

# Bayesian Multivariate Quantile Regression with alternative Time-varying Volatility Specifications

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## Motivation

- ▶ Current global economic and financial situation caused by the **COVID-19 pandemic** and the **Russia's invasion of Ukraine** has renewed the interest of the economic forecasters' and policy institutions in tail risk.
- ▶ Increasing interest in understanding, modeling and forecasting the macroeconomic **downside tail risk** (see Adrian et al. (2019,2022)) and in quantifying the uncertainty around these predictions.
- ▶ Typically time series methods model the **conditional mean** of variable of interest, making them unsuited to capture features such as skewness, fat tails and outliers, that characterize economic and financial time series in turbulent periods.
- ▶ **Quantile regression** (QR) models (see Koenker and Bassett, 1978) have been exploited to study the heterogeneous impact of covariates on different quantile levels of a variable of interest.

## Motivation

- ▶ Ferrara et al. (2022) introduce **mixed-data sampling** (MIDAS) to a Bayesian QR model to leverage on the information content of high-frequency financial conditions indicators
- ▶ Carriero et al. (2022) propose to **nowcast tail risk** to GDP growth by using Bayesian QR with mixed frequency and stochastic volatility. Pfarrhofer (2022) introduces time-varying parameters in QR to trace quantiles of inflation.
- ▶ Chavleishvilli and Maganelli (2021) propose **quantile Vector Autoregressive** (QVAR) to capture nonlinear relations among macroeconomics variables and define quantile impulse response function to perform stress tests.
- ▶ Iacopini et al. (2022) propose a novel **asymmetric continuous probabilistic score** for evaluating and comparing density forecast, which is useful when decision-maker has asymmetric preferences in the evaluation of forecasts.
- ▶ Iacopini et al. (2023) introduce a novel **mixed-frequency QVAR** which combines different frequencies in macroeconomic and financial variables to nowcast conditional quantiles of US GDP.

## Our Contribution

- ❖ We propose two frameworks for modeling **time-varying scale**, which is a multiplicative component of the variance in multivariate quantile regression models by means of
  - **Stochastic Volatility (SV)** - parameter-driven specification
  - **GARCH** - observation-driven specification
- ❖ We define the likelihood of a QVAR model with time-varying volatility via the **multivariate asymmetric Laplace (MAL)** distribution.
- ❖ Coupling SV or GARCH effects with mixture of Gaussian representation of asymmetric Laplace distribution results in the **standard deviation** (no variance) affecting also the conditional mean, but differently from traditional SV- or GARCH-in-mean models, making previous algorithm inefficient.
- ❖ We reformulate the models to make possible the joint sampling of whole trajectory of time-varying volatility, independently along cross-sectional dimension.

## Take home results

- ❖ We compare several univariate and multivariate quantile regression models with constant and alternative time-varying volatility specifications for **forecasting different quantiles** for several US macroeconomic and financial indicators
- ❖ The results show that the proposed methods **beat the constant volatility** QVAR benchmark for all the variables investigated.
- ❖ However no single specification is found to uniformly dominate the other over time, nor across variables or quantiles.
- ❖ We introduce **model combinations** based on quantile score weighting schemes to handle model uncertainty.
- ❖ The **combination weights** show significant variation over time, especially when quantiles corresponding to tails of the distribution are concerned, and at each point in time most of the mass is assigned a single model.
- ❖ QVAR time-varying combinations with **time-varying weights** perform accurately.

## Q(VA)R with constant volatility

### Multivariate quantile regression model

$$\mathbf{y}_t = \mathbf{B}\mathbf{x}_t + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \text{MAL}_n(\mathbf{0}, D\boldsymbol{\theta}_1, D\Theta_2\Psi\Theta_2D), \quad (1)$$

where

- ▶  $\mathbf{y}_t$  and  $\mathbf{x}_t$  are the  $n$ -dim vector of responses and the  $k$ -dim vector of common covariates;
- ▶  $B$  is a  $(n \times k)$  coefficient matrix;
- ▶  $\text{MAL}_n(\mu, \boldsymbol{\xi}, \Sigma)$  denotes the **multivariate asymmetric Laplace** distribution with location  $\mu$ , skew parameter  $\boldsymbol{\xi}$  and positive definite scale matrix  $\Sigma$ .

