The Historical Role of Energy in UK Inflation and Productivity with Implications for Price Inflation

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Abstract

We model UK price and wage inflation, productivity and unemployment over a century and a half of data, selecting dynamics, relevant variables, non-linearities and location and trend shifts using indicator saturation estimation. The four congruent econometric equations highlight complex interacting empirical relations. The production function reveals a major role for energy inputs additional to capital and labour, and although the price inflation equation shows a small direct impact of energy prices, the substantial rise in oil and gas prices seen by mid-2022 contribute half of the increase in price inflation. We find empirical evidence for non-linear adjustments of real wages to inflation: a wage-price spiral kicks in when inflation exceeds about 6–8% p.a. We also find an additional non-linear reaction to unemployment, consistent with involuntary unemployment. A reduction in energy availability simultaneously reduces output and exacerbates inflation.

JEL classifications: C51, C22.
Keywords: Energy; Inflation; Location Shifts; Indicator Saturation Estimation; Equilibrium Correction.

1 Introduction

The recent and relatively sudden increases in inflation rates in many countries, especially in energy and food, have posed serious financial problems for lower-income families with high expenditure shares on those items. The price increases in energy (oil and natural gas) and food have been large, stimulated by a mix of recovery from COVID-19, supply chain issues and the Russian invasion of Ukraine, exacerbated by Brexit in the case of the United Kingdom. At their heights since the start of 2022, natural gas prices have more than tripled, and crude oil prices more than doubled, as did those of corn and wheat.

High rates of inflation are not new to the UK, although their low rates for the last 30 years may have lulled memories. A major advantage of long-run consistent time series is that they include many wars, oil (and other) crises and unanticipated major events like pandemics, so can provide evidence on the role of energy in inflation and how the present situation may pan out. There are growing risks that inflation will spike further in 2023 and beyond with continued conflict in Ukraine and potential further reductions in Russian gas supplies. Expectations of higher energy prices have led some forecasters to predict inflation rates as high as 18% over the next year. Thus, it is of crucial importance to understand how the cost of energy feeds through into inflation and the broader UK economy. The main drawback of our data being annual is that the current situation was not foreseen in 2021 and so could not have been forecast by our models. We circumvent that last problem by calculating projections of inflation based on our empirical models but using recent data observations.

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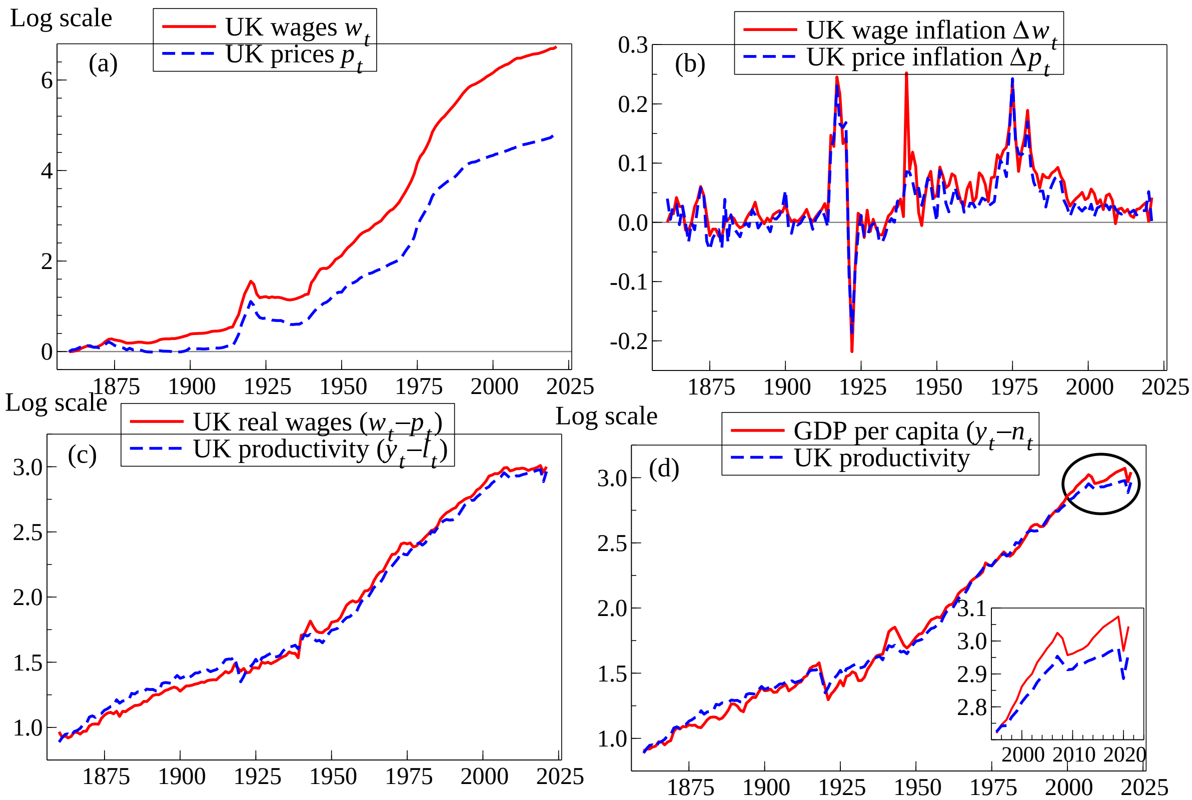

Figure 1: (a) Logs of UK wages and prices, (b) their rates of inflation, 1860–2021, (c) real wages and productivity in logs; (d) GDP divided by population.

Figure 1 reports the time series over 1860–2021 for UK nominal wages (W) and prices (P) and their rates of inflation (calculated as $\Delta \log(X) = \Delta x$ for a level X). There have been huge increases in nominal annual wages and prices (700 fold and 100 fold respectively since 1860, compared to productivity rising 7-fold overall) with annual price inflation rates reaching more than 20% on three occasions. Real wages ($w - p$) have risen at varying rates, closely tracking productivity (measured by GDP per worker per year, $y - l$) and the two clearly cointegrate, yielding the basis for an economic theory model of real wages. Note the flatlining of both real wages and productivity from 2008 onwards, an issue we return to in §3. However, over this century, GDP divided by population (i.e., real income per capita) has risen by 17% (employment growth has exceeded population growth) so UK policies to successfully limit greenhouse gas emissions have not limited economic growth (see Castle and Hendry, 2022).

Figure 2(a) shows that total UK energy use (E_t) rose almost 5-fold from 1860 to 1975, but has not trended since. The mix of sources has altered substantially from 100% coal in 1860 remaining dominant till the 1950s, but essentially none by the end of the period, now replaced by roughly equal amounts of oil, natural gas (both declining) and non-fossil (rising rapidly) all measured in millions of tons of oil equivalent (Mtoe). Energy per unit of capital ($e - k$) shown in Panel (b) has fallen greatly from efficiency improvements, relatively slowly till the mid-1950s then more rapidly since at about 2% p.a. The impacts on nominal oil prices (P_o) of the 1970s oil crises, 2010 speculation and COVID-19 are clearly visible in Panels (c) and (d). In addition, UK natural gas prices have risen more than 200% since 2019 and have increased even faster during 2022. The resulting large rises in UK electricity prices despite extensive renewable supply are partly because wholesale electricity auctions reflect marginal prices, but also the UK mothballed its last gas storage facility in 2017.

One of the most obvious data features is the non-constancy of change, so even log differences are non-stationary from distributional shifts as seen in Figure 3. Thus, the modelling approach undertaken

