

# Multiple Shock Impulse Response Functions

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## Introduction

- Impulse response analysis is a widely employed tool in the field of macroeconomics and econometrics, popularized by Sims (1980).
- Identification issues/what comes first? Koop et al. (1996) introduce generalized impulse response functions.

**We propose: Multiple shock impulse response functions**, which take into account the correlation between the shocks. Incorporates:

- \* Contagion between shocks
- \* Temporal aggregation

**Multiple impulse response functions can shed light on:**

- The interaction and impact of financial shocks.
- The effects of multiple uncertainty sources on economic variables.
- The transmission of shocks across countries and assessing global macroeconomic linkages.

## General Framework

Let  $\mathbf{y}_t$  be a vector with  $n$  endogenous variables, modeled by a function of historical values of  $\mathbf{y}_t$  and variables  $\mathbf{z}_t$ , and a function of  $n$  shocks  $\boldsymbol{\nu}_t$ :

$$\mathbf{y}_t = f(\mathbf{y}_{t-1}, \dots, \mathbf{y}_{t-p}, \mathbf{z}_t, \dots, \mathbf{z}_{t-q}) + g(\boldsymbol{\nu}_t), \quad (1)$$

where  $\boldsymbol{\nu}_t$  have mean zero and finite variances.

## Impulse Response Concepts

Let  $\mathbf{y}_t$  follow a process in accordance with Equation (1).

### Definition 1: Traditional impulse response functions

The traditional impulse response functions of  $\mathbf{y}_{t+h}$  to the  $s$ -th shock  $\nu_{s,t}$  of size  $\delta_s$  are defined as

$$\Psi(h, \delta_s, \boldsymbol{\omega}_{t-1}) = \mathbb{E}[\mathbf{y}_{t+h} \mid \nu_{s,t} = \delta_s, \nu_{j,t} = 0 \forall j \neq s, \boldsymbol{\nu}_{t+1} = \dots = \boldsymbol{\nu}_{t+h} = \mathbf{0}, \boldsymbol{\omega}_{t-1}] - \mathbb{E}[\mathbf{y}_{t+h} \mid \boldsymbol{\nu}_t = \boldsymbol{\nu}_{t+1} = \dots = \boldsymbol{\nu}_{t+h} = \mathbf{0}, \boldsymbol{\omega}_{t-1}],$$

for horizon  $h = 0, 1, \dots, H$ , where  $\boldsymbol{\omega}_{t-1}$  denotes an historical path realization of the stochastic process that generates  $\mathbf{y}_{t+h}$ . This definition implies a linear function of  $g(\cdot)$  and requires identification of the structural relations between shocks.

### Definition 2: Generalized impulse response functions

The one shock generalized impulse response functions (Koop et al., 1996; Pesaran and Shin, 1998) of  $\mathbf{y}_{t+h}$  to the  $s$ -th shock  $\nu_{s,t}$  of size  $\delta_s$  are defined as

$$\Psi^g(h, \delta_s, \mathcal{I}_{t-1}) = \mathbb{E}[\mathbf{y}_{t+h} \mid \nu_{s,t} = \delta_s, \mathcal{I}_{t-1}] - \mathbb{E}[\mathbf{y}_{t+h} \mid \mathcal{I}_{t-1}],$$

for horizon  $h = 0, 1, \dots, H$ , where  $\mathcal{I}_{t-1}$  denotes the information set available at  $t - 1$ . Here, the history is treated random and does not require identification of the structural relations.

### Definition 3: Multiple shock impulse response functions

Let  $\mathcal{S}$  be a set of indices corresponding to the  $1 < m \leq n$  shocks of interest, where  $|\mathcal{S}| > 1$ . The multiple shock impulse response functions of  $\mathbf{y}_{t+h}$  to a set of shocks  $\boldsymbol{\nu}_{\mathcal{S},t}$  of size  $\boldsymbol{\delta}_{\mathcal{S}}$  are defined as

$$\Psi^{\mathcal{S}}(h, \boldsymbol{\delta}_{\mathcal{S}}, \mathcal{I}_{t-1}) = \mathbb{E}[\mathbf{y}_{t+h} \mid \boldsymbol{\nu}_{\mathcal{S},t} = \boldsymbol{\delta}_{\mathcal{S}}, \mathcal{I}_{t-1}] - \mathbb{E}[\mathbf{y}_{t+h} \mid \mathcal{I}_{t-1}],$$

for horizon  $h = 0, 1, \dots, H$ .

