Discussion of

Bayesian Multivariate Quantile Regression with alternative Time-varying Volatility Specifications

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The model

- From a BVAR...

$$\mathbf{y}_t = X_t\boldsymbol{\beta} + \boldsymbol{\epsilon}_t \qquad \boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \Sigma \otimes I_T)$$

- ...to a BQVAR...

$$\begin{aligned}\mathbf{y}_t &= X_t\boldsymbol{\beta} + \boldsymbol{\epsilon}_t \qquad \boldsymbol{\epsilon}_t \sim MAL(\mathbf{0}, D\boldsymbol{\theta}_1, D\Theta_2\Psi\Theta_2 D)\\ &= X_t\boldsymbol{\beta} + D\boldsymbol{\theta}_1 w_t + \sqrt{w_t}\Theta_2 D\Psi^{1/2}\mathbf{z}_t \qquad \mathbf{z}_t \sim \mathcal{N}(0, I) \qquad w_t \sim Exp(1)\end{aligned}$$

where $\Sigma = D\Psi D$ and $D = \text{diag}(\Sigma_{11}^{1/2}, \dots, \Sigma_{nn}^{1/2})$

- ...to BQVAR with time-varying volatility

$$\mathbf{y}_t = X_t\boldsymbol{\beta} + H_t^{1/2}\boldsymbol{\theta}_1 w_t + \sqrt{w_t}\Theta_2 A H_t^{1/2}\mathbf{z}_t$$

with $\Sigma_t = AH_tA$ and $H_t^{1/2} := D_t = \text{diag}(\Sigma_{t,11}^{1/2}, \dots, \Sigma_{t,nn}^{1/2})$.

The time-varying volatility

- Stochastic-volatility

$$H_t = \text{diag}(e^{h_{1,t}}, \ldots, e^{h_{j,t}})$$
$$h_{j,t} = \rho h_{j,t-1} + \varepsilon_{j,t}$$

$\Rightarrow$ similar to a VAR with SV in mean model... but $H_t^{1/2}$ (not H_t).

- GARCH

$$H_t = \text{diag}(\sigma^2_{1,t}, \ldots, \sigma^2_{j,t})$$
$$\sigma^2_{j,t} = \omega_j + \alpha_j \epsilon^2_{j,t-1} + \gamma_j \sigma^2_{j,t-1}$$

$\Rightarrow$ similar to a VAR with GARCH in mean model... but $H_t^{1/2}$ (not H_t).

Main contributions of this paper

1. Consider a QVAR in a Bayesian framework (BQVAR) $\Rightarrow$ build upon the literature and define the likelihood as MAL.
2. Extend the QVAR to time-varying volatility and develop a sampler in pseudo-SV and pseudo-GARCH in mean models.
3. Show that QVAR augmented with time-varying volatility may improve 1-step ahead forecasts.
4. Show that combining homoskedastic and heteroskedastic QVAR forecasts (with TV weights) may improve 1-step ahead forecasts .

Some comments/questions on the model

- **QR with time-varying volatility** : why could it be empirically relevant ?
  - In a linear regression, TV volatility accommodates distributional changes in the shocks.
  - One could argue that the baseline QR accommodates this feature.

$$y_t = \beta_q x_t + \theta_q \sigma_q w_{q,t} + \tau_q \sigma_q \sqrt{w_{q,t}} \epsilon_t$$

    - fat tails generated by asymmetry in the estimated coefficients β_q.
    - the term $\theta_q \sigma_q w_{q,t}$ shifts the location to target the quantile q.
    - the term $\tau_q \sigma_q \sqrt{w_{q,t}}$ rescale the shocks for the quantile q.
  - The paper could elaborate more on the relevance of TV volatility in a QR or QVAR framework.

- **Efficiency of the sampler** : more details on the "efficiency". Monte Carlo simulations ? Convergence diagnostics ?
- **Time-variation** : TV volatility is only part of a broader picture. Accommodate time-variation in model parameters $\boldsymbol{\beta} \Rightarrow$ in this paper ? Or suggest a way forward ?
- **Forecast horizon** : VAR very useful for multi-step "iterative" predictions, but QVAR still limited to 1-step ahead $\Rightarrow$ clearly discuss this issue in the paper.

Some comments/questions on the empirical results

- How well does the model compared to a standard BVAR-SV à la Carriero, Clark, Marcellino (JMCB, forthcoming) ?
- How well does the QVAR model compared to univariate QR which include not only lags of the endogenous variable but also lags of additional (exogenous) predictors ?
- Univariate QR do not seem to benefit so much from TV volatility. Any hint on this outcome ?
- Results only on QS, while an assessment on the entire density forecast would be also useful (CRPS, logS) $\Rightarrow$ implies a choice on the computation of entire density from discrete # of quantiles (fine grid of quantiles ? Skew-t matching approach ?).
- Should the paper discuss a "stress testing" approach ? $\Rightarrow$ forecasting quantile q of variable y conditional on quantile q' of variable x.