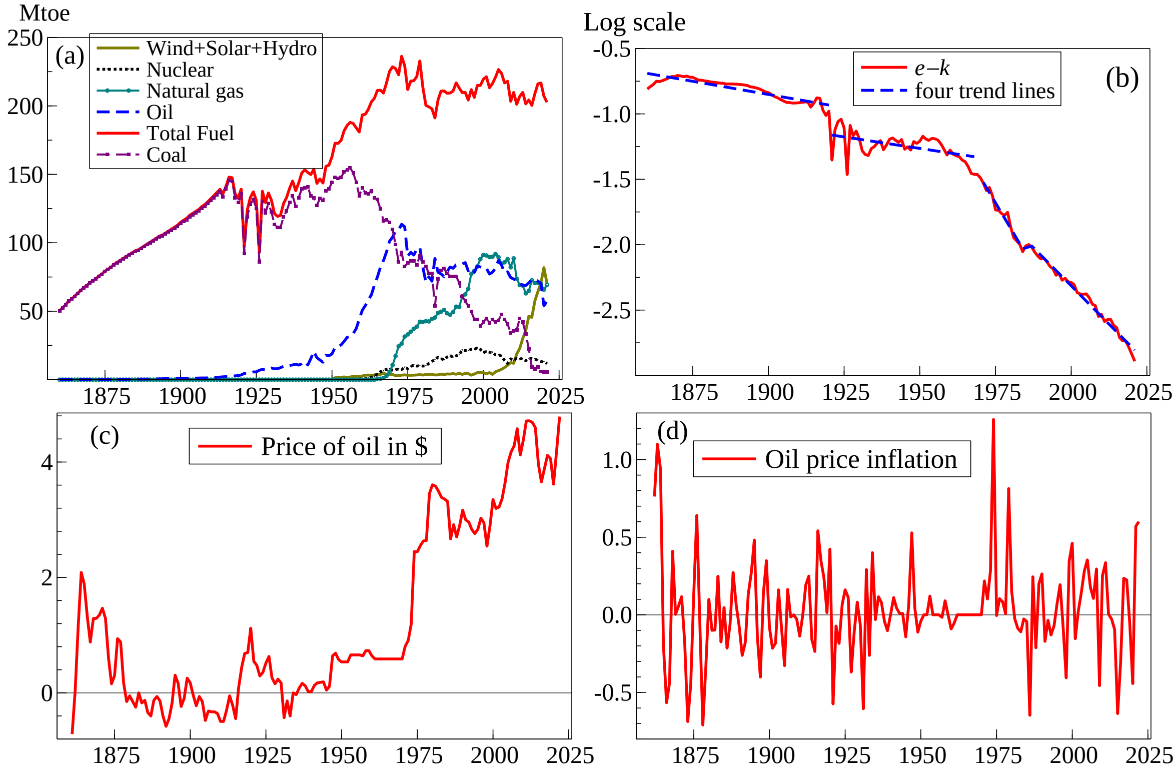

Figure 2: (a) UK total energy use, E_t, calculated as the sum of coal, oil, natural gas, nuclear and wind+solar+hydroelectric all in Mtoe; (b) log energy per unit capital, $e - k$, with four sub-period trends; (c) logs of oil prices in \$; and (d) their rates of change, 1860–2021 (so $1.0 = 100\%$).

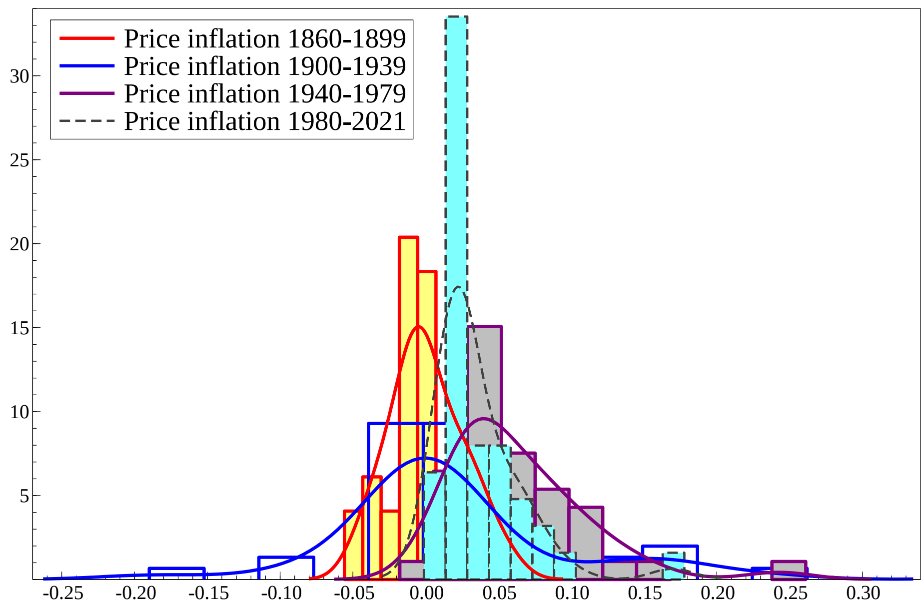

Figure 3: Distributional shifts in UK price inflation by 40-year subsamples.

must be able to handle all forms of change, outlined in Appendix §8 (also see Castle and Hendry, 2019). The next four sections build empirical econometric models of real wages in §2, highlighting the relevance of a wage-price spiral and a non-linear unemployment effect; unemployment in §3 where a profits proxy explains most of the variation in unemployment; productivity in §4 where the crucial role for energy is shown; and price inflation in §5 where many domestic and global factors are found to drive price inflation. Using the models developed, §6 combines the price and wage inflation models to make projections of price inflation based on energy price rises seen during 2022. Section 7 concludes and Appendix §8 sketches our econometric tools for modelling non-stationary time series. A supplementary data file records details of the data series used along with their sources.

2 An empirical model of UK real wages

Real wage models tend to fall into two categories, both of which rely on the underlying theory that real wages are determined by the marginal product of labour, but the first sees inflation expectations accorded a key role in feed-forward mechanisms of the New Keynesian Phillips Curve (NKPC), see, e.g. Galí and Gertler (1999) and Galí, Gertler, and Lopez-Salido (2001), and the second focuses on feed-back mechanisms through dynamic models as in Castle and Hendry (2009).

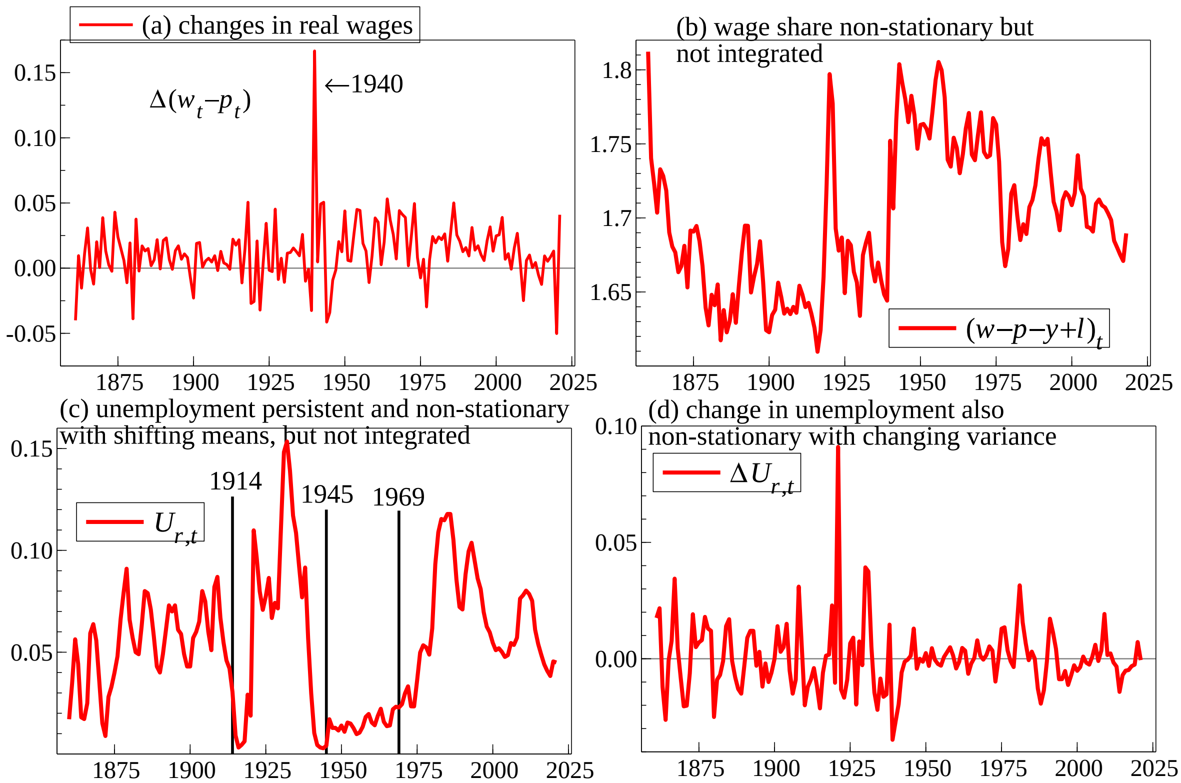

Figure 4: (a) Changes in real wages; (b) wage share; (c) unemployment rate; and (d) changes in the unemployment rate, 1860–2021.

Figure 4(a) records real wages for the UK since 1860. Panel (b) plots the cointegrating relation between real wages and output per worker, which proxies the marginal revenue product of labour with a cointegrating weight of 1. This is also the labour share in national income, and while it is not integrated given the evidence of cointegration, it is clearly not stationary with location shifts, notably at the end of WorldWar II (WWII) and at the beginning of the Thatcher era. Panel (c) plots the unemployment rate, which in a traditional Phillips curve relationship is assumed to be a driver of real wage growth. However, the data is persistent and non-stationary with shifting means attributable to exogenous shocks such as wars and policy, but it is not integrated. The annual change in the unemployment rate, recorded in Panel (d), also shows a changing variance so even the difference is non-stationary.

