production network economy.

The paper relates the macroeconomic literature on production networks. Seminal contributions include Long Jr and Plosser (1983) and Acemoglu et al. (2012) who develop the framework for efficient production network economy and Baqaee and Farhi (2020), Bigio and La'o (2020) who contribute to the analysis of inefficient network economy with exogenous markups. Similarly to monetary models of La'O and Tahbaz-Salehi (2020) and Rubbo (2020), I endogenize markups by introducing a price rigidity framework. However, in contrast to these papers, I use a price rigidity mechanism based on ad hoc heterogeneous inattention, which allows modeling state-dependent price rigidity at a sectoral level.

The empirical evidence of state-dependent pricing is extensive. Nakamura and Steinsson (2008) show that the frequency of price increases positively depends on inflation in the micro-data underlying the U.S. CPI index. Eichenbaum et al. (2011) and Campbell and Eden (2014) report evidence of the state-dependent frequency of price changes in the U.S. scanner data. Cavallo and Rigobon (2016) report a bi-modal distribution of price changes in online price data; bi-modal distribution is a feature of state-dependent models. Carvalho and Kryvtsov (2021) find evidence of strong selection effect into price adjustment in the micro-data underlying CPI of the U.K., Canada, and the U.S. I contribute to the current stock of evidence of state-dependent price adjustment by providing sector-specific (at the BEA code level) measures of state-dependence. While existing evidence largely relies on micro-level data, my estimation method relies on a production network model combined with sector price and wage data.

In terms of approach towards modeling state-dependent price rigidity, this paper belongs to sticky information literature (Mankiw and Reis (2002)) and behavioral inattention literature (Gabaix (2019)) as my state-dependent price rigidity combines these two features. Compared to the two conventional rationality-based frameworks, that is menu-cost approach (Dotsey et al. (1999), Caballero and Engel (2007)) and rational inattention approach (Sims (2003), Reis (2006)) my model remains analytically tractable.

Finally, this paper relates to the literature on money non-neutrality. Nakamura and Steinsson (2010) show that intermediate inputs can fix the weak money non-neutrality feature of menu-cost models brought up by Caplin and Spulber (1987), Golosov and Lucas Jr (2007)). The ability of intermediate inputs to increase money non-neutrality has also been documented for production network models with a heterogeneous but time-invariant degree of price rigidity by Shamloo (2010), Bouakez et al. (2014) and Pasten et al. (2020). The non-neutrality of money literature deals with the real effects of monetary policy. In contrast, this paper focuses on the state-dependence of monetary policy trade-offs.

3 Model description

The model is a multi-sector general equilibrium model with prodution network and state-dependent sectoral price rigidity. Two features are specific to the present model 1) sector-sepcific labor, allowing to make use of sectoral wage data, 2) custom price rigidity framework based on behavioral inattention and sticky information, allowing for a relatively simple treatment of state-dependent sectoral price rigidity. Next, I describe the model setup.

3.1 Firms

There are N production sectors. In each sector, there is a continuous number of monopolistically competitive firms indexed by $k \in [0,\ 1]$. Sectoral output and price indices are the CES sums across all firms within a sector. Sectoral output index is $Y_{t,i} = \left(\int_0^1 Y_{t,i,k}^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ and sectoral price index is $P_{t,i} = \left(\int_0^1 P_{t,i,k}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$. The firm-specific demand is

$$Y_{t,i,k} = \left(\frac{P_{t,i,k}}{P_{t,i}}\right)^{-\epsilon} Y_{t,i} \tag{1}$$

Production technology is constant returns to scale and is given by

$$Y_{t,i,k} = A_{t,i} L_{t,i,k}^{\alpha_i} \prod_j X_{t,ij,k}^{\omega_{ij}(1-\alpha_i)}$$

where $A_{t,i}$ is sector-specific productivity, $L_{t,i,k}$ is labor used by firm k of sector i, $X_{t,ij,k}$ is input of sector j used by firm k in sector i; α_i corresponds to the labor share in production costs and ω_{ij} corresponds to the share of input j in the intermediate input costs. Sectoral productivity follows

$$log(A_{t,i}) = log(\bar{A}_i) + log(A_{i-1}) + \epsilon_{t,i} \tag{2}$$

where $\epsilon_{t,i}$ is productivity shock in sector i; productivity shocks may be correlated across sectors; $log(\bar{A}_i)$ is the growth rate of productivity, which is set to 0 in the model.

The combination of inputs is chosen to minimize the unit cost of production, given input prices. Let $MC_{t,i}$ be the marginal cost in sector i, which is the same for all firms within sector i. Cost-minimizing resource allocation yields sectoral labor demand and intermediate input demand

$$W_{t,i} L_{t,i} = \alpha_i MC_{t,i} Y_{t,i} \tag{3}$$

$$P_{t,j} X_{t,ij} = (1-\alpha_i)\omega_{ij} MC_{t,i} Y_{t,i} \tag{4}$$

Then, marginal cost of production in sector i is

$$MC_{t,i} = \frac{1}{\alpha_i^{\alpha_i} \prod_j (\omega_{ij}(1-\alpha_i))^{\omega_{ij}(1-\alpha_i)}} \cdot \frac{1}{A_{t,i}} \cdot W_{t,i}^{\alpha_i} \prod_j P_{t,j}^{\omega_{ij}(1-\alpha_i)} \tag{5}$$

Input-output matrix Ω is such that Ω_{ij} is a share of input j in total cost of product j, $\Omega_{ij} = (1-\alpha_i)\omega_{ij}$. $L = (I-\Omega)^{-1}$ is the corresponding *Leontief inverse* matrix capturing the total effect of shocks (see Baqaee and Farhi (2020)). The total effect consists of the direct effect and the effect arising through the production network.

Firms have imperfect information (to be precised below) such that firm k in sector i has sectoral marginal cost belief $\widetilde{MC}_{t,i,k}$. Firm k sets the price $P_{t,i,k}$ to maximize its perceived profits

$$P_{t,i,k}Y_{t,i,k} - (1-\bar{\tau})\widetilde{MC}_{t,i,k}$$

subject to demand constraint (1); $\bar{\tau} = \frac{1}{\epsilon}$ is a subsidy correcting the inefficiency stemming from monopoly power. The price set by firm k is

$$P_{t,i,k} = \widetilde{MC}_{t,i,k}$$

Firm price can be expressed as $P_{t,i,k} = \frac{\widetilde{MC}_{t,i,k}}{MC_{t,i}} MC_{t,i}$. Undesired firm-specific markup resulting from information rigidity is $\frac{\widetilde{MC}_{t,i,k}}{MC_{t,i}}$. I define $\mathcal{M}_{t,i}$ to be the average markup in sector i, such that

$$P_{t,i} = \mathcal{M}_{t,i} \cdot MC_{t,i} \tag{6}$$

3.2 Information structure

Information updating by firms relies on sticky information framework (Mankiw and Reis (2002)) altered by an ad hoc heterogeneous inattention across firms to allow for state-dependence in the intensity of information updating.

3.2.1 Sticky information

Let $F_{t,i}$ be share of firms in sector i updating their information in the period t. Those firms who update observe the true sectoral marginal costs $MC_{t,i}$ and set their prices to $P_{t,i|t} = MC_{t,i}$. The share of firms which last updated their information 1 periods ago is $F_{t-1,i} \cdot (1 - F_{t,i})$. The share who has updated h periods ago is $F_{t-h,i} \cdot \prod_{s=0}^{h-1}(1 - F_{t-s,i})$. Those who updated their information h periods ago set their price to the perceived marginal costs under h-periods outdated information $P_{t,i|t-h} = E_{t-h}MC_{t,i}$. The average price in sector i consists of individual prices

$$P_{t,i}^{1-\epsilon} = F_{t,i} \cdot (MC_{t,i})^{1-\epsilon} + \sum_{h=1}^{\infty} \left\{ \left[\prod_{s=0}^{h-1}(1 - F_{t-s,i}) \right] \cdot F_{t-h,i} \cdot (E_{t-h}MC_{t,i})^{1-\epsilon} \right\} \tag{7}$$

3.2.2 Inattention

In a conventional sticky information model the share of firms updating their information at any given period is constant over time. In contrast, I assume that this share is affected by the fluctuations in the underlying *relevant productivity state* as these fluctuations lead to the time-varying the intensity of information acquisition.

Definition 1 (Relevant productivity state)**.** Relevant productivity state for sector i, denoted as $s_{t,i}$, is a combination of sectoral productivities $s_{t,i} = -\sum_j l_{ij} \cdot log(A_{t,j})$, such that each sector enters this combination with the weight corresponding to the strength of its effect on the equilibrium marginal costs in sector i; weights l_{ij} are the elements of the Leontief inverse matrix.

Intuitively, if sector i is strongly connected to sector j through input-output network, then productivity changes in sector j affect marginal cost in sector i, making sector j productivity changes relevant for sector i marginal cost. Later, I show how change in $s_{t,i}$ may be computed in a log-linearized model from productivity innovations and the input-output parameters. For now, let me denote the sector i relevant productivity state by $s_{t,i}$. I assume that changes in relevant productivity state affect the intensity with which firms in sector i update their information. Next, I describe the inattention framework which results in such outcome.

Let firms in sector i have heterogeneous degree of inattention. That is, every period the degree of inattention of firm k in sector i is drawn from a cumulative distribution $x \sim F_i$. Firms in sector i track absolute size of fluctuations is the relevant state $|\Delta s_{t,i}|$ where $\Delta s_{t,i} = s_{t,i} - s_{t-1,i}$. Only firms with low enough degree of inattention $x < |\Delta s_{t,i}|$ update their information set. As a result, the share of firms updating their information set in sector i is

$$F_{t,i} = Pr\{x < |\Delta s_{t,i}|\} = F_i(|\Delta s_{t,i}|) \tag{8}$$

The big-sized changes in the relevant productivity state push more firms to update their information set[2]. The time-varying share of firms updating their information each sector distinguishes the present model from the previous literature and allows addressing the role of state-dependent pricing without losing the tractability of the model.

3.3 Households

Representative household chooses final consumption good Y_t and hours worked $L_{t,i}$ in each sector to maximize utility subject to budget constraint. Household utility is

$$u(Y_t) - \sum_i v(L_{t,i})$$

[2]Similar technique for modeling partial adjustment within a group has been applied in generalized menu-cost models. In these models the cost of price adjustment is heterogeneous across firms which results in partial price adjustment (Caballero and Engel (2007)).

where final consumption Y_t good is a combination of sectoral consumption goods $C_{t,i}$

$$Y_t = \prod_i C_{t,i}^{\beta_i} \tag{9}$$

with $\sum_i \beta_i = 1$. The household's budget constraint is $P_t Y_t = \sum_i P_{t,i} C_{t,i} = \sum_i W_{t,i} L_{t,i} + T_t$, where P_t is the consumer price index, $W_{t,i}$ are sectoral wage rates, T_t are net transfers (including lump sum taxes and subsidies as well as profits from firm ownership). Optimal allocation of consumption across sectors yields sectoral consumption demand

$$P_{t,i} C_{t,i} = \beta_i P_t Y_t \tag{10}$$

Consumer price index is

$$P_t = \prod_i \left(\frac{P_{t,i}}{\beta_i} \right)^{\beta_i} \tag{11}$$

The functional form of utility is $u(Y) = ln(Y)$ and $v(L_i) = \frac{L_i^{1+\gamma}}{1+\gamma}$. Then, optimal consumption-leisure trade-off yields sectoral labor supply

$$W_{t,i} = L_{t,i}^{\gamma} P_t Y_t \tag{12}$$

3.4 Monetary policy

Monetary policy controls money supply which equals nominal spending, that is

$$P_t \cdot Y_t = M_t \tag{13}$$

3.5 Equilibrium

In equilibrium, all markets clear given the described behavior of firms and households. Product market clearing in sector i implies that product of sector i is either consumed or used as intermediate input.

$$Y_{t,i} = C_{t,i} + \sum_j X_{t,ji} \tag{14}$$

3.6 Log-linear model

The model is given by equations (1)-(14). I log-linearize the model around the efficient steady state. Efficient steady state is an equilibrium in which productivities are $A_i = 1$ and markups are $\mathcal{M}_i = 1$ for every sector i.

Throughout the paper, I denote column vectors $[X_1, ..., X_N]'$ with corresponding bold letters $\boldsymbol{X}$. Log-deviation of X is denoted by small x, so that $x = log(X) - log(\bar{X})$. Next, I list the key log-linear equations which are used in the further analysis. All derivations are available in Appendix A.