The parametrization of eq. (1) is such that  $D = \text{diag}(\delta_1, \dots, \delta_n)$  with  $\delta_j > 0$ ,  $\Psi$  is a **correlation** matrix, and  $\Theta_2 = \text{diag}(\boldsymbol{\theta}_2)$ , with:

$$\theta_{1,j} = \frac{1 - 2\tau_j}{\tau_j(1 - \tau_j)}, \quad \theta_{2,j} = \sqrt{\frac{2}{\tau_j(1 - \tau_j)}}, \quad j = 1, \dots, n, \quad (2)$$

where  $\tau_j \in (0, 1)$  is the (marginal) quantile of the  $j$ th series.

## Q(VA)R with constant volatility

- ▶ Building on the mixture representation of the multivariate asymmetric Laplace distribution, one obtains:

$$\mathbf{y}_t = X_t \boldsymbol{\beta} + D \boldsymbol{\theta}_1 w_t + \sqrt{w_t} D \Theta_2 \Psi^{1/2} \mathbf{z}_t, \quad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \text{Exp}(1), \quad (3)$$

where  $\boldsymbol{\beta} = \text{vec}(B) \in \mathbb{R}^{nk}$ ,  $\boldsymbol{\theta}_1 = (\theta_{1,1}, \dots, \theta_{1,n})$ , and  $X_t = (I_n \otimes \mathbf{x}_t)$ .

- ▶ The multivariate QR includes the **quantile VAR** (QVAR) model as special case for  $\mathbf{x}_t = \mathbf{y}_{t-1}$
- ▶ We assume **homoskedastic variance** for the conditional distribution of the response variable  $\mathbf{y}_t$ , which is highly restrictive when modeling economic and financial time series as they are typically characterized by highly persistent and clustered volatility

## Q(VA)R with time-varying volatility

- ▶ Let denote  $\Sigma = D\Psi D$  a positive definite matrix, where  $D = \text{diag}(\Sigma_{11}^{1/2}, \dots, \Sigma_{nn}^{1/2})$ , then we assume **heteroskedasticity**:

$$\Sigma_t = AH_tA', \quad (4)$$

where  $H_t$  is a diagonal matrix with positive elements on the diagonal and  $A$  is a lower triangular matrix with 1 on the main diagonal.

- ▶ Recalling definition of  $D$ , one has  $H_t^{1/2} = \text{diag}(\Sigma_{t,11}^{1/2}, \dots, \Sigma_{t,nn}^{1/2}) = D_t$ . Therefore introducing **time-varying volatility** in the scale matrix,  $\Sigma$ , the multivariate QR model with time-varying volatility is

$$\mathbf{y}_t = X_t\beta + H_t^{1/2}\theta_1w_t + \sqrt{w_t}\Theta_2AH_t^{1/2}\mathbf{z}_t, \quad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \text{Exp}(1). \quad (5)$$



## Parameter Driven: Q(VA)R-SV

- ▶ When dealing with conditional mean multivariate time series models, the **inclusion of stochastic volatility** leads to strong improvements with respect to constant volatility models.
- ▶ The quantile multivariate regression model with **stochastic volatility** (QR-SV) is defined as

$$\begin{aligned} \mathbf{y}_t &= X_t\boldsymbol{\beta} + D\boldsymbol{\theta}_1 w_t + \sqrt{w_t} D\boldsymbol{\theta}_2 \Psi^{1/2} \mathbf{z}_t, & \mathbf{z}_t &\sim \mathcal{N}_n(\mathbf{0}, I_n), & w_t &\sim \text{Exp}(1), \\ \Sigma_t &= A H_t A', \\ H_t &= \text{diag}(e^{h_{1,t}}, \dots, e^{h_{n,t}}), \\ h_{j,t} &= \phi_j h_{j,t-1} + \epsilon_{j,t}^h, & \epsilon_{j,t}^h &\sim \mathcal{N}(0, \sigma_{h,j}^2), \end{aligned}$$

where  $|\phi_j| < 1$  and  $h_{j,1} \sim \mathcal{N}(0, \sigma_{h,j}^2 / (1 - \phi_j^2))$ .