Eyeballing the data on changes in real wages in Figure 4(a) it is hard to discern changes in growth rates, but Step Indicator Saturation (SIS; see Appendix §8) can be applied (here at 0.1%) to check on location shifts unconditionally, recorded in the solid black line in Figure 5(a). This reveals a doubling of the growth rate of real wages post WWII from 0.8% to 1.7%. Applying SIS to the growth rate in productivity in Panel (c) highlights the upwards shift from 1.2% to 1.7%, but at a different time and by a different magnitude to the shift in real wages. As the location shifts do not co-break and the large outliers do not align it suggests a much more complex empirical model of real wages. Panel (b) records many step shifts in price inflation, and Panel (d) shows those in unemployment: the location shifts in $U_{r,t}$ and Δp_t also do not match. In the general model of real wages below, we include both the level and the change in the unemployment rate. The change allows for possible dynamic labour supply effects, i.e. if unemployment is growing the pool of potential labour supply is increasing, lowering wages, and we find the level of inflation enters non-linearly. Price inflation is included as a catch-up mechanism if wages have been eroded due to less than complete adjustment to past inflation, playing an important non-linear role in the form of a wage-price spiral.

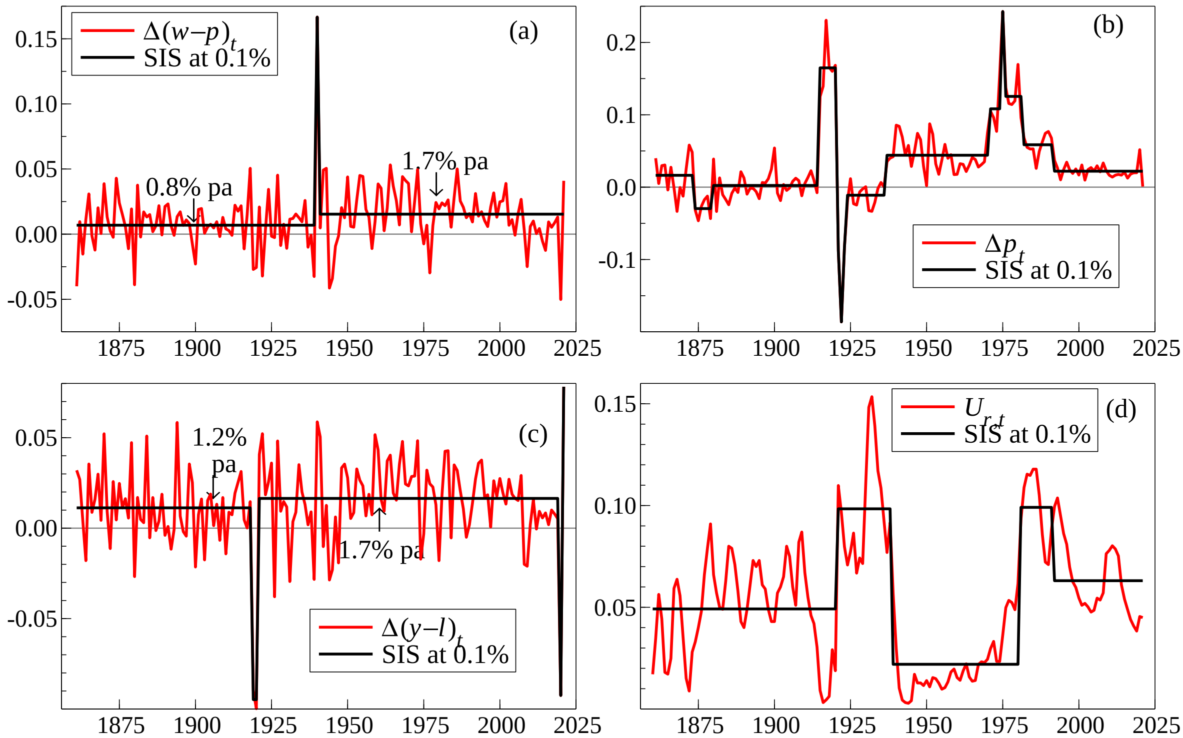

Figure 5: Step indicator saturation (SIS) and regime shifts: selecting at 0.1% (a) change in real wages; (b) price inflation; (c) productivity; (d) unemployment rate.

2.1 Modelling non-linearities in real wage determination

We begin by specifying a general unrestricted model of the change in real wages, $\Delta(w-p)_t$, to include an intercept, two lags of the dependent variable $\Delta(w-p)_{t-i}$ and the labour share of income, $(w-p-y+l)_{t-i}$ for $i = 1, 2$, as well as the contemporaneous values and two lags of labour productivity, $\Delta(y-l)_{t-j}$, the unemployment rate, $U_{r,t-j}$, and price inflation, Δp_{t-j}, for $j = 0, 1, 2$. We include polynomials of price inflation and the unemployment rate to capture non-linearities. We apply Impulse Indicator Saturation (IIS; see Appendix §8) and SIS to detect outliers and location shifts retaining regressors and selecting indicators at 0.1%; then selecting regressors at 1%.

Let x_t be a transition variable with standard deviation σ_x in the logistic transition function:

$$F(z_t) = (1 + \exp\{-z_t\})^{-1}; \quad z_t = \gamma \left(\frac{x_t - c}{\sigma_x} \right), \tag{1}$$

where γ determines the rapidity of transition, and c determines the transition point. The function in (1) can be approximated by the following expansion:

$$F(z_t) \simeq \left(\frac{1}{2} + \frac{z_t}{4} - \frac{z_t^3}{48} \right). \tag{2}$$

To approximate logistic-type functional forms, we therefore include polynomials of $x_t = \Delta p_t$ in selection, retaining significant terms. Building on the findings in Castle and Hendry (2014), let:

$$f_t \Delta p_t = \frac{-\Delta p_t}{1 + 1000(\Delta p_t)^2}, \tag{3}$$

then (3) is close to the logistic smooth transition function in squared annualized inflation as a percentage $100\Delta p_t$, scaling for the same mean and range as $f_t \Delta p_t$:

$$F(z_t) = 2\left(1 + \exp\left(-\gamma (100\Delta p_t)^2\right)\right)^{-1} - 2. \tag{4}$$

Applying a parsimonious encompassing test for higher-order polynomials added to (3) against (4) (demeaned as correlations can be artificially high due to non-zero means, see Castle and Hendry, 2011) reveals (3) is preferred.

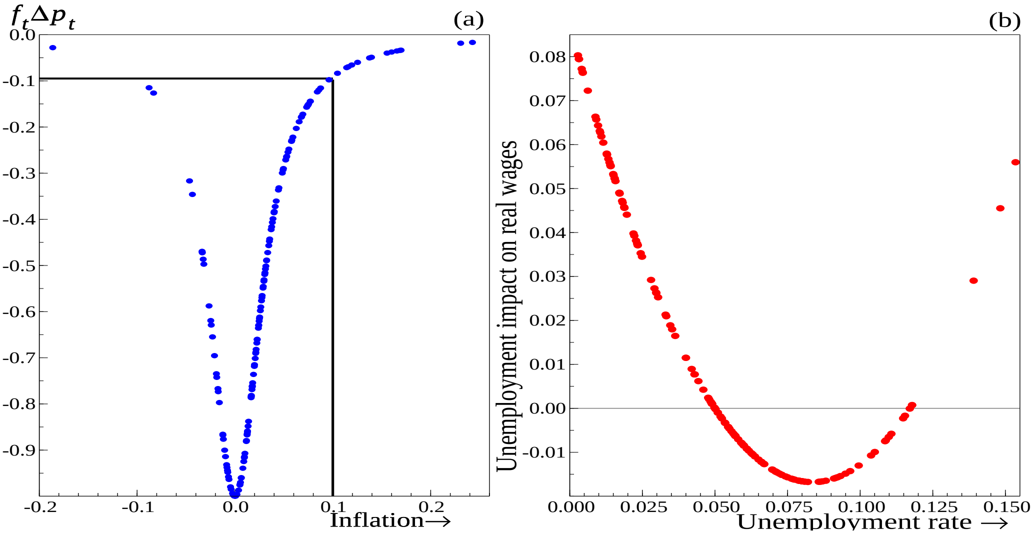

Figure 6: (a) Wage-price spirals and (b) involuntary unemployment

The non-linear mapping (3) is V-shaped as shown in Figure 6(a). There is little reaction of real wages to inflation when it is low, but workers become more attentive as price inflation rises and act to prevent further erosion of their real wages. This is similar to the model of inattentive producers in Reis (2006). The points are calculated from observed data using estimated coefficients and suggest that the non-linearity induces a wage-price spiral if inflation exceeds $6\% - 8\%$ p.a. This has implications for the UK in 2022, when CPI inflation is currently running above 8.2% and is projected to exceed 10%.