Sectoral wages. Log-linear equilibrium link between wages and markups is obtained by combining the product market clearing condition (14) with the conditions for optimal input allocation (3)-(4), and the link between sectoral prices and marginal costs (6). The resulting system of wage equations is

$$\boldsymbol{w_t} = (p_t + y_t) \cdot \boldsymbol{1} - \frac{\gamma}{1+\gamma} I_\xi^{-1} L' I_\xi \cdot \boldsymbol{\mu_t} \tag{15}$$

where $\boldsymbol{\mu_t}$ is vector of log-deviations of markups; $L = (I - \Omega)^{-1}$ is Leontief inverse, Ω is input output matrix; $I_\xi = diag\{\boldsymbol{\xi}\}$ is diagonal matrix with sectoral *Domar weights* $\xi_i = \frac{P_i Y_i}{PC}$ (computed in steady-state) on the diagonal; $\boldsymbol{1}$ is the vector of ones.

Sectoral prices. Sectoral prices expressed through sectoral markups are obtained by combining sectoral marginal cost equations (5), wage equations (15) and the link between sectoral prices and marginal costs (6). The resulting system of price equations is

$$\boldsymbol{p_t} = (p_t + y_t) \cdot \boldsymbol{1} - L\boldsymbol{a_t} + \tilde{L}\boldsymbol{\mu_t} \tag{16}$$

where $\tilde{L} = L(I - \frac{\gamma}{1+\gamma} I_\alpha I_\xi^{-1} L' I_\xi)$, $I_\alpha = diag\{\boldsymbol{\alpha}\}$ is diagonal matrix with labor shares in sectoral costs α_i on the diagonal.

Equilibirum sectoral marginal costs are $\boldsymbol{mc_t} = \boldsymbol{p_t} - \boldsymbol{\mu_t} = (p_t + y_t) \cdot \boldsymbol{1} - L\boldsymbol{a_t} + \tilde{L}\boldsymbol{\mu_t} - \boldsymbol{\mu_t}$ where $m_t = p_t + y_t$ is money supply and $\boldsymbol{s_t} = -\boldsymbol{L a_t}$ is a vector of sector-relevant productivity states, which I defined above. The i-th element of this vector $s_{t,i}$ is the combination of sectoral productivities affecting sector i equilibrium marginal costs, implying that firms in sector i should devote their attention to tracking fluctuations in $s_{t,i}$ rather than the whole vector of sectoral productivities.

Final output. Final output in terms of productivities and markups is obtained by multiplying both sides of price equations (16) and summing across equations with the corresponding consumption weights. This yields

$$y_t = \boldsymbol{\xi'} \cdot \boldsymbol{a_t} - \frac{1}{1+\gamma} \boldsymbol{\xi'} \cdot \boldsymbol{\mu_t} \tag{17}$$

where the first term $y_t^e = \boldsymbol{\xi'} \cdot \boldsymbol{a_t}$ is the efficient output and the second term $\tilde{y}_t = -\frac{1}{1+\gamma} \boldsymbol{\xi'} \cdot \boldsymbol{\mu_t}$ is output gap arising due to non-zero markups.

Price-markup link. The link between prices and markup which endogenously arises due to price rigidity is obtained from Equations (7)-(6) the using partial log-linearization operation, that is, treating all $F_{t-s,i}$ as time-varying coefficients

$$(I - F_t) \cdot (\boldsymbol{p_t} - \boldsymbol{p_{t-1}}) = -F_t \cdot \boldsymbol{\mu_t} + (I - F_t) \cdot \boldsymbol{e_{t-1}} \tag{18}$$

where F_t is a diagonal matrix with sectoral flexibililities $F_{t,i}$ on diagonal; the parameters governing $F_{t,i}$ are estimated from the sectoral price and wage data in the next section; $\boldsymbol{e_{t-1}}$ is vector collecting past expectations about the present marginal cost growth, such that $e_{t-1,i} = F_{t-1,i}E_{t-1}\Delta mc_{t,i} + \sum_{h=1}^{\infty}\left\{F_{t-1-h,i} \cdot \left[\prod_{s=0}^{h-1}(1 - F_{t-1-s,i})\right] \cdot E_{t-1-h}\Delta mc_{t,i}\right\}$ is predetermined in period t; $\Delta mc_{t,i} = mc_{t,i} - mc_{t-1,i}$. Note, that log-linearization is partial for this equation. That is, the sequence $\{F_{h,i}\}_{h=-\infty}^{t}$ of past and present shares of information updating firms is treated as given. Note that in the sticky information framework, prices depend not on current expectations about the future, but on past expectations about the present. This means that the sequence of past price flexibilites, which has already occurred, rather than the expected sequence of future price flexibilites, influences the equilibrium prices.

The system of equations (18) has time-varying coefficients $F_{t,i}$. The time-average of each $F_{t,i}$ determines the average degree of price flexibility in a given sector over time, encountered in non-state-dependent pricing models. The variability of $F_{t,i}$ over time determines the strength of state-dependent pricing mechanism in sector i. Both these characteristics of secrotal price flexibility are estimated below for the disaggregated sectors of the US economy. All further analysis relies on equations (15) - (18).

4 Empirical evidence of state-dependent pricing

In this section, I parameterize and estimate sectoral price flexibility $F_{t,i}$ for the disaggregated sectors of the US economy. To this end, I employ a two-step procedure. In the first step, I combine model equations and sectoral price and wage time series to construct sectoral productivity and markup series and compute sectoral productivity innovations. In the second step, I estimate the average price flexibility and the degree of state dependence in price adjustment for each sector by fitting the contemporaneous model response of sectoral markups to sectoral productivity innovations to a corresponding empirical response. Intuitively, a strong contemporaneous response of markups to shocks points to a low degree average price flexibility and the dependence of the response on the shock size captures the presence of state dependence in price adjustment.

4.1 Methodology

Model-implied markups and productivities From Equations (15) and (16) I construct the unobserved sectoral markups and productivities in terms of the observed wages and prices. Having period t observations of sectoral prices and wages as well as aggregate output and consumer price index, I compute period t sector markups and productivities implied by the model. The caveat is that in any period t, the number of sectors where we observe prices and wages is $k \leq N$. Hence for any t, I compute sectoral markups $\boldsymbol{\mu_t}$ and productivities $\boldsymbol{a_t}$ only for those sectors for which wages and prices are observed. The details of these computations are provided in Appendix C. Having computed sectoral productivities $\boldsymbol{a_t}$, I then compute sectoral productivity innovations $\boldsymbol{\epsilon_t}$ from Equation (2).

Markup response to productivity shocks. Sectoral markups and productivity innovations are then used to estimate sectoral price flexibility and its state dependence. The following proposition establishes the contemporaneous link between sectoral productivity shocks and markups obtained by combining price equations (16) with price rigidity equations (18).

Proposition 1 (Link between productivity shocks and markups.)**.** Productivity shocks $\boldsymbol{\epsilon_t}$ and markups $\boldsymbol{\mu_t}$ are related as

$$(\tilde{L} + (I - F_t)^{-1} \cdot F_t) \cdot \boldsymbol{\mu_t} = L\boldsymbol{\epsilon_t} + \boldsymbol{\tilde{v}_t} \tag{19}$$

where $\boldsymbol{\tilde{v}_t} = -\boldsymbol{p_{t-1}} + m_t \cdot \mathbf{1} - \boldsymbol{e_{t-1}} - L\boldsymbol{\bar{a}} - L\boldsymbol{a_{t-1}}$.

The derivations are available in Appendix C. Note, that the term $\boldsymbol{\tilde{v}_t}$ contains only predetermined variables $\boldsymbol{p_{t-1}}$, $\boldsymbol{a_{t-1}}$, $\boldsymbol{e_{t-1}}$ and monetary policy variable m_t and hence is independent from $\boldsymbol{\epsilon_t}$ as long as monetary policy does not react ti productivity shocks within one month period.

The change in the relevant states for each sector under the shock $\boldsymbol{\epsilon_t}$ is $\Delta \boldsymbol{s_t} = -L\boldsymbol{\epsilon_t}$. The matrix $F_t = diag\{F_{t,i}\}$ is dioganal with $F_{t,i} = F_i(|\Delta s_{t,i}|)$ determined according to Equation (8). I impose a linear functional form on sectoral of F_i such that

$$F_i(|\Delta s_{t,i}|) = \bar{F}_i + f_i \cdot log \frac{|\Delta s_{t,i}|}{E|\Delta s_{t,i}|} \tag{20}$$

where $E|\Delta s_{t,i}|$ is the time average of the absolute size of the relevant productivity state fluctuations. With this functional form of F_i, the parameter $\bar{F}_i$ corresponds to the average price flexibility over time in sector i, that is, the degree of price flexibility under the average size of relevant state fluctuations ins sector i. The parameter f_i measures the degree of state dependence in price adjustment. This parameter shows how much price flexibility varies with the size of the absolute changes in the relevant productivity state.

The goal of the empirical exercise is to estimate the average price flexibility $\bar{F}_i$ and the degree of state dependence f_i for each sector of the US economy, that is, to estimate $2 \times N$ parameters by evaluating System (19) of N interlinked equations. This task is nontrivial. To make estimation possible, I rearrange the terms in System (19) to make equations independent from each other with respect to the estimated parameters in F_t

$$\boldsymbol{L\epsilon_t} - \tilde{L}\boldsymbol{\mu_t} = F_t \cdot \left[\boldsymbol{L\epsilon_t} + (I - \tilde{L})\boldsymbol{\mu_t}\right] + (I - F_t) \cdot \tilde{\boldsymbol{v}}_{\boldsymbol{t}}$$

Since matrix F_t is diagonal, i-th equation in the above system contains only sector i price flexibility parameters, which means that this system can be estimated equation-by-equation, with one equation per sector. Denoting $\boldsymbol{y_t} = \boldsymbol{L\epsilon_t} - \tilde{L}\boldsymbol{\mu_t}$, $\boldsymbol{x_t} = \boldsymbol{L\epsilon_t} + (I - \tilde{L})\boldsymbol{\mu_t}$ and $\boldsymbol{v_t} = (I - F_t) \cdot \tilde{\boldsymbol{v}}_{\boldsymbol{t}}$ and using the expression for F_i in terms of estimated parameters, I get N equations of the form

$$y_{t,i} = \bar{F}_i \cdot x_{t,i} + f_i \cdot log\frac{|\Delta s_{t,i}|}{E|\Delta s_{t,i}|}x_{t,i} + v_{t,i} \tag{21}$$

These equations can be estimated independently from each other. The only complication is that $x_{t,i}$ is endogenous as it contains markups. At the same time, the relevant productivity state changes $\Delta s_{t,i}$ are exogenous and can serve as an instrument for $x_{t,i}$. As long as monetary policy m_t does not react within a month to productivity shocks, $\Delta s_{t,i}$ is uncorrelated with $\tilde{v}_{t,i}$. In Appendix C, I formally show that $\Delta s_{t,i}$ is not correlated with residual $v_{t,i} = (1 - F_i(|\Delta s_{t,i}|)) \cdot \tilde{v}_{t,i}$ and hence is a valid instrument for $x_{t,i}$.

Estimating Equations (21) using IV approach yields a set of average sectoral flexibilities $\{\bar{F}_i\}_{i=1}^N$ and the sensitivities to a state change $\{f_i\}_{i=1}^N$ which measures the degree of state-dependence of price flexibility in each sector. Since $v_{t,i}$ is heteroskedastic and autocorrelated I use consistent standard errors.

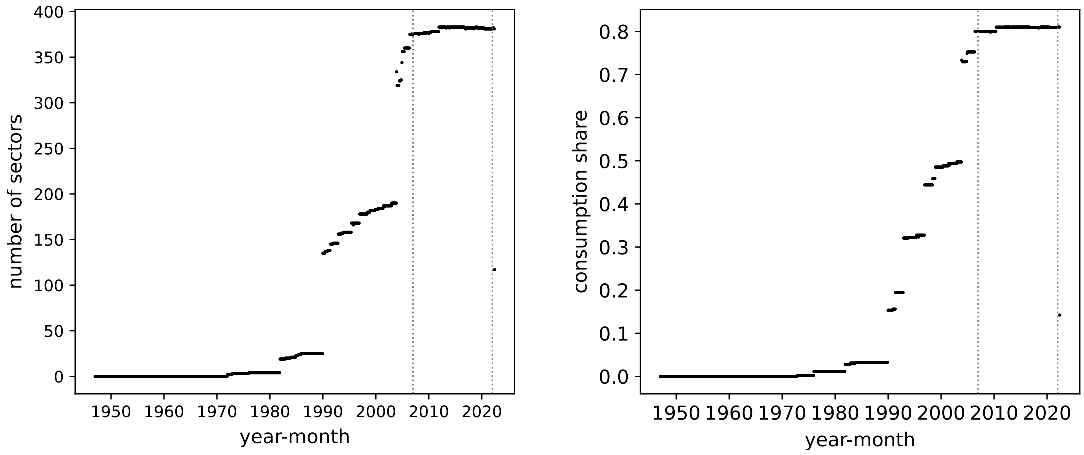
4.2 Data

To compute the intermediate goods, labor, and consumption shares in each sector, I employ the 2007 "Use table" from the BEA inputs-outputs account data. In this table, sectors are classified using BEA codes. I assume that each sector produces only one commodity and remove commodities that do not have a sector correspondence and vice-versa. Further, I remove sectors related to government spending, non-comparable imports, and the rest of the world adjustment. I also remove sectors for which the sum of intermediate and labor costs is zero. I compute labor shares in each sector as a ratio of labor costs to total costs. I compute intermediate input share as a ratio of a given intermediate input cost to the total cost. Finally, I compute consumption shares as the ratio of consumption expenditure on a given commodity to the total consumption expenditure.