- ▶ By introducing lags of the response variable into the covariates, we obtain the **QVAR-SV** model.

## Parameter Driven: Q(VA)R-SV

- ▶ It follows that introducing stochastic volatility in  $\Sigma$  results in a model that includes the **square root of volatility** terms,  $e^{h_{i,t}/2}$ , in the conditional mean equation for  $\mathbf{y}_t$ :

$$\mathbf{y}_t = X_t\boldsymbol{\beta} + w_t\Theta_1 e^{\mathbf{h}_t/2} + \sqrt{w_t}\Theta_2 A \text{diag}(e^{\mathbf{h}_t/2})\mathbf{z}_t, \quad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \mathcal{Exp}(1), \quad (6)$$

where  $\mathbf{h}_t = (h_{1,t}, \dots, h_{n,t})'$ ,  $e^{\mathbf{h}_t/2} = (e^{h_{1,t}/2}, \dots, e^{h_{n,t}/2})'$ , and  $\Theta_1 = \text{diag}(\boldsymbol{\theta}_1)$ .

- ▶ Conditional on  $w_t$ , it resembles a VAR with stochastic volatility in mean (VAR-SVM) model.
- ⇒ The **main difference** is that VAR-SVM model includes vector of log-volatilities  $\mathbf{h}_t$ , whereas we have vector of square roots of volatilities,  $e^{\mathbf{h}_t/2}$ .
- ⇒ We design a **computationally efficient** procedure for making inference on the log-volatility processes  $\mathbf{h}_j = (h_{j,1}, \dots, h_{j,T})'$  for each series  $j = 1, \dots, n$ .

## Computational advantages

- ▶ We remove for simplicity  $X_t\beta$ , and we can rearrange

$$\mathbf{y}_t = w_t\Theta_1 e^{h_t/2} + \sqrt{w_t}\Theta_2 A \text{diag}(e^{h_t/2})\mathbf{z}_t, \quad \mathbf{z}_t \sim \mathcal{N}_n(\mathbf{0}, I_n), \quad w_t \sim \text{Exp}(1)$$

$$\implies \mathbf{y}_t = B_t e^{h_t/2} + A_t \bar{\mathbf{z}}_t, \quad \bar{\mathbf{z}}_t \sim \mathcal{N}_n(\mathbf{0}, H_t),$$

where  $B_t = w_t\Theta_1 \in \mathbb{R}^{n \times n}$  and  $A_t = \sqrt{w_t}\Theta_2 A \in \mathbb{R}^{n \times n}$  are transformation of  $\Theta_1$  and  $\Theta_2$ .

- ▶ After some computations, we obtain the **likelihood** for the vector  $\mathbf{h}_j$

$$\tilde{\mathbf{y}}_t^j = A_t^{-1}\mathbf{y}_t - \sum_{i \neq j} \tilde{A}_{t,:i} e^{h_{i,t}/2}, \quad \text{where } \tilde{A}_{t,:i} \text{ denotes } i\text{-th column of } \tilde{A}_t = A_t^{-1}B_t$$

- ▶ We use an **adaptive RW Metropolis-Hastings** algorithm (with Gaussian proposal) to draw samples from posterior distribution of  $\mathbf{h}_j$ , which allows for computationally more efficient sampling of log-volatility

$\implies$  We substitute a standard forward loop over time  $t$ , with a cycle over series  $j$  which can be easily **parallelized**. Thus replacing a step of  $O(T)$  complexity with one of complexity  $O(n)$ .