A second non-linear term, $(U_{r,t} - 0.05)^2$, is also found to be significant, as discussed in Castle and Hendry (2014). As the unemployment rate is intrinsically positive, but enters the model with a negative coefficient, the combined term is positive at low rates, with an increasingly negative impact until the unemployment rate exceeds approximately 8%, but then increases: see Figure 6(b). Such an effect could initially reflect a loss of worker's bargaining power, but then represent movements along the marginal product curve, raising real wages of those still employed both from more capital per worker and the unemployed being the less productive workers. Importantly, the high real wages are not causing high unemployment, but result from the involuntary unemployment.

The final selected model estimated over the period 1862–2015 is reported in equation (5), where IWWII $= (1_{1942} + 1_{1943} - 1_{1944} - 1_{1945})$ in which 1_{xxxx} is an indicator function taking the value 1 for that observation and 0 otherwise, and S_{xxxx} is a step indicator taking the value 1 till $xxxx$ and 0 after. $\widehat{\mu}$ is the sample mean of $(w-p-y+l)$.[1] Six years of data are retained for forecasting from 2016–2021, which includes the COVID-19 pandemic shock.

$$
\begin{aligned}
\Delta\widehat{(w-p)}_t = \; & \underset{(0.044)}{0.363}\ \Delta(y-l)_t + \underset{(0.046)}{0.135}\ \Delta(y-l)_{t-1} - \underset{(0.030)}{0.134}\ \Delta^2 p_t \\
& - \underset{(0.034)}{0.189}\ (U_{r,t}-0.05) + \underset{(0.68)}{3.09}\ (U_{r,t}-0.05)^2 - \underset{(0.054)}{0.233}\ \Delta_2 U_{r,t} \\
& + \underset{(0.12)}{0.74}\ (f_t\Delta p_t) - \underset{(0.012)}{0.136}\ S_{1939} + \underset{(0.016)}{0.182}\ S_{1940} - \underset{(0.012)}{0.070}\ S_{1941} \\
& - \underset{(0.011)}{0.042}\ I_{1916} - \underset{(0.011)}{0.045}\ I_{1977} + \underset{(0.006)}{0.029}\ \mathit{IWWII} \\
& - \underset{(0.030)}{0.18}\ (w-p-y+l-\widehat{\mu})_{t-2} + \underset{(0.003)}{0.021}\ S_{2012}
\end{aligned} \tag{5}
$$

$$\widehat{\sigma} = 1.09\% \quad \mathsf{R}^2 = 0.79 \quad \mathsf{F}_{\mathsf{ar}}(2,137) = 0.06 \quad \mathsf{F}_{\mathsf{arch}}(1,152) = 1.21$$
$$\chi^2_{nd}(2) = 0.62 \quad \mathsf{F}_{\mathsf{Het}}(19,130) = 2.36^{**} \quad \mathsf{F}_{\mathsf{reset}}(2,137) = 3.48^{*}$$

The model is reasonably well-specified although fails the test for heteroskedasticity at 1%. Figure 7 records the graphical statistics for the model. The forecast Chow test is $\mathsf{F}_{\mathsf{Chow}}(6,139) = 0.76$ and the t-test for zero forecast innovation mean is $\mathsf{t}(5) = 1.49$. Both tests show the model performs well over the forecast horizon. This is a truly *ex ante* forecast exercise as the model was developed prior to the forecast period, and the data has since been updated so allows a test of the model over the period 2016–2021, albeit conditioning on known contemporaneous regressors. Figure 8 computes the 1-step ahead forecasts both in differences and then cumulating to levels. Both show remarkable constancy and pick up the fall in real wages in 2020 due to the pandemic. The results show that stable models can be developed despite highly non-constant data.

2.2 Testing super exogeneity

Indicator saturation estimators (ISEs) can be used to test for the exogeneity of the conditioning variables as in Hendry and Santos (2010). Under the null hypothesis of super exogeneity, the parameters in the conditional model (5) are invariant to shifts in the marginal models of the included regressors, so any indicators or step shifts that are found in the marginal models should not enter the conditional model. Under the null hypothesis, the parameters in the model of $\Delta(w-p)_t$ are invariant to shifts in models of other variables. To test this hypothesis we use a VAR(2) in $(y-l)$, Δp and U_r, retaining all regressors and selecting outliers and shifts using IIS+SIS at $\alpha = 0.001$. Saturation found 10 impulse and 7 step indicators, noting that at such tight significant levels the probability of retaining an indicator or step that was not relevant is negligible. The retained indicators for the three marginal models were then included

[1]Lower case denotes logs, $\Delta^2 = (x_t - x_{t-1}) - (x_{t-1} - x_{t-2})$, and $\Delta_2 = (x_t - x_{t-2})$. Coefficient standard errors are shown in parentheses, $\widehat{\sigma}$ is the residual standard deviation, F_{ar} tests for residual autocorrelation (see Godfrey, 1978), $\mathsf{F}_{\mathsf{arch}}$ tests for autoregressive conditional heteroscedasticity (see Engle, 1982), $\mathsf{F}_{\mathsf{het}}$ tests for residual heteroskedasticity (see White, 1980), $\chi^2_{\mathsf{nd}}(2)$ tests for non-Normality (see Doornik and Hansen, 2008), $\mathsf{F}_{\mathsf{reset}}$ tests non-linearity (see Ramsey, 1969) and $\mathsf{F}_{\mathsf{chow}}$ tests for parameter constancy (see Chow, 1960). One star indicates test significance at 5%, two at 1%.

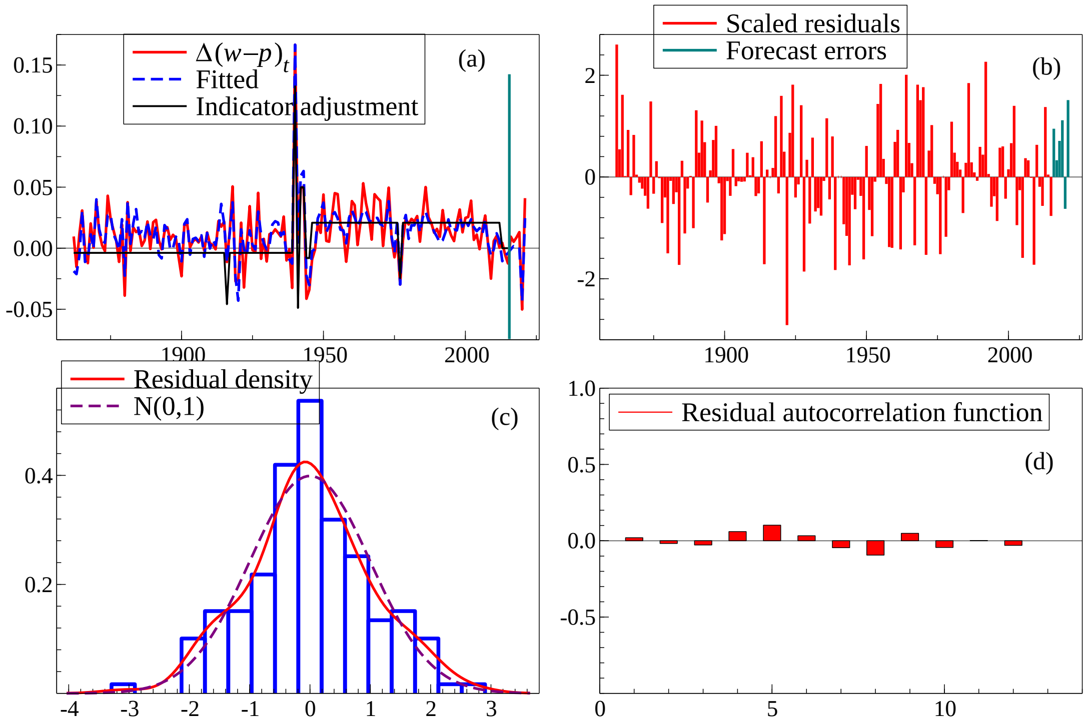

Figure 7: Real-wage model (5) graphical statistics including (a) fitted and actual values along with the indicator adjustment path; (b) residuals and forecast errors; (c) residual density; and (d) residual autocorrelation function.