To compute model-implied sectoral productivities and markups I employ monthly time series for sectoral wages and prices. Monthly wages by sector are available from the "Current Employment Statistics" (CES) from the US BLS and classified with a specific CES classification. Monthly sectoral producer price indices are from the US BLS and classified according to the NAICS classification. Since BEA input-output matrix uses BEA sector classification codes, I convert the wage and price data to the BEA classification to match the sectors of the input-output matrix. The details are provided in Appendix C.

The final dataset contains BEA-coded monthly wages and prices. Figure 2 plots the number of sectors for which both price and wage are available in a given year and month (left Panel) and the consumption share coverage (right Panel) for each year and month. The data availability improves over time and starts covering the majority of sectors by 2007.

Figure 2: Availablility of sectoral price and wage data



Left Panel: number of sectors for which price and wage observations are available in a given month. **Right Panel**: share of consumption covered by the available sectors in a given month. Vertical dotted lines mark the period for which the large and stable number of sectors is available (2007-2023).

4.3 Estimation results

The estimation procedure yields two sets of sectoral parameters: sectoral average price flexibility measures $\bar{F}_i$ and sectoral state dependence of price flexibility f_i. These parameters determine sectoral price flexibility $F_{t,i}$ at time t according to Equation (20). Table 1 shows the share of sectors with statistically significant parameter estimates. Around 87% of sectors have a statistically significant degree of price flexibility. suggesting that even within a short one month period most sectors react to shocks to a certain extent. Around 70% sectors have a statistically significant degree of state dependence, meaning that many sectors in the US economy exhibit some degree of sstate dependence in price adjustment.

Table 1: Share of statistically significant estimates

	signif. at 90% level	signif. at 95% level
Average flex. ($\bar{F}_i$)	0.87	0.86
State-dep. param. (f_i)	0.70	0.65

Note: Sectors are weighted by their corresponding consumption shares β_i

Table 2 plots a summary of cross-sectoral distribution of estimated parameters. The estimates of average price flexibility vary between 0 and 1 with the median of around 0.27, which means that, on average, around 27% of firms reset their information within one month period; in other words, the half of prices remain unchanged at least for four months, which corresponds to the evidence of Bils and Klenow (2004) who report median price duration of 4.3 months. However, the range of the average price flexibility estimates across sectors is quite broad. Figure 3 Panel (a) plots the histogram of the average price flexibility estimates in each sector. The pattern of average price flexibility suggests that commodity-related and upstream sectors such as oil and metals have more flexible prices, while various manufacturing sectors have less flexible prices.

The distribution of state dependence parameters in Table 2 suggests the median degree of state dependence of 0.17, which means that the absolute relevant productivity state fluctuation of 1 percentage point above its average leads to an increase in price flexibility by 0.0017 price flexibility units. Figure 3 Panel (b) plots the histogram of the cross-sectoral distribution of state dependence estimates. Sectors with both low and high degree of state dependence include manufacturing and services, hence this histogram does not reveal any pattern for the link between state dependence and the broad type of sector.

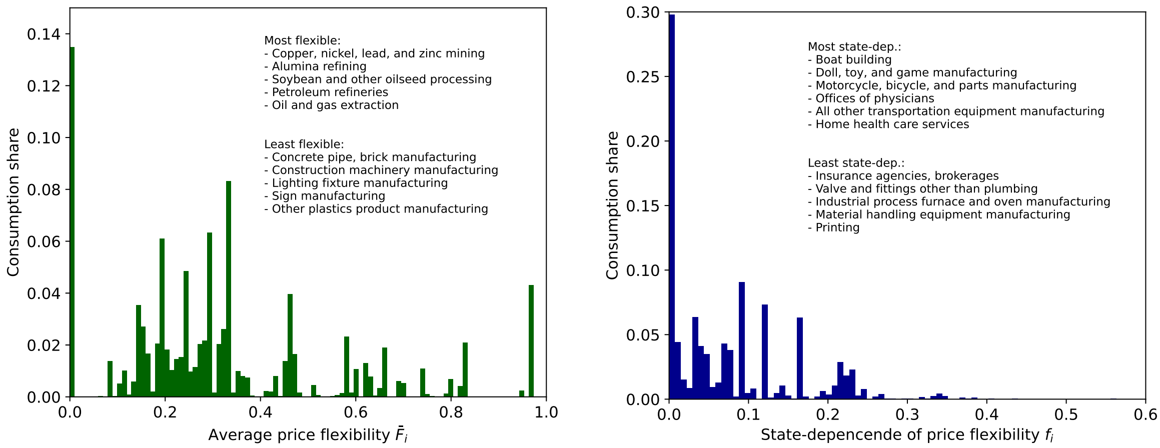
Table 2: Distribution of statistically significant estimates

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Average flex. ($\bar{F}_i$)	0.061	0.180	0.275	0.352	0.481	0.993
State-dep. param. (f_i)	0.012	0.081	0.172	0.192	0.269	0.731

Note: Only sectors with statistically significant estimates at 90% level

Figure 3: Price flexibility estimates

(a) Average price flexibility | (b) State-dependence of price flexibility

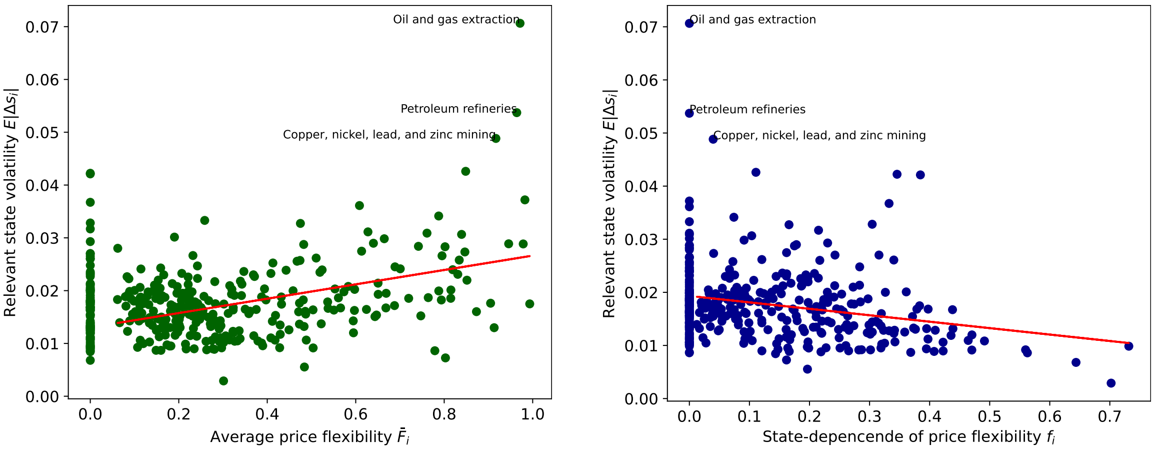


Histogram of average price flexibility estimates $\bar{F}_i$ (a) and state-dependence parameter estimates f_i (b) across 364 sectors; sectors are weighted by consumption shares β_i; variation is plotted only for 90%-level significant estimates; **estimates insignificant at 90% level are forced to zero**; interpretation of state-dependence parameter f_i: 1.p.p. increase in $|\Delta s_{t,i}|$ above its time average leads to price flexibility increase of $0.01 \cdot f_i$.

Next, I analyze how the average price flexibility and the state dependence parameters relate to the average volatility of the sectoral relevant productivity state. Figure 4 plots the parameter estimates and the corresponding relevant state volatilities. Figure 4 Panel (a) plots the relevant state volatilities against average price flexibility estimates. We observe that the higher average volatility in a sector is associated with higher average price flexibility with a correlation of 0.44. This suggests that sectors existing under more volatile conditions have higher price flexibility on average. Panel (b) plots sector-relevant productivity state volatilities against the corresponding state dependence parameters. We observe that higher volatility in a sector is associated with a lower degree of state dependence with a negative correlation of -0.25, suggesting that more volatile sectors have less state dependence in their pricing. This result implies that the less volatile (and hence less flexible) sectors tend to adjust their price flexibility more to the changing conditions, meaning that sectors with overall rigid prices may temporarily have larger price flexibility in the face of exceptionally large shocks.

Figure 4: Relevant state volatility and price flexibility

(a) Average price flexibility (b) State-dependence of price flexibility



Average price flexibility estimates $\bar{F}_i$ and state-dependence parameter estimates f_i are plotted against the time average volatility of sector-relevant productivity state $E|\Delta s_i|$; sectors are weighted by consumption shares β_i; estimates insignificant at 90% level are forced to zero; red lines correspond to linear regressions within the group of significant estimates; **correlation coefficient for Panel (a) is 0.44 and correlation coefficient for Panel (b) is -0.25**.

5 Phillips curve and cost-push inflation

I this section I establish the theoretical role of state-dependent pricing for the cost-push inflation. To this end, I derive the consumer price inflation in terms of aggregate demand and cost-push factors, the relationship known as Phillips curve. Then I offer a decomposition of the Phillips curve residual which allows analyzing the contribution of state-dependent pricing.

5.1 Phillips curve

The Phillips curve residual captures the cost-push effect on consumer price inflation. Let the vector of relative prices be the sectoral prices measured relatively to the consumer price index p_t such that $\hat{\boldsymbol{p}}_{\boldsymbol{t}} = \boldsymbol{p}_{\boldsymbol{t}} - p_t \cdot \mathbf{1}$. The efficient relative prices in sector i denoted by $\hat{p}^{\star}_{t,i}$ are relative prices that obtain under zero markups (all $\mu_{t,i} = 0$), that is $\hat{\boldsymbol{p}}^{\star}_{\boldsymbol{t}} = \boldsymbol{p}^{\star}_{\boldsymbol{t}} - p^{\star}_t \cdot \mathbf{1}$ with $\boldsymbol{p}^{\star}_{\boldsymbol{t}}$ being the vector of efficient prices and $p^{\star}_t = \sum_i \beta_i p^{\star}_{t,i}$ efficient consumer price. Next I define sectoral price gaps in the spirit of menu-cost literature

Definition 2 (Sectoral price gaps)**.** Vector of sectoral price gaps $\hat{\boldsymbol{\pi}}^{\star}_{\boldsymbol{t}}$ is the difference of the current efficient relative prices $\hat{\boldsymbol{p}}^{\star}_{\boldsymbol{t}}$ and the privious period true relative prices $\hat{\boldsymbol{p}}_{\boldsymbol{t-1}}$, that is $\hat{\boldsymbol{\pi}}^{\star}_{\boldsymbol{t}} = \hat{\boldsymbol{p}}^{\star}_{\boldsymbol{t}} - \hat{\boldsymbol{p}}_{\boldsymbol{t-1}}$.

Sectoral price gaps indicate the difference between the true prices and the efficient prices and reflect the potential of price adjustment towards efficiency. Note, that sectoral price gaps do not depend on the true prices in period t but only on the lagged true prices. Next proposition establishes the Phillips curve in terms of price gaps.

Proposition 2. (Consumer price inflation Phillips curve). The Phillips curve for consumer price inflation is

$$\pi_t = \underbrace{\kappa_t \cdot \tilde{y}_t}_{\text{demand inflaton}} + \underbrace{(1-\kappa_t) \cdot \boldsymbol{\beta'} M_t F_t \cdot \boldsymbol{\hat{\pi}_t^\star}}_{\text{cost inflation}} + \underbrace{(1-\kappa_t) \cdot \boldsymbol{\beta'} M_t F_t \cdot \boldsymbol{\tilde{e}_{t-1}}}_{\text{predetermined in period } t \text{ factors}} \tag{22}$$

where $\boldsymbol{\hat{\pi}_t^\star} = \boldsymbol{\hat{p}_t^\star} - \boldsymbol{\hat{p}_{t-1}}$ is a vector of sectoral price gaps, the slope of Phillips curve is $\kappa_t = \frac{\boldsymbol{\beta'} M_t F_t \mathbf{1}}{1-\boldsymbol{\beta'} M_t F_t \mathbf{1}}$ with $M_t = (I + \tilde{L} F_t^{-1}(I - F_t))^{-1} F_t^{-1}$ and expectation-related terms are $\boldsymbol{\tilde{e}_{t-1}} = \tilde{L} F_t^{-1}(I - F_t) \boldsymbol{e_{t-1}}$.

See proof in Appendix B.