## Illustration: VAR(1) process

Let  $\mathbf{y}_t$  denote the  $n$  variables of interest. The vector autoregression (VAR) with one lag is then

$$\mathbf{y}_t = \mathbf{B}\mathbf{y}_{t-1} + \mathbf{u}_t, \quad \mathbf{u}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}). \quad (2)$$

We assume i.i.d. residuals  $\mathbf{u}_t$  and stability of the VAR.

### Impulse response functions for Equation (2)

Let  $\sigma_{ss}$  be the  $(s, s)$ -th element of  $\boldsymbol{\Sigma}$ ,  $\mathbf{e}_s$  an  $s$ -th element unit vector, and  $\mathbf{P}$  an  $n \times m$  permutation matrix, with  $m$  unit vectors, then:

**Generalized impulse response functions (GIRF)** for one shock  $s$ :

$$\Psi^g(h, \delta_s, \mathcal{I}_{t-1}) = \mathbf{B}^h \boldsymbol{\Sigma} \mathbf{e}_s (\sigma_{ss})^{-1} \delta_s. \quad (3)$$

**Multiple shock impulse response functions** for  $m > 1$  shocks:

$$\Psi^{\mathcal{S}}(h, \boldsymbol{\delta}_{\mathcal{S}}, \mathcal{I}_{t-1}) = \mathbf{B}^h \boldsymbol{\Sigma} \mathbf{P} (\mathbf{P}' \boldsymbol{\Sigma} \mathbf{P})^{-1} \boldsymbol{\delta}_{\mathcal{S}}. \quad (4)$$

## Simulation

Consider a data generating process (DGP):