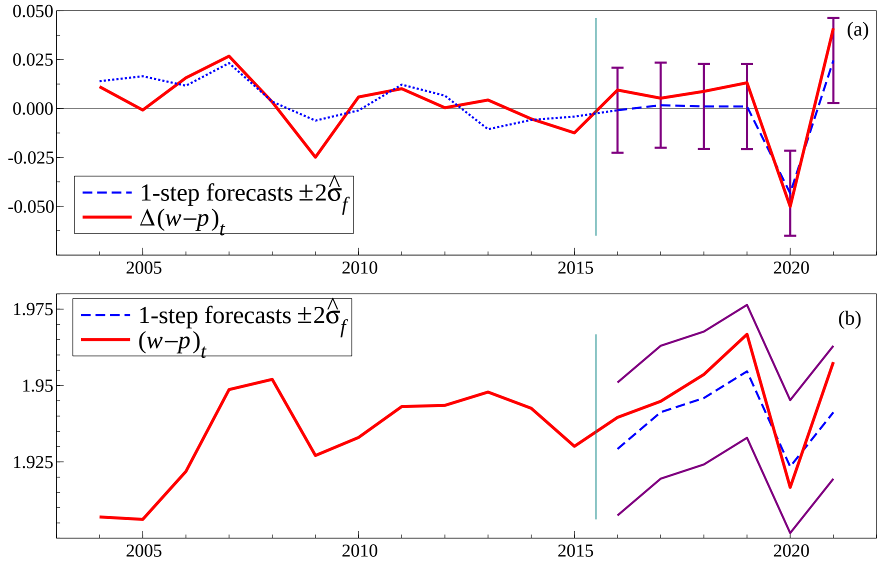

Figure 8: 'Forecasts' for real wages in differences and levels.

in (5) and tested for their significance. The resulting super exogeneity test of $\mathsf{F}_{\mathsf{SE,IIS+SIS}}(15, 111) = 1.56$ is insignificant. Hence, we can conclude that it is valid to condition on the contemporaneous regressors in (5) and we can proceed to interpret the empirical results.

2.3 Interpretation

The short-run impact of changes in productivity on real wages is ≈ 0.5, which is a rather rapid incorporation of productivity increases into real wage increases but is symmetric, so also reflects a dampening of real wages due to the productivity slowdown since 2008. There is a strong equilibrium correction of -0.18 from the labour share of income reflecting a long-run feedback to real unit labour costs of about 20% p.a, or a half life of just under 4 years. The coefficient of $f_t\Delta p_t$ is highly significant reflecting the importance of the non-linearity, but the effect is < 1 so is dampened, possibly avoiding an additional unit root although just $2SE$ from unity. The non-linear unemployment effect can be re-written as $-0.5U_{r,t}(1 - 6.0U_{r,t})$, which is negative until the unemployment rate exceeds $\approx 15\%$, but is then positive. This non-linearity was not detected in the initial Castle and Hendry (2009) paper, highlighting the importance of the model selection method allowing for general non-linearities and hence discovering its important role in an explanation of real wages.

Figure 7(a), records the fitted and actual values of $\Delta(w-p)_t$ with indicators affecting real wages, as IIS and SIS were essential to identifying a stable model of real wage growth. There is a key role for the step indicators in explaining the higher growth rate of real wages post WWII (1.7% p.a., versus 0.8% p.a. pre-1945), even though $\Delta(y-l)$ is included and displays a similar pattern, suggesting the spike in $\Delta(w-p)$ in 1940 induced a permanent location shift which is not explained by the variables in the model. One possible explanation could be the increase in female labour force participation after WWII following a rapid up-skilling of the labour force during the war; see e.g. Bernstein and Martinez (2021). The step shift in 2012 suggests this increase has been reversed over the last decade; steps are defined as taking the value 1 prior to the date, so real wage inflation has experienced a level shift down by 2%p.a. since 2012. None of the economic variables in the model explain this step shift which poses serious policy questions, but the shift is fundamental for the forecast performance of the model over 2016–2021.

The equation standard error of 1.09% compares to an unconditional standard deviation of 2.3% for real wage growth over the same period, although the equation standard error also reflects the steps and impulses. The model is remarkably constant over the period of the 'Great Recession' and the 'flat lining' of real wages. Given such constancy over a period of structural change, we can derive insights into possible effects on real wages of the current economic pressures in the UK. Rapidly rising price inflation is a key area of concern, but low productivity derived from a recession is likely to dampen this positive effect on real wage growth. The tight labour market driving very low unemployment levels suggests the second non-linearity is not likely to kick in, and if we think of unit labour costs as being closely related to energy costs for production this may have a long run dampening effect on real-wage growth via a smaller labour share of income.

3 An empirical model of UK unemployment

Unemployment plays an key role in determining real wages, both as the change in unemployment and as a non-linear relationship with real wages, possibly capturing movements along the marginal product curve. As such, a model of the determinants of unemployment is useful for a well specified marginal model to test, for example, super exogeneity, as well as an understanding of the system more generally. Following from Hendry (2001), Castle, Clements, and Hendry (2016) formulated and estimated a model of unemployment estimated up to 2014, and here we extend the analysis to 2021, testing parameter constancy over the extended sample from 2018.

The empirical model from Hendry (2001) assumes the unemployment rate, $U_{r,t}$, is the outcome of supply and demand for labour, aggregated across all prospective workers, with labour demand derived from the demand for goods and services. This implies a highly complex data generating process (DGP) which is approximated by assuming employment increases if hiring is profitable, and falls if not. As there is no good annual data over the last century and a half for profits we use a proxy. Changes in revenues are linked to changes in GDP, Δy_t, reflecting the demand side, and the close link between $(w-p)_t$ and

$(y-l)_t$ (seen above) suggests labour costs and revenues are equilibrated. On the supply side, capital costs depend on real borrowing costs, $(R_L - \Delta p)_t$, where R_L is the nominal long-term interest rate. Combining, we approximate changes in profits by the difference between the proxies for costs and for revenues:

$$R_{r,t} = (R_L - \Delta p - \Delta y)_t. \tag{6}$$

Figure 9 records this measure of the profits proxy along with the unemployment rate. While there are some deviations between the two series they tend to move closely together suggesting a 'cointegrating' relation between the unemployment rate and the profits proxy: see Castle, Doornik, and Hendry (2021) for a monthly model of the unemployment rate including non-linear transformations and ISEs.

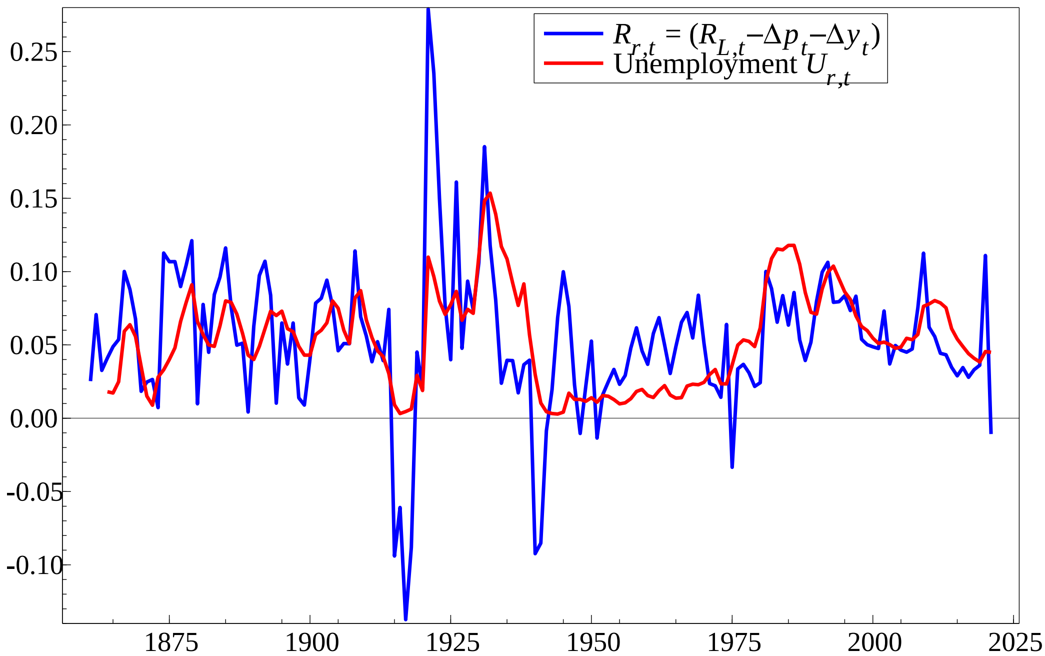

Figure 9: Profits proxy and the unemployment rate, 1860–2021.