## Observation Driven: Q(VA)R-GARCH

- ▶ The quantile multivariate regression model with **GARCH** (QR-GARCH) is defined as

$$\begin{aligned} \mathbf{y}_t &= X_t \boldsymbol{\beta} + D \boldsymbol{\theta}_1 w_t + \sqrt{w_t} D \boldsymbol{\Theta}_2 \boldsymbol{\Psi}^{1/2} \mathbf{z}_t, & \mathbf{z}_t &\sim \mathcal{N}_n(\mathbf{0}, I_n), & w_t &\sim \text{Exp}(1), \\ \boldsymbol{\Sigma}_t &= A H_t A', & H_t &= \text{diag}(\sigma_{1,t}^2, \dots, \sigma_{n,t}^2), \\ \sigma_{j,t}^2 &= \omega_j + \alpha_j \epsilon_{j,t-1}^2 + \gamma_j \sigma_{j,t-1}^2 = \omega_j + \alpha_j (y_{j,t-1} - X_{t-1} \boldsymbol{\beta} - w_{t-1} \boldsymbol{\theta}_{1,j} \sigma_{j,t-1})^2 + \gamma_j \sigma_{j,t-1}^2, \end{aligned}$$

where parameters need to ensure stationarity:  $\omega_j > 0$ ,  $\alpha_j \geq 0$ ,  $\gamma_j \geq 0$ , and  $(\alpha_j + \gamma_j) < 1$ .

- ▶ It follows that introducing GARCH in  $\boldsymbol{\Sigma}$  results in a model that includes the **square root of volatility terms**,  $\sigma_{i,t}$ , in the conditional mean equation for  $\mathbf{y}_t$ :

$$\mathbf{y}_t = X_t \boldsymbol{\beta} + w_t \boldsymbol{\Theta}_1 \boldsymbol{\sigma}_t + \sqrt{w_t} \boldsymbol{\Theta}_2 A \text{diag}(\boldsymbol{\sigma}_t) \mathbf{z}_t, \text{ where } \boldsymbol{\sigma}_t = (\sigma_{1,t}, \dots, \sigma_{n,t})'$$

- ▶ Conditional on  $w_t$ , it resembles a VAR with GARCH in mean (VAR-GARCH-M) model.
- ⇒ The **main difference** is that VAR-GARCH-M includes vector of volatilities  $\boldsymbol{\sigma}_t^2$ , whereas we have the vector of square roots of volatilities,  $\boldsymbol{\sigma}_t$ .

## Bayesian Inference

- ▶ For vectorized **matrix of coefficients** and vector containing non zero elements of  $j$ -th row of the  $A$  matrix,  $\bar{\mathbf{a}}_j$ , we assume

$$\beta \sim \mathcal{N}_{nk}(\underline{\mu}_\beta, \underline{\Sigma}_\beta), \quad \bar{\mathbf{a}}_j \sim \mathcal{N}_{j-1}(\underline{\mu}_{a,j}, \underline{\Sigma}_{a,j}), \quad j = 2, \dots, n,$$

where hyperparameters are chosen such as to be noninformative.

- ▶ For QR-SV, prior for **persistence parameter** and innovation variance of log-volatility are

$$\left(\frac{1 + \phi_j}{2}\right) \sim \mathcal{Be}(\underline{a}_\rho, \underline{b}_\rho), \quad \sigma_{h,j}^2 \sim \mathcal{IG}(\underline{a}_\sigma, \underline{b}_\sigma).$$

- ▶ For QR-GARCH, prior for log-transformation of **parameters** with stationarity condition

$$\begin{aligned} \log(\omega_j) &\sim \mathcal{N}(\underline{\mu}_\omega, \underline{\sigma}_\omega^2) \\ \begin{pmatrix} \log(\alpha_j) \\ \log(\gamma_j) \end{pmatrix} &\sim \mathcal{N}_2 \left( \begin{pmatrix} \underline{\mu}_\alpha \\ \underline{\mu}_\gamma \end{pmatrix}, \begin{pmatrix} \underline{\sigma}_\alpha^2 & 0 \\ 0 & \underline{\sigma}_\gamma^2 \end{pmatrix} \right) \mathbb{I}(\alpha_j + \gamma_j < 1). \end{aligned}$$