The first term in the Phillips curve (22) relates inflation to the output gap and corresponds to a demand component of inflation. The second term is the Phillips curve residual $u_t = \boldsymbol{\beta'} M_t F_t \boldsymbol{\hat{\pi}_t^\star}$ and measures the cost-push component of inflation. The third term contains predetermined past expectations about the marginal cost growth rate. Next, I focus the properties of the Phillips curve residual term u_t.

5.2 Cost-push effect: main and input-output components

Presence of price rigidity prevents prices from adjustment to their efficient level for two reasons. First reason is that price rigidity does not allow prices to adjust to match the marginal cost. Second reason is that marginal cost itself differs from the efficient level due to equilibrium input-output links. This means that even those firms who adjust their prices do not set them to the efficient level. To separate these two effects I decompose the cost-push inflation u_t into two components, which I label "main" and "input-output" components. The main component captures the effect of heterogeneous price rigidity across final goods sectors given that marginal cost are at their efficient level. The input-output component captures the effect of price rigidity propagation through input-output links which leads to the deviation of marginal cost (and hence reset price) from its efficient level.

Proposition 3. (Phillips curve residual decomposition). Cost-push effect $u_t = \boldsymbol{\beta'} M_t F_t \boldsymbol{\hat{\pi}_t^\star}$ can be decomposed to the sum of the horizontal component and the vertical component

$$u_t = \underbrace{\boldsymbol{\beta'} F_t \cdot \boldsymbol{\hat{\pi}_t^\star}}_{\text{main component } = u_t^h} - \underbrace{\boldsymbol{\beta'}(I - M_t) F_t \cdot \boldsymbol{\hat{\pi}_t^\star}}_{\text{i-o component } = u_t^v} \tag{23}$$

See proof in Appendix B.

To understand the nature of the above decomposition consider a vector of sectoral reset prices (prices set in equilibrium by those who reset their price)

$$\boldsymbol{p_t^{reset}} = \boldsymbol{mc_t} = (p_t + y_t) \cdot \mathbf{1} - L\boldsymbol{a_t} + (\tilde{L} - I)\boldsymbol{\mu_t} = \\ = \underbrace{m_t \cdot \mathbf{1} - L\boldsymbol{a_t}}_{\text{efficient price}} + \underbrace{(L - I)\boldsymbol{\mu_t}}_{\text{interm. cost effect}} \underbrace{- \frac{\gamma}{1+\gamma} L I_\alpha I_\xi^{-1} L' I_\xi \boldsymbol{\mu_t}}_{\text{labor. cost effect}} \quad (24)$$

The reset price equals marginal cost and consists of the efficient price and the effect of markups. In multi-sectoral model reset prices differ from efficient prices because inefficiency caused by price stickiness propagates through production links leading to ”real rigidities”, that is a situation when marginal costs of production deviate from its efficient (flexible price) level. The main component of the decompostion given in Proposition 3 describes the residual arising when all reset prices are at their efficient levels. The input-output component gives the effect of propagation of inefficiency through input-output links.

Further, from Equation 24 we see that the effect of markups on reset price consists of the effect markups have on intermediate goods cost and on sector-specific labor costs. While higher markups lead to higher intermediate good cost, they lead to lower labor costs.

Next, I provide properties of production structures featuring only one component of cost-push effect. First, I describe an economy featuring only input-output component.

Corollary 1. (Single final good economy (only I-O component)). Consider an economy with only one final good such that consumption shares are $\beta_1 = 1$ and $\beta_i = 0$ for all $i \neq 1$. 1) In such economy only input-output component is present, that is $u_t^h = 0$. 2) If the only rigid price sector is the final good sector the cost-push effect is zero, $u_t = 0$.

See proof in Appendix B.

In an economy with a single final good the only possible source of cost-push effect is distortion in marginal cost of this good caused by upstream price rigidity. For this reason productivity hocks in one-sector textbook NK model with flexible wages do not create any cost push effect while in a one-sector sticky wage economy (rigidity in marginal costs) cost-push effect emerges (see Galí (2015) book).

Next, I describe an economy featuring only main component. From Equation 24 we see that in order to exclude the effect of markups on reset prices we need to have an economy in which the effect of markups on intermediate goods exactly offsets the effect of markups on labor costs. Next, I describe properties of such economy.

Corollary 2. (Quasi-horizontal economy (only horizontal component)). Consider an economy with multiple final sectors and no vertical links except the roundabout production in each sector (meaning that each sector uses part of its own output as its intermediate input) such that $W = I - I_\alpha$ and $\alpha_i = \frac{1}{1+\gamma}$ fro all i. Such economy features only a horizontal component of cost push effect, that is $u^v = 0$.

See proof in Appendix B.

The particular degree of roundabout production is needed so that the change in marginal cost due to change in intermediate good price is exactly offset by the change in labor cost. Note that in purely horizontal economy with no roundabout production such that Leontief inverse is $L = I$ input-output component still exists because labor input gets distorted by the presence of price rigidity.

5.3 Main component around steady state

The presence of multiple consumption goods is necessary for having main component. The main component captures the fact that for a given marginal cost distribution the "cost" of final consumption basket may be inefficiently high or low due to the fact that prices of different consumption goods have different degree of price flexibility. Indeed, if price rigidities are the same $F_{i,t} = F$ for all i main component disappears since te sum of price gaps weighted by consumption shares is zero by construction. Later, I will show that main component is quantitatively more important in shaping cost-push effect than the input-output component. Now, I derive some theoretical properties of main component around the undistorted steady-state.

Proposition 4. (Main component around steady state). Consider the economy being in steady state at $t-1$ such that $\boldsymbol{p_{t-1}} = 0$ and the productivity shocks $\boldsymbol{\epsilon_t}$ occur at time t. The main component of cost-push effect can be expressed as:

(a) under non-state-dependent pricing with heterogeneous price flexibility across sectors

$$u_t^h = cov_\beta(F,\ \boldsymbol{\hat{\pi}_t})$$

where $cov_\beta(F,\ \boldsymbol{\hat{\pi}_t})$ is cross-sectoral covariance between price flexibilities and price gaps.

(b) under common state-dependent pricing across sectors

$$u_t^h = k \cdot Asym_\beta(\boldsymbol{\hat{\pi}_t})$$

where k is positive constant and $Asym_\beta(\boldsymbol{\hat{\pi}_t}) = \frac{cov_\beta(|\Delta s_t|,\ \Delta s_t)}{var_\beta(|\Delta s_t|)}$ is the asymmetry of price gaps.

See proof in Appendix B.

Under non-state-dependent pricing, the cost-push effect is given by correlation of price flexibility with price gaps across sectors. If desirable upward price adjustment expressed by positive price gap happens in more flexible sectors, we observe positive cost-push effect and vise versa.

Under common state-dependent pricing the cost-push effect arises as long as price gap distribution is asymmetric. If certain sector have disproportionately large positive price gap this sector (or a group of sectors) will create the positive of cost-push effect and vise versa. The dependence of cost-push effect on asymmetry in sectoral price gaps has been first pointed out in Ball and Mankiw (1995) who have argued that asymmetry in price changes across sectors can serve as a proxy for cost-push effect. They treated the distribution of the desired price changes as exogenously given which corresponds to the main component of cost-push inflation in my model.

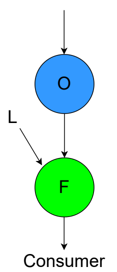
Finally, the above results remain purely theoretical even though they provide an intuition about the effect of pricing framework. In a realistic calibration both non-state-dependent and state-dependent pricing components of price rigidity co-exist and the degree of state-dependence might differ across sectors. In the quantitative section I calibrate price rigidity framework to feature the empirically plausible heterogeneity in pricing across sectors.

5.4 Illustrative examples

Next I provide examples illustrating the role of state-dependent pricing for cost-push inflation in various production networks.

Example 1. Two-sector vertical chain

Figure 5: Two-sector vertical chain



Consider a two-sector vertical chain economy. Let the upstream sector be Oil sector and the downstream sector be Final good sector (Figure 5). Oil sector has fully flexible prices $F^O = 1$ while final good sector has partially rigid prices $F^F \leq 1$. The economy is initially at the steady state and that the productivity shock in oil sector ϵ^{Oil} occurs. In this case cost-push effect is

$$u_t = u(F^O, F^F) \cdot \frac{1 - F^O}{F^O} \cdot \alpha^F \cdot \epsilon^{Oil}$$

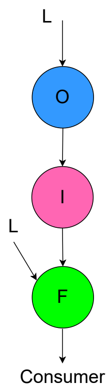
where $u(.,.) > 0$ as long as $(1 - \alpha^F)\gamma < 1$, α^F labor share in F; for derivation see Appendix

B.4. Note that since the consumption consists on a single sector, main component of cost-push inflation is absent in this economy.

Cost-push inflation $u_t = 0$ as long as Oil sector has fully flexible prices $F^O = 1$. The Oil shock does not cause cost-push effect since there is no distortion in the marginal cost of production.

Example 2. Intermediate good

Figure 6: Three-sector chain



Consider a vertical chain with intermediate good sector (Figure 6). Oil sector has fully flexible prices $F^{Oil} = 1$ but the intermediate sector has partially rigid prices $F^I \leq 1$. Price distortion in intermediate good sector creates cost distortion in final good sector. The cost-push effect of Oil productivity shock is

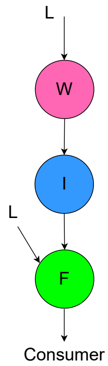
$$u_t = \tilde{u}(F^I,\ F^F) \cdot (1 - \alpha^F)\alpha^F \cdot \epsilon^{Oil}$$

where $\tilde{u}(.,.) \geq 0$ and $\tilde{u}'_1 \geq 0$, $\tilde{u}'_2 \geq 0$ as long as $(1 - \alpha^F)\gamma < 1$, α^F labor share in F; for derivation see Appendix B.4.

When productivity in Oil sector goes down we have cost-push *deflation* and state-dependent pricing (the fact that F^I and F^F change with shock size) amplify the cost-push effect of the shock. The negative cost push effect of a negative oil productivity shock goes against the intuition that negative shocks in oil industry lead to a positive cost push inflation. Nevertheless, this example illustrates the basic mechanism of why cost-push effect emerges. After a negative productivity shock prices of Oil go up. Intermediate sector uses Oil as input meaning that optimal price of intermediate good should also go up. But since prices in the intermediate good sector are sticky, they increase by less then they should. As a result, marginal cost in final good sector are smaller than they should be resulting in a negative cost-push effect.

Example 3. "Sticky wages" economy

Figure 7: Sticky wages



Consider a vertical chain economy in which the most upstream sector has partially rigid prices while intermediate sector has fully flexible prices. Let the upstream sector be the "sticky wages" sector, intermediate sector be the Oil sector and the final sector be the consumption good sector. This is the case of a so-called sticky wage economy (Figure 7). The corresponding price flexibility is $F^{Oil} = 1$ (fully flexible), $F^W \leq 1$ and $F^F \leq 1$. The cost-push effect of Oil productivity shock is

$$u_t = -\tilde{u}(F^W,\ F^F) \cdot (1 - \alpha^F) \cdot \epsilon^{Oil}$$

where $\tilde{u}(.,.) \geq 0$ and $\tilde{u}'_1 \geq 0$, $\tilde{u}'_2 \geq 0$ as long as $(1-\alpha^F)\gamma < 1$, α^F labor share in F; for derivation see Appendix B.4.

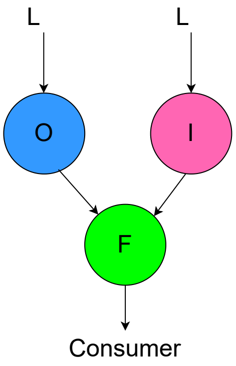
When oil productivity goes down we have cost-push *inflation* in line with the intuition that the negative productivity shock in oil industry should create cost-push effect. Upon negative Oil productivity shock, the level of production decreases and less labor is demanded. As a result, wages should optimally go down. But since wages are sticky they remain too high and the marginal cost of producing Oil and ultimately final good remains higher than it should be. The inefficiently high marginal cost lead to a positive cost-push inflation. State-dependent price flexibility affects the size of cost-push effect through the adjustment in F^F.

Example 4. Multiple inputs

Consider an economy in which a single final good is produces using two material inputs: Oil and Intermediate good (Figure 8). Oil sector has fully flexible prices $F^{Oil} = 1$ while price flexibility in intermediate good sector is partial $F^I \leq 1$. Also for the exposition purposes I assume that final good sector also has fully flexible prices $F^F = 1$. Aster the oil shock ϵ^{Oil} the cost-push inflation is

$$u_t = -\alpha^I(1-\alpha^I) \cdot (1-F^I) \cdot \epsilon^{Oil}$$

Figure 8: Multiple inputs



where α^I share of input I in F; for derivation see Appendix B.4.