3.1 Modelling unemployment by the profits proxy

Here we formulate a dynamic model of $U_{r,t}$ by commencing with a model in levels with two lags of both $U_{r,t}$ and $R_{r,t}$. Non-linear functions are not included in the initial specification as the index test of non-linearity provides no evidence of non-linear functional form at the 1% significance level ($\chi^2_{nl}(12) = 22.7^*$, see Castle and Hendry, 2010). The model in levels with saturation over 1863–2017 yields:

$$\begin{aligned}
\widehat{U}_{r,t} \;=\; & \underset{(0.07)}{1.26}\, U_{r,t-1} - \underset{(0.06)}{0.36}\, U_{r,t-2} + \underset{(0.002)}{0.006} + \underset{(0.02)}{0.15}\, \Delta R_{r,t} - \underset{(0.02)}{0.08}\, \Delta R_{r,t-1} \\
& + \underset{(0.001)}{0.052}\, \Delta 1_{1922} + \underset{(0.008)}{0.036}\, 1_{1930} - \underset{(0.008)}{0.035}\, 1_{1939} \\
& \widehat{\sigma}_\epsilon = 0.83\% \;\; \mathsf{R}^{*2} = 0.94 \;\; \mathsf{F}_{\mathsf{ar}}(2,146) = 1.93 \;\; \chi^2_{\mathsf{nd}}(2) = 9.85^{**} \;\; \mathsf{F}_{\mathsf{Chow}}(4,147) = 0.44 \\
& \mathsf{F}_{\mathsf{arch}}(1,153) = 0.08 \;\; \mathsf{F}_{\mathsf{Het}}(10,142) = 1.16 \;\; \mathsf{F}_{\mathsf{Reset}}(2,145) = 2.55
\end{aligned} \tag{7}$$

All the pairs of impulse and step saturation indicators cancelled as differences so were essentially equivalent to including $\Delta 1_{1922}$, 1_{1930} and 1_{1939}. The model's graphical statistics are recorded in Figure 10.

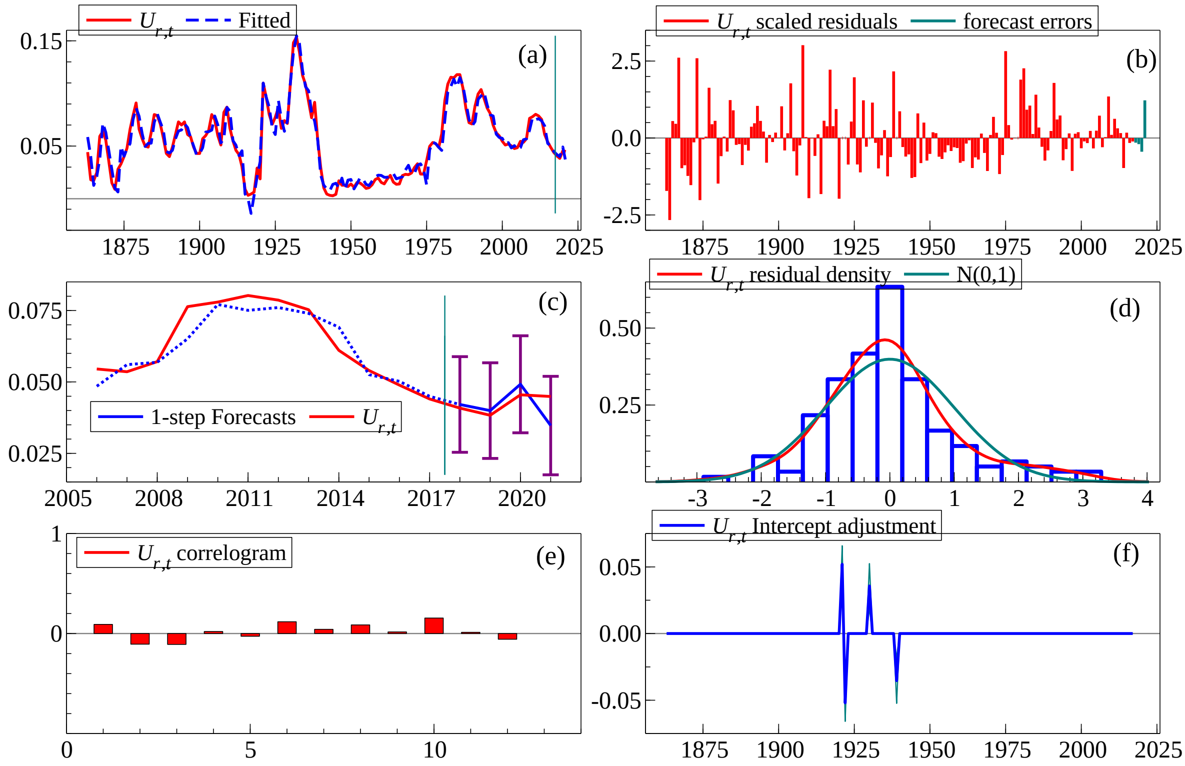

Figure 10: Graphical statistics of the dynamic unemployment model in levels (7), including (a) fitted and actual values; (b) scaled residuals; (c) 1-step ahead forecasts; (d) residual density; (e) residual correlogram; and (f) the implied intercept adjustment from the retained indicators.

We then solve for the long-run 'cointegrating' relation and transform the dynamic model to differences including the lagged long-run relationship. The resulting selected model over 1863–2017 yields:

$$\begin{aligned}\Delta\widehat{U}_{r,t} = \; & \underset{(0.05)}{0.36}\ \Delta U_{r,t-1} + \underset{(0.016)}{0.15}\ \Delta R_{r,t} - \underset{(0.022)}{0.10}\ E_{Ur,t-1} \\ & - \underset{(0.007)}{0.052}\ \Delta 1_{1922} + \underset{(0.008)}{0.036}\ 1_{1930} - \underset{(0.008)}{0.035}\ 1_{1939} \\ & \widehat{\sigma}_{\epsilon} = 0.82\% \;\; \mathsf{R}^{*2} = 0.67 \;\; \mathsf{F}_{\mathsf{ar}}(2,147) = 1.9 \;\; \chi^2_{\mathsf{nd}}(2) = 9.86^{**} \\ & \mathsf{F}_{\mathsf{arch}}(1,153) = 0.08 \;\; \mathsf{F}_{\mathsf{Het}}(8,144) = 0.90 \;\; \mathsf{F}_{\mathsf{Reset}}(2,147) = 0.85\end{aligned} \tag{8}$$

where the long-run relation is given by:

$$E_{Ur} = U_r - 0.054 - 0.72 R_r. \tag{9}$$

We test the constancy of the model by computing the 1-step *ex post* forecasts for 2018–2021 which delivers a Chow test of $\mathsf{F}_{\mathsf{Chow}}(4,149) = 0.45$, demonstrating remarkable stability of the simple model over the COVID-19 pandemic.

Figure 11 records the graphical statistics. Although there is one diagnostic failure, the simple model can explain a large amount of the variation in the unemployment rate over the past century and a half.

3.2 Interpretation

The underlying economic theory is found to be empirically consistent over a century and a half; when the real long-term interest rate, $R_L - \Delta p$, equals Δy, then $R_r = 0$, and equilibrium U_r is about 5%, close to