## Evaluation: Quantile Score (QS)

- ▶ To assess the quality of quantile forecasts, we rely on **Quantile score (QS, see Giacomini and Komunjer, 2005)** as tail metric.
- ▶ The QS for model  $k = 1, \dots, K$ , where  $K$  is total number of models estimated in forecasting exercise, at forecasting horizon  $h = 1, \dots, H$  and quantile  $\tau$ , is defined as:

$$QS_{k,\tau,t+h} = (\mathbf{y}_{t+h} - \hat{Q}_{k,\tau,t+h}) \odot (\tau - \mathbb{I}_{\{\mathbf{y}_t \leq \hat{Q}_{k,\tau,t+h}\}}),$$

where

- $\odot$  denotes Hadamard product,
  - $\mathbf{y}_{t+h}$  is observed value of vector response to be forecasted,
  - $\hat{Q}_{k,\tau,t+h}$  is forecast of quantile  $\tau$  under model  $k$ ,
  - $\mathbb{I}_{\{C\}}$  is vector-valued indicator function, whose  $j$ th element has value of 1 if outcome  $y_{j,t+h}$  is at or below forecasted quantile  $\hat{Q}_{j,k,\tau,t+h}$  and 0 otherwise.
- ▶ Notice that **better performances** are associated to **lower values** of the QS.

## Combination

We propose a **combination** of different models based on QS:

- Forecast combination based on **time-varying weights** (T-V):

$$Q_{\tau,t+h}^{c,tv} = \sum_{k=1}^K w_{k,\tau,t+h} \times QS_{k,\tau,t+h},$$

where weights of model  $k$  at horizon  $h$  and quantile  $\tau$  are function of past performance of each model  $k$  known when the forecast is made

$$w_{k,\tau,t+h} = \frac{\sum_{t=t_0}^{t_i+t_0-h} QS_{k,\tau,t}^{-1}}{\sum_{j=1}^K \sum_{t=t_0}^{t_i+t_0-h} QS_{j,\tau,t}^{-1}}, \text{ where } t_i = \text{in-sample, } t_0 = \text{out-of-sample length}$$

- Forecast combination based on **constant (average) weights** (AVG):

$$Q_{\tau,t+h}^{c,avg} = \sum_{k=1}^K \bar{w}_{k,\tau} \times QS_{k,\tau,t+h},$$

where we use temporal average of weights,  $\bar{w}_{k,\tau} = \frac{1}{t_0} \sum_{t=1}^{t_0} w_{k,\tau,t+h}$ .

## Data Description

- ▶ Eight quarterly **macroeconomic** variables and one **financial** variable (NFCI) as in Iacopini et al. (2022):

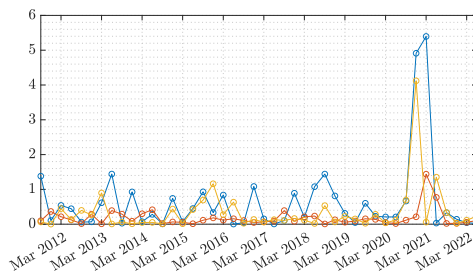
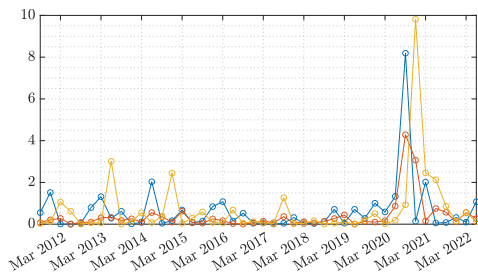
Description	Fred Mnemonic	Transformation
Average Weekly Hours	AWHMAN	$0.1x_t$
CPI Inflation	CPIAUCSL	$100\Delta \ln(x_t)$
Industrial Production	INDPRO	$100\Delta \ln(x_t)$
S& P 500	S&P500	$100\Delta \ln(x_t)$
Federal Funds Rate	FEDFUNDS	$\Delta x_t$
10y Government Treasury yield	GS10	$\Delta x_t$
Unemployment Rate	UNRATE	$\Delta x_t$
Real Gross Domestic Product	GDPC1	$400\Delta \ln(x_t)$
Chicago Fed National Financial Condition Index	NFCI	Level

- ▶ **In-sample** analysis: 1971Q1-2022Q2.
- ▶ **Out-of-sample** analysis based on both rolling and expanding window of length 160 quarters (alias 40 years): 2011Q1-2022Q2