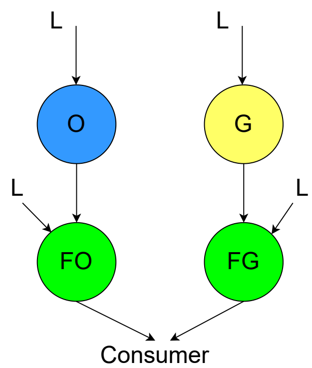
Negative oil productivity shock leads to a *positive* cost-push inflation and the state-dependence of price flexibility influences the size of cost-push effect by changing F^I.

The mechanism of cost-push effect of oil shock some what differs from the previous examples. In this economy when negative oil productivity shock occurs the marginal cost of producing final good go up and the demand for intermediate input goes down as long as substitutability between oil and intermediate good is not too high. Hence prices in the intermediate good sector should optimally go down which they do not do because of price rigidity in this sector. As a result, the price of intermediate good is inefficiently high and the resulting marginal cost of producing final good is also inefficiently high, which creates cost push inflation.

The examples 1-4 have illustrated the amplification role of state dependence. Now, I turn to the example in which state dependence leads to *sign reversal* of cost-push effect.

Example 5. Multiple final good sectors Consider an economy consisting of two up-

Figure 9: Multiple final good sectors



stream goods (Oil and Grain) and two final goods (Oil-intensive and Grain-intensive) with equal share in consumption. Oil-intensive final good uses oil as input while grain-intensive final good uses grain as input (Figure 9). Upstream commodity sectors have fully flexible prices $F^{Oil} = F^{Grain} = 1$ and final good sectors have partially rigid prices $F^{FO} \leq 1$ and $F^{FG} \leq 1$. As before, the economy is initially in steady state and is perturbed simultaneously by two commodity shocks - oil and grain shocks $\epsilon^{Oil}, \epsilon^{Grain}$. The corresponding cost-push effect is

$$u_t = -\frac{1}{4} \cdot (F^{FO} - F^{FG}) \cdot (\epsilon^{Oil} - \epsilon^{Grain})$$

Assume first that price rigidity is non-state-dependent such that $F^{FO} > F^{FG}$. Than negative oil shock leads to a positive cost push effect. However the negative grain shock leads to a *negative* cost-push effect. This behavior is not plausible as there are no obvious reasons why shock in one commodity sector should lead to cost-push inflation while similar shock in another commodity sector should lead to cost-push deflation.

But what happens if price flexibility is state-dependent? In this case we have larger price flexibility in oil-intensive sector $F^{FO} > F^{FG}$ under oil shock and larger price flexibility in grain-intensive sector $F^{FG} > F^{FO}$ under grain shock. Hence, under state-dependent pricing a negative shock in any commodity sector leads to a positive cost-push effect. The presence of state-dependence reverses the sign of cost-push effect of grain shock.

The mechanism of cost-push effect in this economy is as follows. When negative oil shock hits, oil price goes up and production of oil and oil-intensive good drops which leads to lower level of household income. With the lower level of income household decreases its demand for grain-intensive good as well (as long as this good is not an inferior good) which should cause prices of grain-intensive good to optimally drop. However, price rigidity in grain-intensive industry prevents grain-intensive good price from dropping meaning that the *relative* price of grain-intensive good is higher than it should optimally be which leads to cost-push inflation.

6 Quantitative analysis

In this section, I compute the monthly cost-push effect in the US implied by the model and analyze the role of the state-dependent component of price flexibility. The calibration of the model block unrelated to price flexibility is described in Section 4. Each sector's price flexibility and state dependence are calibrated according to the estimates obtained in Section 4. The period of quantitative analysis covers the years 2007-2023 for which sufficiently large number of sectors is available (see data availability on Figure 2 of 4).

6.1 Cost-push effect and state-dependence

This section computes the residual for each month from Equation (22), using the model-implied productivities and sectoral prices from the empirical section. To evaluate the quantitative role of state dependence over time, I also compute counterfactual residual without the state-dependent component of price flexibility. Figure 10 shows the result.

Overall, an empirically plausible degree of state dependence in each sector yields a more volatile cost-push effect than the non-state-dependent pricing model. Two episodes are worth investigating to analyze the role of state dependence: 2009, just after the Great Recession, and the period after 2019, during and after the Covid crisis. In 2009, both state-dependent and non-state-dependent pricing models produced a positive spike in the cost-push effect, and state dependence played an amplification role. In 2019, starting from the Covid crisis, the US economy entered a turbulent period with large fluctuations of cost-push effect. The state-dependent model yields a negative cost-push effect at the start of the Covid crisis, followed by a positive effect just after the crisis when the supply chain disruption issue emerged. In 2022, when the full-scale Russia-Ukraine war broke out, the state-dependent pricing model gives a strong growth of the cost-push effect. In contrast, the non-state-dependent pricing model gives quite different predictions for this period: no negative cost-push effect during the Covid crisis, no strong positive effect after the Covid crisis, and no increase in cost-push inflation in 2022 when the Russia-Ukraine war started.

Note that none of the models predict a long-lasting positive cost-push effect during the post-Covid period, suggesting that the persistent post-Covid inflation cannot be entirely characterized as cost-push but instead has demand or expectation-driven features, which justifies a strong monetary response undertaken by the FED.

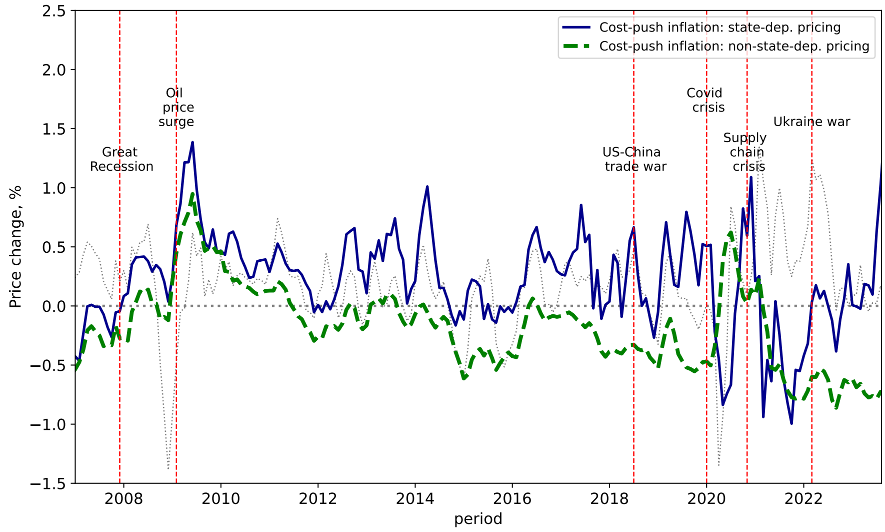
6.2 Cost-push effect decomposition

Now, I look into the quantitative importance of the main and input-output components by commuting the former separately. Figure (11) shows that the main component largely shapes the fluctuations of the cost-push effect, meaning that the I-O component merely plays an amplifying/dampening role during different episodes.

6.3 Phillips curve fit

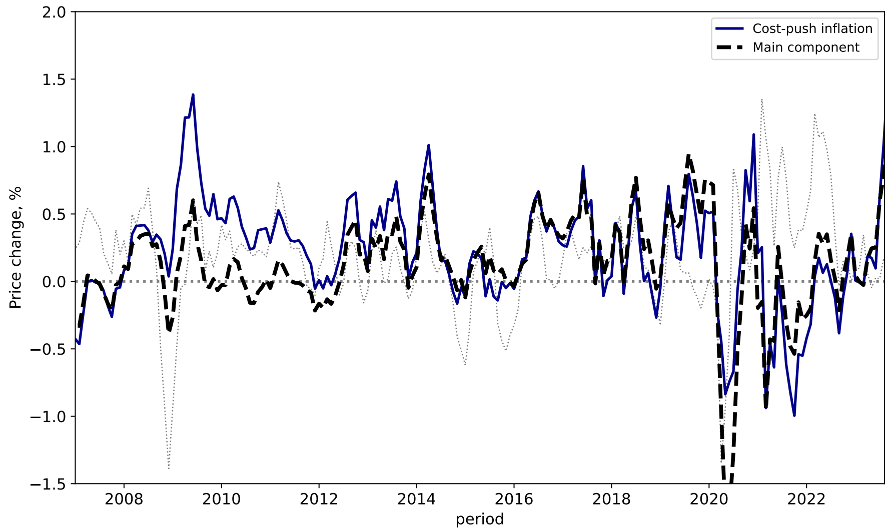
Now, I investigate if the Phillips curve residual implied by the state-dependent model outperforms its non-state-dependent counterpart in explaining inflation in a conventional Phillips curve regression. For this, I regress CPI inflation on the standard Phillips curve variables: unemployment, expected and lagged inflation, and oil prices. Then, I sequentially add the non-state-dependent and state-dependent residual computed from the model. Table 3 shows the regression results. The regression with a non-state-dependent residual outperforms the

Figure 10: Cost-push inflation and state-dependent pricing



Grey line plots CPI inflation; **blue line** plots the Phillips curve residual implied by the model under estimated degree of price flexibility; **dashed green line** plots the Phillips curve residual when the effect of state-dependent pricing is absent (all $f_i = 0$).

Figure 11: Cost-push inflation and main component



Grey line plots CPI inflation; **blue line** plots the Phillips curve residual implied by the model under estimated degree of price flexibility; **dashed black line** plots the main component of Phillips curve residual. CPI inflation and residual series are smoothed with a 3-month moving average.

regression with only oil price inflation, but adding a state-dependent residual improves the fit. Moreover, a state-dependent residual effect is statistically significant even when a non-state-dependent residual is already accounted for.

Table 3: Phillips curve estimation with model implied residual

	Dependent variable:		
	CPI inflation		
	(1)	(2)	(3)
Unempl.	0.0001	−0.0004**	−0.0002
	(0.0001)	(0.0002)	(0.0002)
Lagged inflation	0.132*	0.108	0.093
	(0.076)	(0.072)	(0.069)
Expect. inflation	−0.00003	0.0004	0.001
	(0.001)	(0.001)	(0.001)
Oil inflation	0.023***	0.016***	0.016***
	(0.004)	(0.004)	(0.004)
u(non-st.-dep.)		0.399***	0.179
		(0.094)	(0.114)
u(st.-dep)			0.197***
			(0.062)
Constant	0.0005	0.004**	0.001
	(0.002)	(0.002)	(0.002)
R^2	0.284	0.382	0.433
Adjusted R^2	0.259	0.354	0.403
Note:	*p<0.1; **p<0.05; ***p<0.01		

6.4 Analysis by sector

Now, I turn to the analysis of the contribution of particular sectors to the cost-push effect on average and during particular episodes. To investigate the importance of particular groups of sectors, I group the disaggregated sectors into the 2-digit BEA-coded groups. Table 4 gives the list of these groups.

Table 4: 2-digit BEA sector names

2-digit BEA	Sector description
11	Agriculture, Forestry, Fishing and Hunting
21	Mining, Quarrying, and Oil and Gas Extraction
22	Utilities
23	Construction
31	Manufacturing (non-durable goods)
32-33	Manufacturing (durable goods)
42	Wholesale Trade
44 - 45	Retail Trade
48 - 49	Transportation
51	Information
52	Finance and Insurance
53	Real Estate and Rental and Leasing
54	Professional, Scientific, and Technical Services
55	Management of Companies and Enterprises
56	Administrative and Support and Waste Management and Remediation Services
61	Educational Services
62	Health Care and Social Assistance
71	Arts, Entertainment, and Recreation
72	Accommodation and Food Services
81	Other Services (except Public Administration)
92	Public Administration

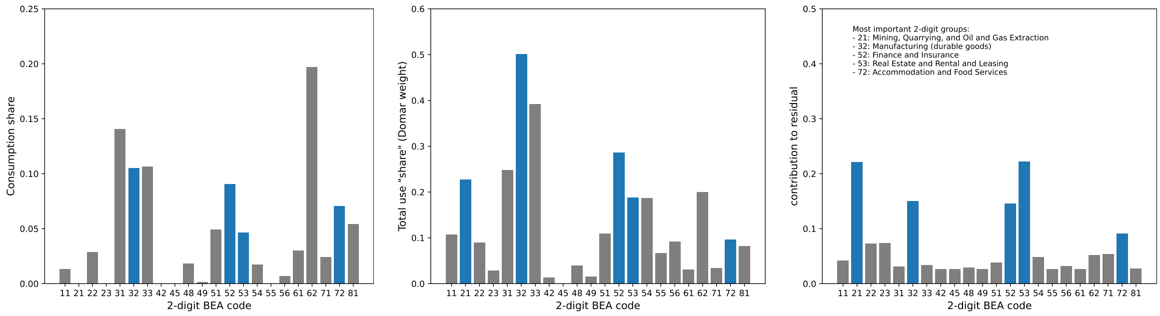
6.4.1 Most important sectoral groups

First, I compute the marginal importance of each group in explaining the cost-push effect. For each sector group, I recompute the residual, excluding the contribution of sectors in this group, and compare this new residual with the full residual by regressing the latter on the former; I compute the importance of each sector group as (1 minus R-squared) of this regression. Figure 12 plots the importance of each sector group in consumption (panel A), production (panel B), and in explaining Phillips curve residual (panel C). The five most important sector groups emerge on panel c.