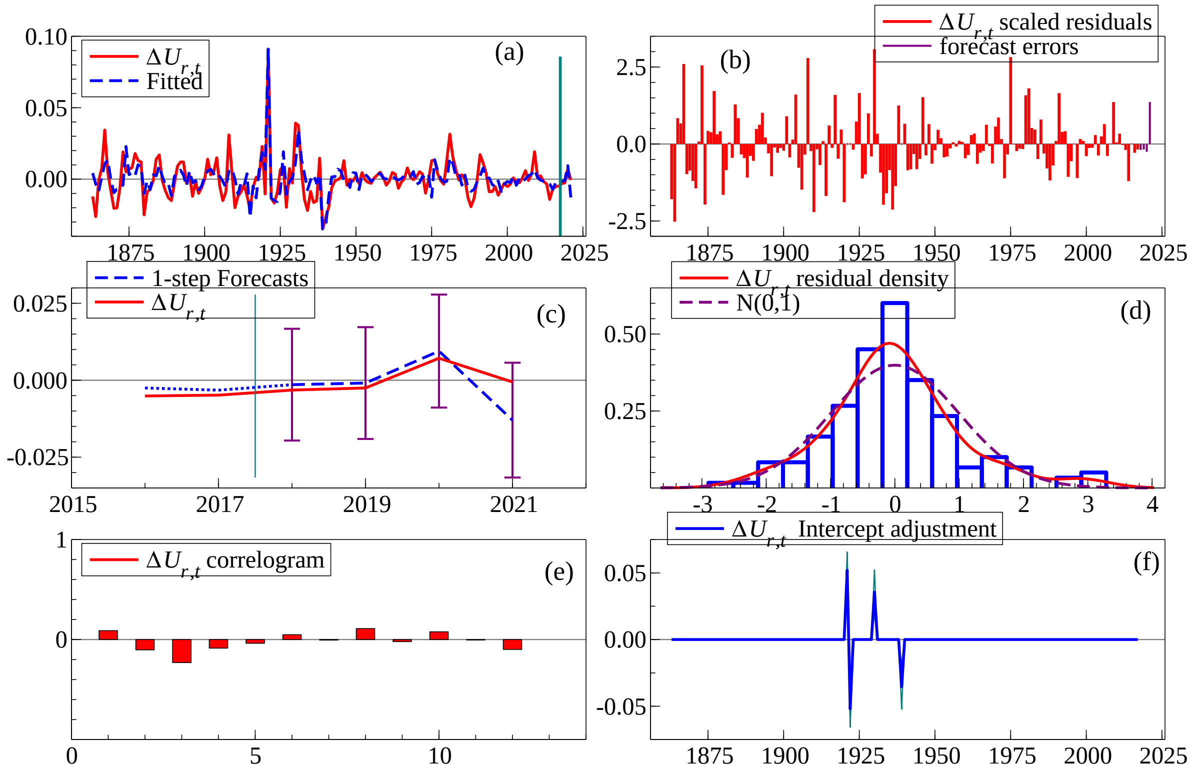

Figure 11: Graphical statistics of the dynamic unemployment model in differences (8), including (a) fitted and actual values; (b) scaled residuals; (c) 1-step ahead forecasts; (d) residual density; (e) residual correlogram; and (f) the implied intercept adjustment from the retained indicators.

the historical average. Since the financial crisis, quantitative easing has lowered $R_L - \Delta p$, offsetting a large fall in Δy, so $R_{r,t}$ only rose briefly, which goes some way to understanding why U_r rose less than anticipated over the Great Recession. There is a rise in unemployment during 2008, which nevertheless was much smaller than expected, given a fall of more than 6% in real GDP. The earlier in-sample period saw many key changes, including two world wars, unemployment benefits, and vast industrial changes, yet only 1 differenced and 2 impulse indicators are needed with just one explanatory variable. Castle, Clements, and Hendry (2016) compared (8) to more-conventional models (e.g.) using Δy and showed the latter were far poorer. However, despite the success of the model, *ex ante* forecasts would require forecasts of the profits proxy and so we next investigate a system model of unemployment and the profits proxy.

3.3 System model of unemployment and profits proxy

The two variable system in levels for $U_{r,t}$ and $R_{r,t}$ over 1864–2017, with 4 observations held for forecasts up to 2021, applying IIS and SIS at 0.001 to select outliers and steps, yields:

$$
\begin{aligned}
\widehat{U}_{r,t} = \; & \underset{(0.082)}{1.39}\; U_{r,t-1} - \underset{(0.074)}{0.48}\; U_{r,t-2} - \underset{(0.023)}{0.025}\; R_{r,t-1} + \underset{(0.002)}{0.0045} \\
& + \underset{(0.010)}{0.092}\; 1_{1921} - \underset{(0.012)}{0.046}\; 1_{1922} + \underset{(0.010)}{0.008}\; 1_{1926} + \underset{(0.010)}{0.042}\; 1_{1930} \\
& - \underset{(0.010)}{0.039}\; 1_{1939} - \underset{(0.010)}{0.010}\; 1_{1940} + \underset{(0.005)}{0.008}\; S_{1914} - \underset{(0.005)}{0.008}\; S_{1918} \\
\chi^2_{\mathsf{nd}}(2) = \; & 13^{**} \quad \mathsf{F}_{\mathsf{ar}}(2,140) = 1.50 \quad \mathsf{F}_{\mathsf{arch}}(1,152) = 0.35 \quad \mathsf{F}_{\mathsf{Het}}(8,139) = 1.05
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\widehat{R}_{r,t} &= \underset{(0.069)}{0.298}\ R_{r,t-1} + \underset{(0.244)}{0.722}\ U_{r,t-1} - \underset{(0.222)}{0.626}\ U_{r,t-2} - \underset{(0.0057)}{0.0076} \\
&\quad + \underset{(0.030)}{0.247}\ 1_{1921} + \underset{(0.035)}{0.054}\ 1_{1922} + \underset{(0.030)}{0.107}\ 1_{1926} + \underset{(0.030)}{0.048}\ 1_{1930} \\
&\quad - \underset{(0.030)}{0.020}\ 1_{1939} - \underset{(0.030)}{0.119}\ 1_{1940} + \underset{(0.017)}{0.114}\ S_{1914} - \underset{(0.016)}{0.105}\ S_{1918} \\
\chi^2_{\mathsf{nd}}(2) &= 5.6\ \ \mathsf{F}_{\mathsf{ar}}(2,140) = 1.7506\ \ \mathsf{F}_{\mathsf{arch}}(1,152) = 0.027\ \ \mathsf{F}_{\mathsf{Het}}(8,139) = 1.06
\end{aligned}
\tag{11}
$$

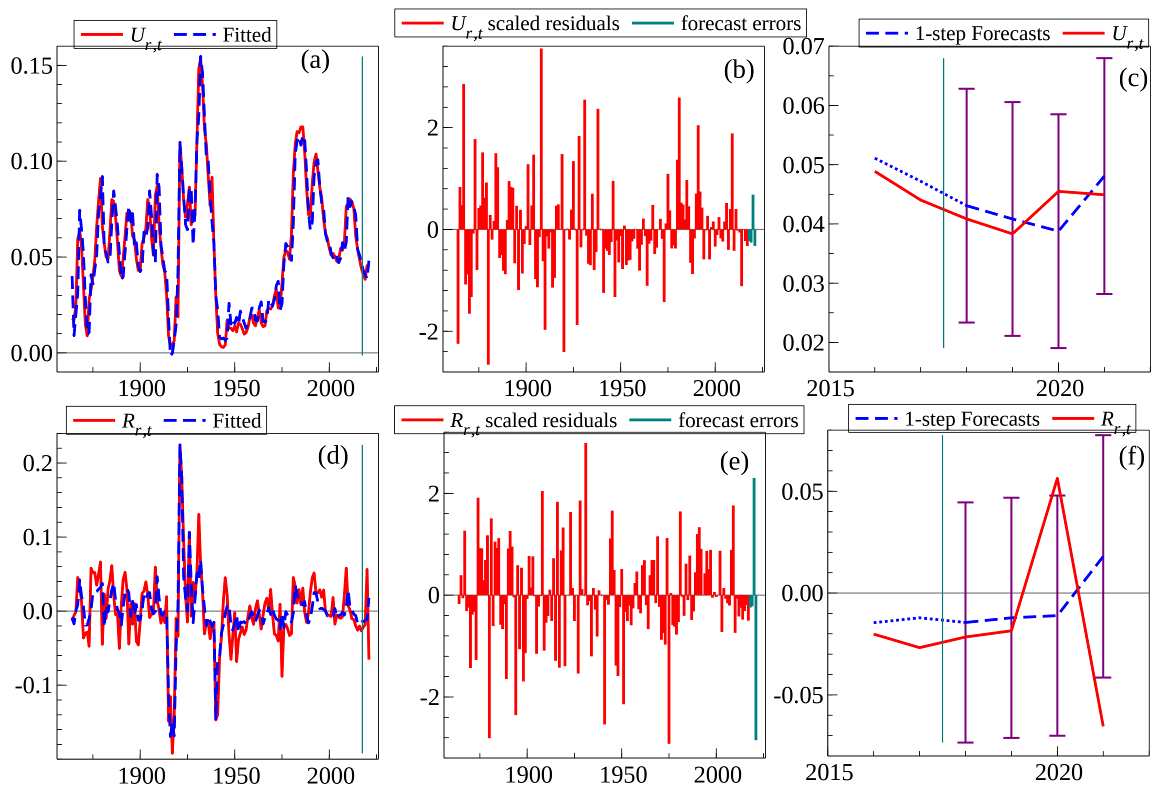

Figure 12: Graphical statistics of the two variable system of U_r and R_r, including (a) fitted and actual values for U_r; (b) scaled residuals for U_r; (c) 1-step ahead forecasts for U_r; (d) fitted and actual values for R_r; (e) scaled residuals for R_r; and (f) 1-step ahead forecasts for R_r.