## Quantile Score over-time for QVAR-SV for GDP (left) and NFCI (right)

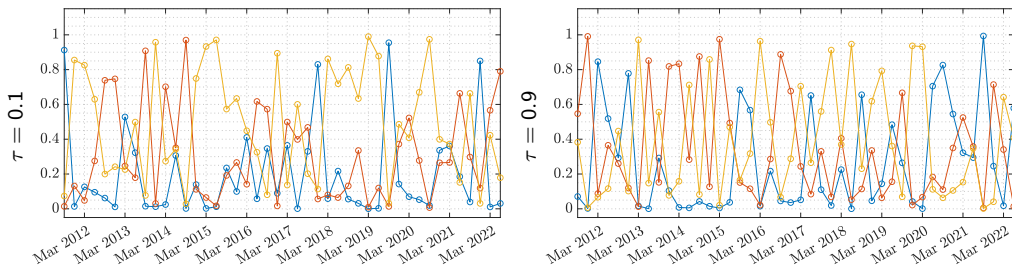
Different **quantiles**:  $\tau = 0.1$  (blue);  $\tau = 0.5$  (red) and  $\tau = 0.9$  (yellow).



- ❖ QS for **GDP** at **90th** pct peaks at 2014:Q4 in correspondence to drop of GDP from 5.0 to 2.5 percent  $\implies$  driven by upturn in imports + downturn in federal government spending
- ❖ Covid-19 pandemic: **10th** pct worsened during 2021:Q4, while 90th occurred in 2022:Q1 at outbreak of Russian-Ukraine war
- ❖ QS for **NFCI** has sinusoidal trajectory at 10th and 90th. Left tail peaks around 2013, 2016 and 2018 related to US debt-ceiling
- ❖ Conditional mean and median miss meaningful changes of macro & financial tail risks

## T-V Combination weights for NFCI for $\tau = 0.1$ (left) and $\tau = 0.9$ (right)

Different models: QVAR-SV (red line); QVAR-GARCH (yellow) and QVAR (blue)



- ❖ **QVAR** model has almost zero weight for all percentiles
- ❖ **Left** quantile evidence QVAR-GARCH is best performing except between 2014–15, where QVAR-SV was the best
- ❖ **Right** quantile less persistent across time, where QVAR-GARCH best from 2018–22, while QVAR-SV best from 2014–15.

## Table interpretations

- ▶ Report QS score for baseline Q(V)AR(1) model with constant volatility and Ratios between computed metric of the current model over baseline Q(V)AR model with expanding window.
- ⇒ **Entries of less than 1** indicate that given current model yields forecasts more accurate than those provided by baseline.
- ▶ Perform **Diebold and Mariano (1995) t-test** for equality of QS to compare predictions of alternative models with the benchmark (QVAR and QAR) if differences in forecast accuracy are significant
- ⇒ **\*, \*\*, \*\*\*** mean significance at 10%, 5%, 1% levels.
- ▶ Perform the **Model Confidence Set** procedure Hansen et al (2011) to jointly compare the predictive power of all models
- ⇒ **Bold number** indicate models that belong to Superior Set of Models delivered by the MCS at confidence level 10%.

Quantile Score for different variables and percentiles ( $\tau = 0.1, 0.9$ )