For these five most important groups, I compute counterfactual cost-push effects generated exclusively by fluctuations in sectors belonging to these groups. Figure 13 panel A plots the residual induced by sector group 21 (Mining, Quarrying, and Oil and Gas Extraction) and indicates that this sector group alone can partially explain the cost-push effect of 2009 but does not explain any other episode. Adding other important groups 52, 53 (Finance and Insurance, Real Estate, and Rental and Leasing) on panel B, and 32, 72 (Manufacturing of durable goods, Accommodation and Food Services) on panel C, improves the fit to full residual - many fluctuations can be attributed to these most important sectors.

Figure 12: Most important sector groups

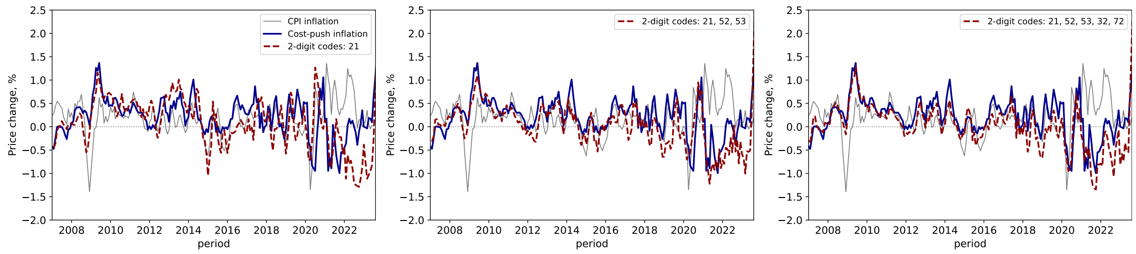
(a) Share in consumption (b) Share in total use (c) Cos-push effect explained



Panel (a): sum of sectoral consumption shares within each group; Panel (b): sum of sectoral Domar weights (shares in total use) within each group; Panel (c): the share of Phillips curve residual explained by a given 2-digit BEA sector group; computed by forcing the shocks in a given sector of interest to zero and calculating the (1- r-squared) from a total Phillips curve residual regression on the resulting counterfactual Phillips curve residual; **blue** highlights the group of sectors most important in explaining the dynamics of cost-push inflation.

Figure 13: Cost-push inflation due to 2-digit sector groups

(a) Sectors: 21 (b) Sectors: 21, 52, 53 (c) Sectors: 21, 52, 53, 32, 72



Red dashed line plots counterfactual residuals computed by shutting down the shocks in all sectors except a given 2-digit sector group.

6.4.2 Sectoral contribution during particular episodes

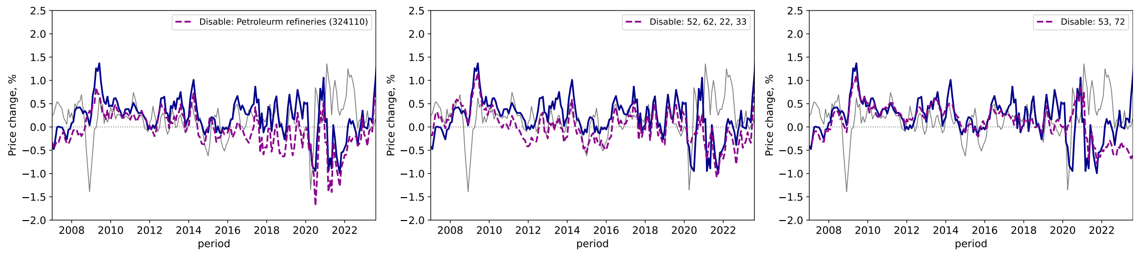
Now, I investigate which sectors have contributed the most during three important historical episodes: the post-Great Recession, the post-Covid episode, and the Ukraine war. For this, I find the largest-seized elements of the sum constituting the main component of the cost-push effect within each episode of interest. Then, I compute counterfactual residual by switching off these sectors.

In 2009, a lot of cost-push effect was attributed to the "Petroleum refineries" sector alone. Figure 14 (panel A) shows that switching off this sector substantially reduces the 2009 cost-push effect. The Covid and post-Covid episode was not attributed to any particular sector but rather to several groups simultaneously 52, 62, 22, 33 (Finance and Insurance, Health Care and Social Assistance, Utilities, Manufacturing (durable goods). Figure 14 (panel B) shows that these groups explain most of the cost-push effect in 2020-2021. The

2022 surge of the cost-push effect is largely attributed to sector groups 53 and 72 (Real Estate and Rental and Leasing, Accommodation and Food Services) as shown on 14 (panel C).

Figure 14: Cost-push inflation due to 2-digit sector groups

(a) disable 2009 sectors (b) disable covid sectors (c) disable Ukraine war sectors



7 Conclusions

This paper investigates the implications of state-dependent pricing for cost-push inflation in a multi-sectoral New Keynesian economy with a production network. To this end, I estimate the sector-specific degree of state dependence and evaluate its importance for cost-push inflation in the US.

My empirical approach allows the use of the model to estimate sector-specific price flexibility and its degree of state dependence from the sectoral price and wage data. The estimates reveal that the majority of sectors in the US economy have a statistically significant degree of state dependence.

Theoretically, I show that state-dependent pricing may lead to cost-push inflation having different size and even sign compared to a non-state-dependent pricing framework. This important implication of state-dependent pricing obtains even if one excludes the effect of inefficiency propagation through the production network.

In the model with an empirically plausible degree of state dependence, the importance of state dependence for the cots-push effect is different for different historical periods. After the Great Recession, state dependence amplified the positive cost-push effect, while during and after the Covid crisis, it often led to a sign reversal of cost-push inflation.

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Appendices

A Model log-linearization appendix

A.1 Sectoral wages

The product market clearing condition in sector i (14) can be written as $P_{t,i}Y_{t,i} = P_{t,i}C_{t,i} + \sum_j P_{t,i}X_{t,ji}$. Using the conditions for optimal input allocation (3), (4), and the link between sector price and sector marginal cost (6), we get $\frac{P_{t,i}X_{t,ij}}{MC_{t,i}\cdot Y_{t,i}} = \frac{\mathcal{M}_{t,i}P_{t,i}X_{t,ij}}{P_{t,i}\cdot Y_{t,i}} = (1-\alpha_i)\omega_{ij}$, we have $P_{t,j}X_{t,ji} = (1-\alpha_j)\omega_{ji}\frac{P_{t,j}Y_{t,j}}{\mathcal{M}_{t,j}}$. Substituting this result into the market clearing condition

$$P_{t,i}Y_{t,i} = P_{t,i}C_{t,i} + \sum_j (1-\alpha_j)\omega_{ji}\frac{P_{t,j}Y_{t,j}}{\mathcal{M}_{t,j}} \tag{A.1}$$

Consumption shares and Domar weights are connected through a well-known link (see Baqaee and Farhi (2020)).

Proposition (Consumption shares to Domar weights link)**.** $\boldsymbol{\xi} = L'\boldsymbol{\beta}$.

Proof. First, let us compute (A.1) at the efficient steady state and divide by $\bar{P}\bar{Y}$. We have the $\frac{\bar{P}_i\bar{Y}_i}{\bar{P}\bar{C}} = \frac{\bar{P}_i\bar{C}_i}{\bar{P}\bar{C}} + \sum_j (1-\alpha_j)\omega_{ji}\frac{\bar{P}_j\bar{Y}_j}{\bar{P}\bar{C}}$. Then, the steady state product market clearing condition can be expressed as $\xi_i = \beta_i + \sum_j (1-\alpha_j)\omega_{ji}\xi_j$, or in matrix form $\boldsymbol{\xi} = \boldsymbol{\beta} + W'\boldsymbol{\xi}$. This gives us the link between consumption shares and Domar weights: $\boldsymbol{\xi} = L'\boldsymbol{\beta}$. □

Log-linearizing (A.1) and dividing by $\bar{P}\bar{Y}$ yields

$$\xi_i(p_{t,i} + y_{t,i} - \mu_{t,i}) = \beta_i(p_{t,i} + c_{t,i}) - \xi_i\mu_{t,i} + \sum_j (1-\alpha_j)\omega_{ji}\xi_j(p_{t,j} + y_{t,j} - \mu_{t,j})$$

The demand for i-th sector consumption is $p_{t,i} + c_{t,i} = p_t + y_t$. Hence, we have

$$(p_i + y_i - \mu_i) = \frac{1}{\xi_i}\sum_j l_{ji}(\beta_j(p_t + y_t) - \xi_j\mu_j) = p_t + y_t - \frac{1}{\xi_i}\sum_j l_{ji}\xi_j\mu_j \tag{A.2}$$

where l_{ij} is $(i,\ j)$-th element of matrix L.

Labor demand in log-deviations is $w_{t,i} + l_{t,i} = p_{t,i} + y_{t,i} - \mu_{t,i}$ and labor supply is $w_{t,i} = p_t + y_t + \gamma l_{t,i}$. Combining labor demand and labor supply, we get the following expression for equilibrium wage

$$w_{t,i} = \frac{1}{1+\gamma}(p_t + y_t) + \frac{\gamma}{1+\gamma}(p_{t,i} + y_{t,i} - \mu_{t,i}) \tag{A.3}$$

Combining (A.2) and (A.3) yields

$$w_{t,i} = p_t + y_t - \frac{\gamma}{1+\gamma}\frac{1}{\xi_i}\sum_j l_{ji}\xi_j\mu_{t,j} \tag{A.4}$$

which in vector form gives equation 15.

A.2 Sectoral prices

From (5) log-linear marginal cost deviation is sector i is

$$mc_{t,i} = -a_{t,i} + \alpha_i w_{t,i} + (1-\alpha_i)\sum_j \omega_{ij} p_{t,j} \tag{A.5}$$

The link between sector price and sector marginal cost is $p_{t,i} = \mu_{t,i} + mc_{t,i}$. Combining these two results yields the following system of equations for sector prices

$$p_{t,i} = \mu_{t,i} - a_i + \alpha_i w_{t,i} + (1-\alpha_i)\sum_j \omega_{ij} p_{t,j} \tag{A.6}$$

This system of price equations can be written in matrix form as

$$\boldsymbol{p_t} = \boldsymbol{\mu_t} - \boldsymbol{a_t} + I_\alpha \boldsymbol{w_t} + W\boldsymbol{p_t} \tag{A.7}$$

Substituting wage (15) into (A.7), moving parts containing $\boldsymbol{p_t}$ to the left side and multiplying by matrix $L = (I-W)^{-1}$ gives

$$\boldsymbol{p_t} = L\boldsymbol{\mu_t} - L\boldsymbol{a_t} + (p_t + y_t)\cdot L\boldsymbol{\alpha} - \frac{\gamma}{1+\gamma} L I_\alpha I_\xi^{-1} L' I_\xi \boldsymbol{\mu_t} \tag{A.8}$$

Next, I establish a link between labor shares vector and Leontief inverse matrix.

Proposition (Labor shares and Leontief inverse.)**.** $L\boldsymbol{\alpha} = \mathbf{1}$.

Proof. Indeed, $L\boldsymbol{\alpha} = \mathbf{1} \Leftrightarrow (I-W)^{-1}\boldsymbol{\alpha} = \mathbf{1} \Leftrightarrow \boldsymbol{\alpha} = (I-W)\cdot\mathbf{1} = \mathbf{1} - (\mathbf{1}-\boldsymbol{\alpha}) = \boldsymbol{\alpha}$. □

Then, the system of price equations can be expressed as

$$\boldsymbol{p_t} = (p_t + y_t)\cdot\mathbf{1} - L\boldsymbol{a_t} + \tilde{L}\boldsymbol{\mu_t} \tag{A.9}$$

where $\tilde{L} = L(I - \frac{\gamma}{1+\gamma} I_\alpha I_\xi^{-1} L' I_\xi)$.