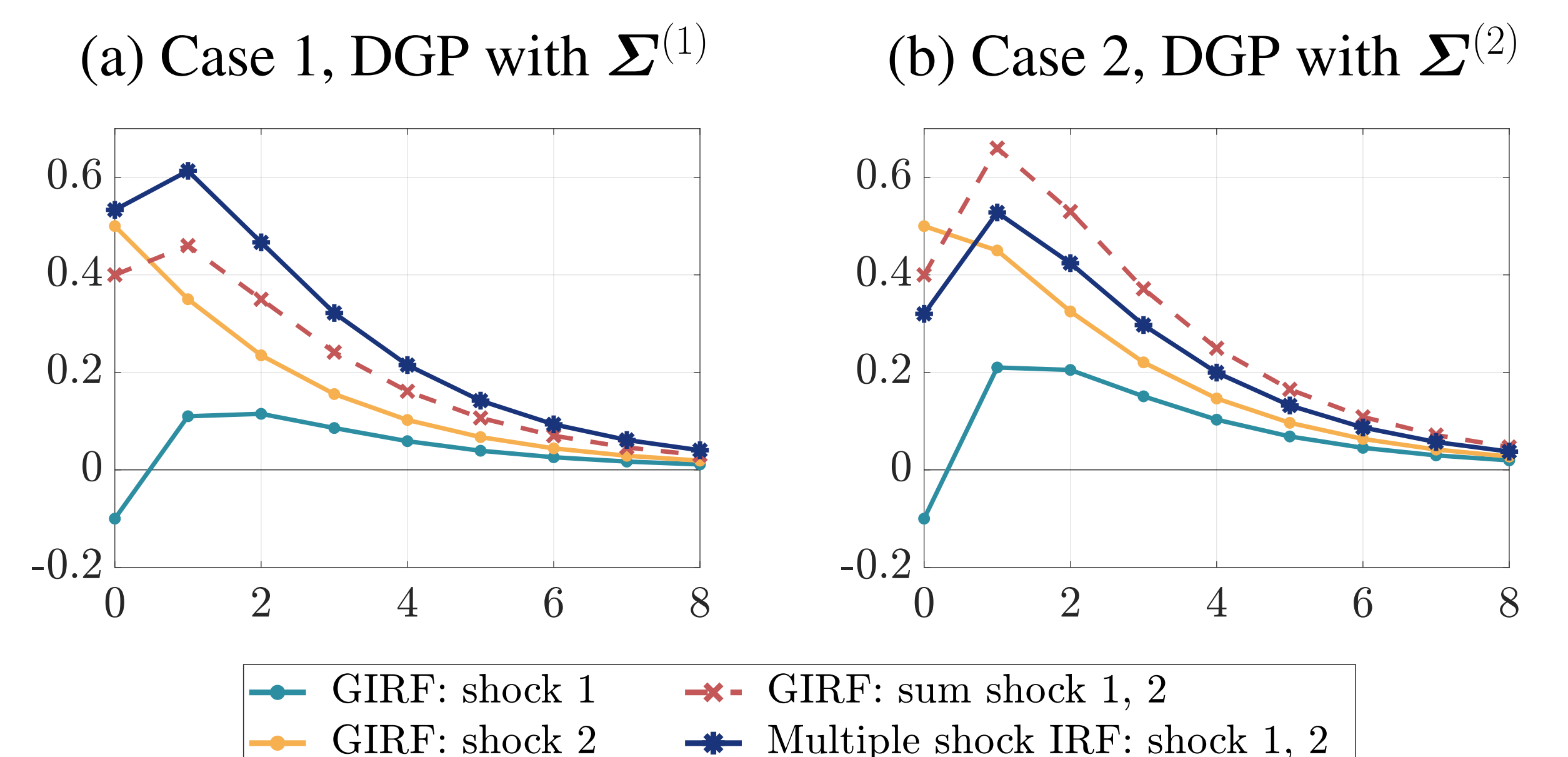
- DGP:  $n = 3$  variables, Equation (2), with  $\mathbf{B} = \begin{bmatrix} 0.4 & 0.1 & 0.1 \\ 0.1 & 0.4 & 0.1 \\ 0.2 & 0.2 & 0.4 \end{bmatrix}$ .

- Consider 2 cases:

$$\boldsymbol{\Sigma}^{(1)} = \begin{bmatrix} 1 & -0.25 & -0.1 \\ -0.25 & 1 & 0.5 \\ -0.1 & 0.5 & 1 \end{bmatrix}, \text{ and } \boldsymbol{\Sigma}^{(2)} = \begin{bmatrix} 1 & 0.25 & -0.1 \\ 0.25 & 1 & -0.5 \\ -0.1 & -0.5 & 1 \end{bmatrix}.$$

- Analyze effect of first two shocks  $\mathcal{S} = \{1, 2\}$  on variable 3.

### Figure: Impulse Response Functions



The sum of the one-shock GIRFs  $\sum_{\ell \in \mathcal{S}} \Psi^g(h, \delta_{\ell}, \mathcal{I}_{t-1})$  (dashed red line) **underestimates** (case 1) or **overestimates** (case 2) the total effect,  $\Psi^{\mathcal{S}}(h, \boldsymbol{\delta}_{\mathcal{S}}, \mathcal{I}_{t-1})$  (solid blue line).

## Summary and Further Research

- **Multiple shock impulse response functions are necessary to accurately analyze the combined effect of shocks.**
- Summing the one-shock generalized impulse response functions can lead to either over- or underestimation of the total effect.
- **Further research:**
  - \* Empirical analysis
  - \* Non-linear specifications, second order dynamics

## References

- Koop, G., M. H. Pesaran, and S. M. Potter (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics* 74(1), 119–147.
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- Sims, C. A. (1980). Macroeconomics and reality. *Econometrica* 48(1), 1–48.