Variable	AWHMAN	CPIAUCSL	INDPRO	S&P500	FEDFUNDS	GS10	UNRATE	NFCI	GDPC1
<b>Quantile: <math>\tau = 0.1</math></b>									
QVAR	1.699	1.128	1.598	1.806	1.855	2.078	1.901	1.497	1.793
QVAR-SV	<b>0.601***</b>	<b>0.568**</b>	<b>0.627***</b>	<b>0.425***</b>	0.618***	<b>0.591***</b>	<b>0.963***</b>	<b>0.643***</b>	<b>0.643***</b>
QVAR-GARCH	<b>1.091</b>	<b>0.476***</b>	<b>0.932</b>	<b>0.523***</b>	<b>0.347***</b>	<b>0.570***</b>	<b>1.126*</b>	<b>0.349***</b>	<b>0.974*</b>
QVAR Combin (AVG)	1.136**	<b>0.538***</b>	<b>0.731***</b>	<b>0.512***</b>	<b>0.482***</b>	<b>0.574***</b>	<b>1.050**</b>	<b>0.450***</b>	0.828***
QVAR Combin (T-V)	<b>0.360***</b>	1.100	<b>0.511***</b>	<b>0.415***</b>	<b>0.480***</b>	<b>0.463***</b>	<b>0.581***</b>	<b>0.591***</b>	<b>0.480***</b>
QAR	1.763	1.869	1.728	1.514	1.277	1.732	1.937	1.604	1.788
QAR-SV	1.721	2.261	1.643	1.933	1.950	1.864	1.979	2.315	1.836
QAR-GARCH	2.054	2.562	2.279	2.370	1.754	2.242	1.953	1.651	2.163
QAR Combin (AVG)	1.792	2.224	1.690	1.673	1.441	2.048	1.937	1.843	1.886
QAR Combin (T-V)	1.705	1.974	1.565	0.997***	0.905***	0.758***	<b>0.841***</b>	<b>0.657***</b>	<b>0.405***</b>
<b>Quantile: <math>\tau = 0.9</math></b>									
QVAR	2.115	1.822	1.804	2.050	1.618	1.853	<b>1.881</b>	1.180	1.814
QVAR-SV	0.665***	0.488***	0.553***	0.754***	0.417***	0.526***	<b>0.616***</b>	0.344***	0.684***
QVAR-GARCH	<b>0.725***</b>	0.396***	0.696***	0.893***	0.564***	0.587***	<b>1.892</b>	0.564*	1.022*
QVAR Combin (AVG)	<b>0.717***</b>	1.492***	1.207***	0.759***	0.484***	0.515***	<b>1.364</b>	0.568**	0.792***
QVAR Combin (T-V)	<b>0.238***</b>	<b>0.187***</b>	<b>0.147***</b>	0.304***	<b>0.140***</b>	0.198***	<b>0.322***</b>	<b>0.111***</b>	<b>0.315***</b>
QAR	1.745	1.415	1.666	1.815	1.370	1.518	1.667	1.319	1.734
QAR-SV	1.964	1.911	1.980	1.911	1.502	1.792	1.407**	1.647	1.939
QAR-GARCH	2.205	1.099**	2.012	2.356	1.558	1.842	<b>1.608</b>	0.832***	1.751
QVAR Combin (AVG)	1.917	1.218**	1.915	1.906	1.466	1.696	1.519*	1.519	1.810
QVAR Combin (T-V)	<b>0.203***</b>	<b>0.225***</b>	<b>0.170***</b>	<b>0.115***</b>	<b>0.105***</b>	<b>0.127***</b>	<b>0.318***</b>	<b>0.077***</b>	<b>0.277***</b>

## Conclusion

- ❖ We compare several univariate and multivariate quantile regression models with constant and alternative **time-varying volatility** specifications for forecasting different quantiles for several US macroeconomic and financial indicators
- ❖ The results show that the proposed methods **beat the constant volatility** QVAR benchmark for all the variables investigated.
- ❖ However no single specification is found to uniformly dominate the other over time, nor across variables or quantiles.
- ❖ We introduce **model combinations** based on quantile score weighting schemes to handle model uncertainty.
- ❖ QVAR time-varying combinations with **time-varying weights** perform accurately.

## References

THANK YOU FOR THE ATTENTION

The talk was based on

- ❖ Iacopini, M., Ravazzolo, F. & Rossini, L. (2023) – [Bayesian Multivariate Quantile Regression with alternative Time-varying Volatility Specifications](#), arXiv:2211.16121

Other project on Asymmetry/ Quantile regression

- ❖ Iacopini, M., Ravazzolo, F. & Rossini, L. (2022) – [Proper Scoring Rules for Evaluating Density Forecasts with Asymmetric Loss Functions](#), *Journal of Business and Economic Statistics*, 41(2), 482-496
- ❖ Iacopini, M., Poon, A., Rossini, L. and Zhu, D. (2023) – [Bayesian Mixed-Frequency Quantile Vector Autoregression: Eliciting tail risks of Monthly US GDP](#), arXiv:2209.01910
- ❖ Iacopini, M., Poon, A., Rossini, L. and Zhu, D. (2023) – [Quantile Responsiveness](#), Work in progress