A.3 Final output

Log-linearization of consumer price index yields $p_t = \sum_i \beta_i p_{t,i} = \boldsymbol{\beta'} \cdot \boldsymbol{p_t}$. Multiplying both sides of price equations (16) by vector $\boldsymbol{\beta'}$ and noticing that $\boldsymbol{\beta'} \cdot \mathbf{1} = \sum_i \beta_i = 1$, we get

$$0 = y_t - \boldsymbol{\beta'} \cdot L \cdot a_t + \boldsymbol{\beta'} \tilde{L} \cdot \boldsymbol{\mu_t} \tag{A.10}$$

Next, as shown shown before $\boldsymbol{\beta}' L = \boldsymbol{\xi}'$. Then, $\boldsymbol{\beta}' \tilde{L} = \boldsymbol{\xi}' - \frac{\gamma}{1+\gamma} \boldsymbol{\xi}' I_\alpha I_\xi^{-1} L' I_\xi = \boldsymbol{\xi}' - \frac{\gamma}{1+\gamma} \boldsymbol{\alpha}' L' I_\xi = \boldsymbol{\xi}' - \frac{\gamma}{1+\gamma} \mathbf{1}' \cdot I_\xi = \frac{1}{1+\gamma} \boldsymbol{\xi}'$, where in the third step I use the previous result that $L\boldsymbol{\alpha} = \mathbf{1}$. Hence, we have the expression for output as a function of productivities and markups.

$$y_t = \boldsymbol{\xi'} \cdot \boldsymbol{a_t} - \frac{1}{1+\gamma} \boldsymbol{\xi'} \cdot \boldsymbol{\mu_t} \tag{A.11}$$

A.4 Price-markup link

Log-linearizing Equation (18), while treating all $F_{t-s,i}$ as time-varying coefficients

$$p_{t,i} = F_{t,i} \cdot mc_{t,i} + \sum_{h=1}^{\infty} \left\{ \left[\prod_{s=0}^{h-1} (1 - F_{t-s,i}) \right] \cdot F_{t-h,i} \cdot E_{t-h} mc_{t,i} \right\} \tag{A.12}$$

Let $mc_{t,i} = mc_{t-1,i} + \Delta mc_{t,i}$. Then, we can write

$$p_{t,i} = F_{t,i} mc_{t,i} + (1 - F_{t,i}) \left[F_{t-1,i} E_{t-1} mc_{t,i} + \sum_{h=1}^{\infty} \left\{ \left[\prod_{s=0}^{h-1} (1 - F_{t-1-s,i}) \right] \cdot F_{t-1-h,i} mc_{t,i} \right\} \right] =$$
$$= F_{t,i} mc_{t,i} + (1 - F_{t,i}) p_{t-1,i} + (1 - F_{t,i}) e_{t-1,i}$$

where $e_{t-1,i} = F_{t-1,i} E_{t-1} \Delta mc_{t,i} + \sum_{h=1}^{\infty} \left\{ \left[\prod_{s=0}^{h-1} (1 - F_{t-1-s,i}) \right] \cdot F_{t-1-h,i} \Delta mc_{t,i} \right\}$ is predetermined at period t. Markup is $\mu_{t,i} = p_{t,i} - mc_{t,i}$. Hence, the price-markup link is

$$(1 - F_{t,i}) \cdot (p_{t,i} - p_{t-1,i}) = -F_{t,i} \mu_{t,i} + (1 - F_{t,i}) e_{t-1,i} \tag{A.13}$$

B Cost-push inflation theoretical appendix

This appendix contains proofs for Section 5.

B.1 Phillips curve

Proof of Proposition 2 (Consumer price inflation Phillips Curve). Rewriting price equations (16) in terms of sectoral inflations gives

$$\boldsymbol{\pi_t} = -\boldsymbol{p_{t-1}} + p_{t-1} \mathbf{1} + (\pi_t + y_t) \mathbf{1} - L a_t + \tilde{L} \mu_t$$

where $\tilde{L} = L(I - \frac{\gamma}{1+\gamma} I_\alpha I_\xi^{-1} L' I_\xi)$.
On the other hand, the markup-inflation link through price prigdity (18) can be written as

$$(I - F_t)\boldsymbol{\pi_t} = -F_t \boldsymbol{\mu_t} + (I - F_t)\boldsymbol{e_{t-1}}$$

where $F_t = diag\{F_{t,i}\}$, $\boldsymbol{e_{t-1}}$ is such that
$e_{t-1,i} = F_{t-1,i} E_{t-1} \Delta mc_{t,i} + \sum_{h=1}^{\infty} \left\{ \left[\prod_{s=0}^{h-1} (1 - F_{t-1-s,i}) \right] \cdot F_{t-1-h,i} \Delta mc_{t,i} \right\}$ is predetermined at period t.

Efficient relative prices are

$$\boldsymbol{\hat{p}_t^\star} = \boldsymbol{p_t^\star} - p_t^\star \cdot \mathbf{1} = y_t^e \cdot \mathbf{1} - L \cdot \boldsymbol{a_t}$$

In terms of price gaps $\boldsymbol{\hat{\pi}_t^\star} = \boldsymbol{\hat{p}_t^\star} - \boldsymbol{\hat{p}_{t-1}}$, price equation can be rewritten as

$$\boldsymbol{\pi_t} - \pi_t \cdot \mathbf{1} = \tilde{y}_t \cdot \mathbf{1} + \boldsymbol{\hat{\pi}_t^\star} + \tilde{L} \cdot \mu_t$$

Substituting markup-rigidity link into the previous equation and rearranging, we get

$$F_t(I + \tilde{L} F_t^{-1}(I - F_t))\boldsymbol{\pi_t} - F_t \mathbf{1} \pi_t = F_t \mathbf{1} \tilde{y}_t + F_t \boldsymbol{\hat{\pi}_t^\star} + \tilde{L} F_t^{-1}(I - F_t)\boldsymbol{e_{t-1}}$$

Let $M_t^{-1} = F_t(I + \tilde{L} F_t^{-1}(I - F_t))$. Multipling previous equation by M_t and then by $\boldsymbol{\beta'}$, we get Phillips curve

$$\pi_t(1 - \boldsymbol{\beta'} M_t F_t \mathbf{1}) = \boldsymbol{\beta'} M_t F_t \mathbf{1} \tilde{y}_t + \boldsymbol{\beta'} M_t F_t \boldsymbol{\hat{\pi}_t^\star} + \boldsymbol{\beta'} M_t F_t \tilde{L} F_t^{-1}(I - F_t)\boldsymbol{e_{t-1}}$$

Let $\kappa_t = \frac{\boldsymbol{\beta'} M_t F_t \mathbf{1}}{1 - \boldsymbol{\beta'} M_t F_t \mathbf{1}}$. Then, Phillips curve takes the form stated in proposition. □

B.2 Cost-push effect decomposition

Proof of Proposition 3 (Phillips curve residual decomposition). Absence of input-output effect in price setting means that firms set their prices ignoring the inefficient component of their marginal costs. Instead they consider marginal costs being equal to the efficient prices $\boldsymbol{p_t^\star}$. Hence, the resulting sector prices are $\boldsymbol{p_t} = F_t \cdot \boldsymbol{p_t^\star} + (I - F_t)(\boldsymbol{p_{t-1}} + \boldsymbol{e_{t-1}})$, which yields $(I - F_t) \cdot (\boldsymbol{p_t} - \boldsymbol{p_{t-1}}) = F_t \cdot (\boldsymbol{p_t^\star} - \boldsymbol{p_t}) + (I - F_t) \cdot \boldsymbol{e_{t-1}}$.

Since $\boldsymbol{p_t^\star} - \boldsymbol{p_t} = -\tilde{L} \cdot \boldsymbol{\mu_t}$, we have $(I - F_t) \cdot (\boldsymbol{p_t} - \boldsymbol{p_{t-1}}) = -F_t \tilde{L} \cdot \boldsymbol{\mu_t} + (I - F_t) \cdot \boldsymbol{e_{t-1}}$. Under this link between inflation and markups, the Phillips curve is

$$\pi_t(1 - \boldsymbol{\beta'} F_t \mathbf{1}) = \boldsymbol{\beta'} F_t \mathbf{1} \tilde{y}_t + \boldsymbol{\beta'} F_t \boldsymbol{\hat{\pi}_t^\star} + \boldsymbol{\beta'} F_t \tilde{L} F_t^{-1}(I - F_t)\boldsymbol{e_{t-1}}$$

and the Phillips curve residual not-related to inefficiency in marginal cost is $u_t^h = \boldsymbol{\beta'} F_t \boldsymbol{\hat{\pi}_t^\star}$
□

Proof of Corollary 1 (Single final good economy(only I-O component)). Let $\boldsymbol{\pi_t^\star}$ be desired price changes. $\boldsymbol{\beta'\pi_t^\star} = \pi_{1,t}^\star$ is the desired consumer price change. Then, price gaps (relative desired price changes) are $\boldsymbol{\hat{\pi}_t^\star} = \left[0, \hat{\pi}_{2,t}^\star,\ ..., \hat{\pi}_{N,t}^\star\right]$. As a result $u_t^h = \boldsymbol{\beta'} F_t \boldsymbol{\hat{\pi}_t^\star} = 0$

If $F_{1,t} < 1$ and $F_{i,t} = 1$ for all $i \neq 1$ then we have $[F_t^{-1}(I - F_t)]_{1,1} \neq 0$ and $[F_t^{-1}(I - F_t)]_{i,j} = 0$ otherwise. Then $M_t F_t = [I + \tilde{L} F_t^{-1}(I - F_t)]^{-1}$ is such that it has non-zero first column, ones on the diagonal and zeros otherwise. Then $\boldsymbol{\beta'} M_t F_t$ is a row vector with the first element being the only non-zero element. Hence, we have $\boldsymbol{\beta'} M_t F_t \boldsymbol{\hat{\pi}_t^\star} = 0$ since $\hat{\pi}_{1,t}^\star = 0$. □

Proof of Corollary 2 (Quasi-horizontal economy (only horizontal component)). If $\tilde{L} = I$ the net effect of markups on marginal cost is zero as intermediate cost effect exactly compensates the labor cost effect. In this case, $M_t = (I + \tilde{L}F_t^{-1}(I - F_t))^{-1} F_t^{-1} = I$ and the vertical component disappears.

In the case described by corollary, Leontief inverse is $L = I_\alpha^{-1}$, which gives $\tilde{L} = \frac{1}{1+\gamma} I_\alpha^{-1}$. To eliminate vertical component we need to have $\alpha_i = \frac{1}{1+\gamma}$ for all sectors i. □

B.3 Horizontal component around steady state

Proof of Proposition 4 (Horizontal component around steady state). The economy is in the efficient steady state at time $t-1$. After a productivity shock, price gaps equal $\boldsymbol{\hat{\pi}_t} = -L\boldsymbol{\epsilon_t} + \boldsymbol{\beta'} L \boldsymbol{\epsilon_t} = \boldsymbol{\Delta s_t} - \boldsymbol{\beta'}\boldsymbol{\Delta s_t} = \boldsymbol{\Delta \hat{s}_t}$ where $\boldsymbol{\Delta s_t}$ are the changes in the relevant productivity states caused by productivity shocks at time i.

Consider the horizontal component $u_t^h = \boldsymbol{\beta'} F_t \cdot \boldsymbol{\hat{\pi}_t^\star}$ and let price flexibility be state-dependent with common degree of state-dependence such that $F_{t,i} = \bar{F}_i + f \cdot |\Delta s_{t,i}| + v_{i,t}$ where $v_{i,t}$ is a part of sectoral price flexibility unexplained by this state-dependent framework. Note, that $f = \frac{cov_\beta(F_t - \bar{F},\ |\Delta s_t|)}{var_\beta(|\Delta s_t|)}$. In this case the horizontal component of the cost-push inflation can be written as

$$u_t^h = \boldsymbol{\beta'} F_t \cdot \Delta \boldsymbol{\hat{s}_t} = cov_\beta(\bar{F},\ \Delta s_t) + cov_\beta(F_t - \bar{F},\ |\Delta s_t|) \cdot Asym_\beta(\Delta s_t) + cov_\beta(v_t,\ \Delta s_t)$$

where $Asym_\beta(\Delta s_t) = \frac{cov_\beta(|\Delta s_t|,\ \Delta s_t)}{var_\beta(|\Delta s_t|)}$ is the asymmetry of price gaps (equal to the efficient price changes).

Part (a): Under purely non-state dependent pricing $f = 0$ and $v_{i,t} = 0$ so that only the first term is preset yielding the Part (a) of proposition.

Part (b) : Under pure common state-dependent pricing, only second term is present ($\bar{F} = 0$ and $v_{i,t} = 0$) and asymmetry in efficient price changes describes cost-push effect with $k = cov_\beta(F_t,\ |\Delta s_t|)$; this yields the Part (b) of proposition.

The presence of heterogeneity in state-dependence adds additional term to the proposition, which doesn't have an analytical interpretation.

□

B.4 Illustrative examples derivations

Consider a general case of a two-sector vertical chain. U - upstream sector, D - downstream sector. F^U - upstream price flexibility, F^D - downstream price flexibility. The share of upstream input in downstream production is w. Let productivity vector be $a' = [\epsilon^U,\ \epsilon^D]$. Price flexibility matrix is $F_t = \begin{pmatrix} F^U & 0 \\ 0 & F^D \end{pmatrix}$. I-O matrix is $W = \begin{pmatrix} 0 & 0 \\ w & 0 \end{pmatrix}$. Leontief inverse is $L = \begin{pmatrix} 1 & 0 \\ w & 1 \end{pmatrix}$. Consumption shares are $\boldsymbol{\beta'} = [0,\ 1]$ and Domar weigths are $\boldsymbol{\xi'} = \boldsymbol{\beta'}L = [w,\ 1]$. Labor shares $\alpha' = [1,\ (1-w)]$. Phillips curve residual is $u_t = \boldsymbol{\beta'}M_tF_t\boldsymbol{\hat{\pi}_t^\star}$ where $M_tF_t = (I + \tilde{L}F_t^{-1}(I - F_t))^{-1}$ and $\tilde{L} = \frac{1}{1+\gamma} \cdot \begin{pmatrix} 1 & -\gamma \\ w & 1 \end{pmatrix}$.

$M_tF_t = \frac{1+\gamma}{Det} \cdot \begin{pmatrix} 1+\gamma+f^D & \gamma f^D \\ -wf^U & 1+\gamma+f^U \end{pmatrix}$ where $f^U = \frac{1-F^U}{F^U}$, $f^D = \frac{1-F^D}{F^D}$ and $Det = (1+\gamma+f^U)\cdot(1+\gamma+f^D) - w\gamma f^U \cdot f^D > 0$. $\boldsymbol{\beta'}M_tF_t = \frac{1+\gamma}{Det}\cdot[-wf^U,\ 1+\gamma+f^U]$. Desired price changes are $\boldsymbol{\hat{\pi}_t^\star} = -[(1-w)\epsilon^U - \epsilon^D,\ 0]'$. Then, Phillips curve residual

$$u_t = \frac{1+\gamma}{Det} \cdot w((1-w)\epsilon^U - \epsilon^D) \cdot \frac{1-F^U}{F^U} \tag{A.14}$$

Example 1: two-sector vertical chain. In this example Oil sector is Upstream and Final good sector is Downstream. We have $F^U = F^O = 1$, $\epsilon^U = \epsilon^O$ and $\epsilon^D = 0$. As a result we have $u = \frac{1+\gamma}{Det} \cdot w((1-w)\epsilon^O) \cdot \frac{1-F^O}{F^O} = 0$.

Example 2: Intermediate good. Consider a three-sector vertical chain Oil → Intermediate good → Final good. Assume that intermediate good uses only oil and no labor. Let price flexibilities be $F^O = 1$, $F^I < 1$ and $F^F < 1$. Then, Oil and Intermediate good can be combined in one Upstream sector such that $F^U = F^I$ and $F^D = F^F$. Under the oil shock ϵ^O, we have $\epsilon^U = \epsilon^O$ and $\epsilon^D = 0$. Then, the residual is is $u = \frac{1+\gamma}{Det} \cdot w(1-w) \cdot \epsilon^O \cdot \frac{1-F^I}{F^I}$. When oil productivity goes down (oil price goes up), Phillips curve residual also goes down (consumer prices go down).

Example 3: "Sticky wage" economy. Consider a three-sector vertical chain Labor sector→ Oil→ Final good. Assume that final good uses only oil and no labor. Let price flexibilities be $F^L < 1$, $F^O = 1$ and $F^F < 1$. Then, Oil and Final good can be combined in one Downstream sector such that $F^U = F^L$ and $F^D = F^F$. Under the oil shock ϵ^O, we have $\epsilon^U = 0$ and $\epsilon^D = \epsilon^O$. Then, the residual is is $u = \frac{1+\gamma}{Det} \cdot -w\epsilon^O \cdot \frac{1-F^L}{F^L}$. When oil productivity goes down (oil price go up), Phillips curve residual goes up (consumer prices go up).

Multiple inputs. Next, consider a two-sector horizontal economy with good 1 (G1) and (G2) such that only labor and own output is used for production of each good. Then production network is $W = I - I_\alpha$, $L = I_\alpha^{-1}$, $\tilde{L} = I$ and $M_t = I$ which eliminates vertical component of cost-push inflation. The shares of each good in consumption are s_1 and s_2 such that $s_1 + s_2 = 1$. Let each of these sectors be hit by a respective shock ϵ_1 and ϵ_2 and the respective price flexibilities be F_1 and F_2. Then $L\boldsymbol{a_t} = [(1-\alpha_1)^{-1} \cdot \epsilon_1,\ (1-\alpha_2)^{-1} \cdot \epsilon_2]'$. Then, $\hat{\boldsymbol{\pi}}_{\boldsymbol{t}}^{\star} = -[s_2((1-\alpha_1)^{-1} \cdot \epsilon_1 - (1-\alpha_2)^{-1} \cdot \epsilon_2),\ -s_1((1-\alpha_1)^{-1} \cdot \epsilon_1 - (1-\alpha_2)^{-1} \cdot \epsilon_2)]'$. Then cost-push inflation is $u = -s_1 \cdot s_2 \cdot (F_1 - F_2) \cdot ((1-\alpha_1)^{-1} \cdot \epsilon_1 - (1-\alpha_2)^{-1} \cdot \epsilon_2)$.

Example 4: Multiple inputs economy. Consider an economy where single final good is produced using two inputs Oil and Intermediate good. If price flexibility in final good sector is 1 and no labor is used in this sector, then this economy is a special case of a horizontal economy described above. We have $F_1 = F^O = 1$, $F_2 = F^I$, $s_1 = 1 - \alpha^I$, $s_2 = \alpha^I$, $\alpha_1 = \alpha_2 = 1$ and $\epsilon_1 = \epsilon^{Oil}$, $\epsilon_2 = 0$. As a result we have cost-push effect $u = -\alpha^I(1-\alpha^I) \cdot (1-F^I) \cdot \epsilon^{Oil}$.

Example 5: Multiple final good economy. Consider an economy consisting of two commodities: Oil and Grain and two final goods: Oil-intensive final good and Grain-intensive final good. Commodity sectors have fully flexible prices, while final good sectors have partially rigid prices. If final goods sectors do not use any labor and use only respective commodities, then this economy can be represented as a special case of a two-sector horizontal economy described above with Oil commodity and Oil intensive final good representing the first sector and Grain commodity and Grain-intensive final good representing the second sector. Then, we have $F_1 = F^{FO}$, $F_2 = F^{FG}$, $\alpha_1 = \alpha_2 = 1$, $s_1 = s_2 = 0.5$ are consumption shares, $\epsilon_1 = \epsilon^{Oil}$ and $\epsilon_2 = \epsilon^{Grain}$. Then, cost-push effect is $u = -\frac{1}{4} \cdot (F^{FO} - F^{FG}) \cdot (\epsilon^{Oil} - \epsilon^{Grain})$

C Empirical evidence appendix

C.1 Sectoral productivities and markups

Let all industies be indexed by $i \in \{1, ..., N\}$. At any period t the available k sectors have indices $\{i^1, ..., i^k\} \subseteq \{1, ..., N\}$. I construct $N \times k$ selection matrix S, such that $S[i^j, j] = 1$ and zero otherwise. Note, that $S^T S = I$. Then transformation $S\boldsymbol{u}$ transforms k-sized vector $\boldsymbol{u}$ to N-sized vector with zeros for unavailable sectors; $S^T\boldsymbol{v}$ transforms N-sized vector $\boldsymbol{v}$ to k-sized, by choosing only elements for available industires. Hence, we can write a system of k equations for k markups and productivities in terms of k wages and prices

$$\boldsymbol{\mu} = \frac{1+\gamma}{\gamma} \cdot S^T (I_\xi^{-1} L^T I_\xi)^{-1} S \cdot ((p+y) \cdot \mathbf{1} - \boldsymbol{w}) \tag{A.15}$$

$$\boldsymbol{a} = \boldsymbol{\mu} + S^T I_\alpha S \cdot \boldsymbol{w} - S^T L^{-1} S \cdot \boldsymbol{p} \tag{A.16}$$

C.2 Estimating price flexibility

Proof: (Link between productivity shocks and markups). Let sectoral price change be $\pi_{t,i} = p_{t,i} - p_{t-1,i}$. Log-linear equation for money supply is $m_t = y_t + p_t$. Moreover, productivies are $\boldsymbol{a_t} = \bar{\boldsymbol{a}} + \boldsymbol{a_{t-1}} + \boldsymbol{\epsilon_t}$. Combining these with price system (16) we get

$$\boldsymbol{\pi_t} = -\boldsymbol{p_{t-1}} + m_t \cdot \mathbf{1} - L\boldsymbol{\epsilon_t} + \tilde{L}\boldsymbol{\mu_t} - L(\bar{\boldsymbol{a}} + \boldsymbol{a_{t-1}})$$

On the other hand, the link between inflation and markups (18) can be written as

$$(I - F_t) \cdot \boldsymbol{\pi_t} = -F_t\boldsymbol{\mu_t} + (I - F_t)\boldsymbol{X_{t-1}}$$

Substituting for $\boldsymbol{\pi_t}$, and rearranging yields the result. □

Proof: Validity of instruments. . Note that $\tilde{v}_{t,i}$ is independent of $z_{t,i}$ as long as monetary policy does not react within a month to a productivity shock. Furthermore, $F_i(|z_{t,i}|)z_{t,i}$ has mean zero, since $z_{i,t}$ is zero mean normally distributed. Hence, we have

$$Cov(F_i(|z_{t,i}|)z_{t,i},\ F_i(|z_{t,i}|)\tilde{v}_{t,i}) = E(F_i(|z_{t,i}|)^2 z_{t,i}\tilde{v}_{t,i}) =$$
$$= \int\int F_i(|z_{t,i}|)^2 z_{t,i}\tilde{v}_{t,i} f_z f_{\tilde{v}} dz d\tilde{v} = \int_{\tilde{v}} \left[\int_z F_i(|z_{t,i}|)^2 z_{t,i} f_z dz\right] \tilde{v}_{t,i} f_{\tilde{v}} d\tilde{v} = 0$$

The last equality follows as inner integral equals to zero due to zero mean symmetric distribution of $z_{i,t}$. Hence, instruments constructed in this matter are valid. □

C.3 Dataset construction

This Appendix describes the construction of BEA-coded sectoral price and wage data set.

Sectoral wages (form CES to NAICS). Sectoral wages are initially classified with CES codes, with available correspondence from CES to NAICS codes. So first I transform the wages classification to NAICS-based. The main complication is that CES to NAICS mapping is not one-to-one as at least for some NAICS codes more than one CES sector exists.To overcome this complication I compute the weighted average wage for each NAICS sector as $w^{NAICS} = \sum \alpha_i w_i^{CES}$ where w_i^{CES} are CES-sector wages corresponding to a given NAICS sector code. Each weight α_i is computed as a ratio of the number of workers employed in sector i to the total number of workers in all CES sectros corresponding to a given NAICS sector. The number of employed workers is taken form the same CES dateset as the average number for the year 2012, to correspond to the year of the Input-Output table used.

Sectoral consumer prices (from NIPA to BEA). Sectoral consumer prices are initially classified by NIPA codes. BEA provides a bridge table between NIPA codes and

BEA codes. The complication is that one BEA code sometimes corresponds to multiple NIPA codes. For such cases I compute the BEA sector price as the weighted average of the corresponding NIPA sector prices $p^{BEA} = \sum \alpha_i p_i^{NIPA}$. Weights α_i are computed using the "Purchasers value" quantity available in the bridge table for each NIPA sector. I use the 2012 bridge table.

NAICS to BEA concordance. The producer prices data is classified by NAICS codes as well as wages data (after the transformation from CES to NAICS described above). To apply this data to the available input-output tables I convert NAICS based sectoral data to BEA based sectoral data. BEA Bridge tables have a rough BEA-NAICS code correspondence, from which I make use to establish a concordance between NAICS codes and BEA codes. The problem is that the BEA-NAICS codes correspondence is not one-to-one. For those cases when one BEA code corresponds to several NAICS codes I need weights to evaluate the BEA-based price as a weighted average of the NAICS based prices. For this I need to compute the relative sector size of each NAICS sector withing a given BEA sector. The primary data source I use to compute NAICS sector sizes is the Annual survey of manufacturers from the US Census. I use the corresponding "Shipment value" quantities for the survey of 2012. The secondary data source is the Current Employment Survey. I use the number of employed people as an sector size variable, translated from CES into NAICS codes in the same manner as wages. First I try to compute NAICS sector weights in each BEA code using ASM data. If ASM data is unavailable, I use CES data. For those sectors, that are not covered by either dataset I use the uniformal weights.

NAICS to BEA matching procedure. Having constructed the mapping from NAICS to BEA codes with corresponding weights, I convert the NAICS data into the BEA data. I want to find a corresponding NAICS code for as many NAICS sectors from the NAICS-BEA mapping as possible. First, I find the the NAICS codes in the data that have the identical NAICS codes in the NAICS-BEA mapping. For the remaining NAICS codes from the BEA-NAICS mapping I try to find the correspondence at the more aggregated level. I subsequently 1,2 and 3 last digits of NAICS codes form the mapping and try to find the corresponding more aggregated sector in the data.