The models are well-specified and satisfy the single-equation diagnostic tests, apart from normality for the unemployment rate equation, as well as the system statistics reported in Table 1. System residuals and root mean square forecast errors (RMSFEs) are reported in Table 2 where both the 1-step ahead and 2-step ahead forecast errors for $U_{r,t}$ are smaller than the in-sample residuals. The 1-step ahead forecast errors for $U_{r,t}$ are very similar to those for the single equation model despite not including the contemporaneous profits proxy. The forecasts for R_r are poor over 2020 and 2021 due to the Covid-19 pandemic. Figure 12 records the model fit and residuals along with 1-step ahead forecast errors. Few indicators and steps are needed other than for the Great Depression and World Wars. The highly significant step indicators for 1914 and 1918 in (12) are not needed in (7) nor are the impulse indicators for 1926 and 1940, effectively demonstrating the super exogeneity of $R_{r,t}$ in (7). Overall, the results suggest that the relationship between the unemployment rate and the profits proxy has been remarkably stable over the past century and a half despite many shocks including wars, recessions, technology and policy changes, but Covid has highlighted the difficulty of forecasting the profits proxy when there is an unanticipated large fall in Δy_t from lockdowns. Conversely, the UK job retention scheme (furlough), mitigated the impact of lockdowns on recorded unemployment (see Castle, Doornik, and Hendry, 2021).

Table 1: System statistics for (10) and (11).

Statistic	outcome
$\mathsf{F}_{\mathsf{VAR}}(8, 274)$	1.62
$\mathsf{F}_{\mathsf{VRESET}}(8, 274)$	0.78
$\chi^2_{\mathsf{VND}}(4)$	24.3^{**}
$\mathsf{F}_{\mathsf{VHET}}(24, 397)$	1.14
$\mathsf{Corr}(\widehat{e}_{U_r}, \widehat{e}_{R_r})$	0.56

Table 2: System residual $\widehat{\sigma}$s and RMFSEs for 1 and 2-steps ahead forecasts over 2018–2021.

Statistic	U_r	R_r
$\widehat{\sigma}$	0.0098	0.0293
RMFSE_1	0.0041	0.0539
RMFSE_2	0.0040	0.0437

4 Productivity

Rates of technical progress have varied greatly over time, inducing changing trends in the relationships between output and capital. The real-wage data showed its close relation to productivity, so we model the 'production function' as relating $(y - l)$ to $(k - l)$ with changing trends and location shifts.[2]

One part of the recent UK 'productivity puzzle' is that real GDP per capita has risen although productivity has not. This is due to a large rise in employment relative to population leading to a ratio that is at its highest recorded in peacetime. This also leads to the very different trends since 1995 in output per worker and per person highlighted in the box in Figure 1(d). As the proportion of the population employed rises, many of the jobs left for the new workers are lower productivity, but by raising total output, incomes rise.

4.1 Including energy in the productivity model

Given the concern about the rise in the prices of oil and gas since 2021, and the possibility of severe supply cuts, we reconsider the role of energy in determining the UK 'production function' developed in Hendry (2001),(2022). Both studies modelled the relation between output per employee per annum and the capital labour ratio, and found several trend shifts. Here we augment the dynamic model in the second paper with a measure of total energy use, E_t, calculated as the sum of coal, oil, natural gas, renewables (wind, solar and hydroelectric) and nuclear all measured in millions of tons of oil equivalent (Mtoe), shown in Figure 2(a).

Net trends in the relationship between $(y-l)$ and $(k-l)$ depend on the extent and rapidity with which technical progress is embodied in the capital stock both for labour productivity and energy efficiency, or is 'disembodied' as in organizational improvements. Figure 2(b) shows the dramatic drop in energy relative to capital, with three sub-period trends for 1860–1920, 1921–1968, and 1969–2017, corresponding roughly to coal only, coal plus oil, then all fuels. The large swings in the inter-war period are 'captured'

[2]Previous models used $(k - n)$ where n is the log of the measured 'working population', namely $N = L + U$. This corrects for the potential artefact that while Y responds very rapidly to increases in U, K cannot, as can be seen in the 1920s, but proved problematic in more recent data.

as transient events by impulse indicators, after which the trend indicators for the first two sub-periods were essentially the same so were combined with a coefficient of -0.005 compared to -0.023 for the remainder of the sample. Overall the efficiency of energy use improved by more than 80%.

The means and standard deviations (SDs) of the growth rates measured by $\Delta(y-l)$, $\Delta(k-l)$ and $\Delta(e-k)$ have changed greatly over time, as Table 3 records for a mid-period split, as have their correlations, although the mean growth rates and SDs of $\Delta(y-l)$ and $\Delta(k-l)$ have been similar in each period.

Date	Statistic	$\Delta(y-l)$	$\Delta(k-l)$	$\Delta(e-k)$
	Mean	0.9%	0.9%	-0.6%
1863 – 1945	SD	2.7%	2.6%	8.0%
	Correlation	0.06	0.21	-0.44
	Mean	1.8%	2.0%	-1.9%
1946 – 2017	SD	1.7%	1.6%	3.3%
	Correlation	0.32	0.34	-0.17
	Mean	1.3%	1.4%	-1.2%
1863 – 2017	SD	2.3%	2.2%	6.3%
	Correlation	0.17	0.21	-0.40

Table 3: Correlations are between $\Delta(y-l)$ & $\Delta(k-l)$, $\Delta(y-l)$ & $\Delta(e-k)$, and $\Delta(k-l)$ & $\Delta(e-k)$

To model the production relation, we formulated a general model over 1863–2021 of y_t on l_t, k_t, e_t and their first lagged values, together with the trend shift and outlier indicators from Hendry (2022). These had been selected using trend-indicator saturation (TIS, which allows for a potential trend shift at every point in time: see Castle, Doornik, Hendry, and Pretis, 2019). The homogeneity restriction to a model of $(y-l)_t$ on $(k-l)_t$ and $(e-k)_t$, their lags and indicators yielded $\mathsf{F}(3,140) = 1.95$ so that reduction was imposed. Transforming to an equation of $(y-l)_t$ on a constant, t, $(k-l)_t$, $(e-k)_t$ and lags, plus all retained indicators, selecting at 1% resulted in the following model for 1863–2021:

$$
\begin{aligned}
\widehat{(y-l)}_t = \; & \underset{(0.043)}{0.47}\,(y-l)_{t-1} + \underset{(0.026)}{0.216}\,(k-l)_t + \underset{(0.019)}{0.146}\,(e-k)_t + \underset{(0.27)}{1.08} \\
& + \underset{(0.0015)}{0.0089}\,t - \underset{(0.006)}{0.040}\,\tau_{1939} + \underset{(0.008)}{0.058}\,\tau_{1941} - \underset{(0.003)}{0.022}\,\tau_{1946} + \underset{(0.001)}{0.006}\,\tau_{2006} \\
& - \underset{(0.016)}{0.101}\,\tau_{2010} + \underset{(0.008)}{0.034}\,S_{1918} - \underset{(0.015)}{0.109}\,1_{1920} - \underset{(0.017)}{0.100}\,1_{2020} \\
& \widehat{\sigma} = 1.46\% \;\; \mathsf{R}^2 = 0.9995 \;\; \mathsf{F}_{\mathsf{ar}}(2,144) = 1.96 \;\; \chi^2_{\mathsf{nd}}(2) = 0.96 \\
& \mathsf{F}_{\mathsf{arch}}(1,157) = 1.68 \;\; \mathsf{F}_{\mathsf{Het}}(19,137) = 1.05 \;\; \mathsf{F}_{\mathsf{Reset}}(2,144) = 0.08
\end{aligned}
\tag{12}
$$

τ_{xxxx} denotes a segmented trend that commences with a negative value at the beginning of the sample and increases to zero at date xxxx so trends are not carried forward. Figure 13 records the graphical statistics of the model. Although the simple correlation between $(y-l)$ and $(e-k)$ is -0.97, the partial correlation in (12) is $+0.53$. The 1926 general strike and longer miners' strike led to a 36% fall in coal output, and this supply shock produced a 4% fall in GDP, matching the effect expected from the coefficient of $(e-k)$ in (12). Unlike the very sharp GDP falls of around 10% in 1919 and 1920 induced by massive reductions in the government deficit from about 40% of GDP to near zero, no indicator was needed for the 1926 drop, so the change in energy supply captured the fall. To test the invariance of the coefficient of $(e-k)_t$ in (12), we modelled it as a function of $(g-l)_{t-1}$, $(k-l)_{t-1}$ and $(e-k)_{t-1}$ (retained) selecting SIS+TIS at $\alpha = 0.01\%$, then applied IIS at 0.1% and finally selected over all retained variables at 1%. The indicators retained were S_{1920}, S_{1922}, τ_{1965}, 1_{1922}, 1_{1926}, 1_{1927}, 1_{1981}, and testing their significance in (12) yielded $\mathsf{F}(7,82) = 1.4$, so super exogeneity is not rejected.

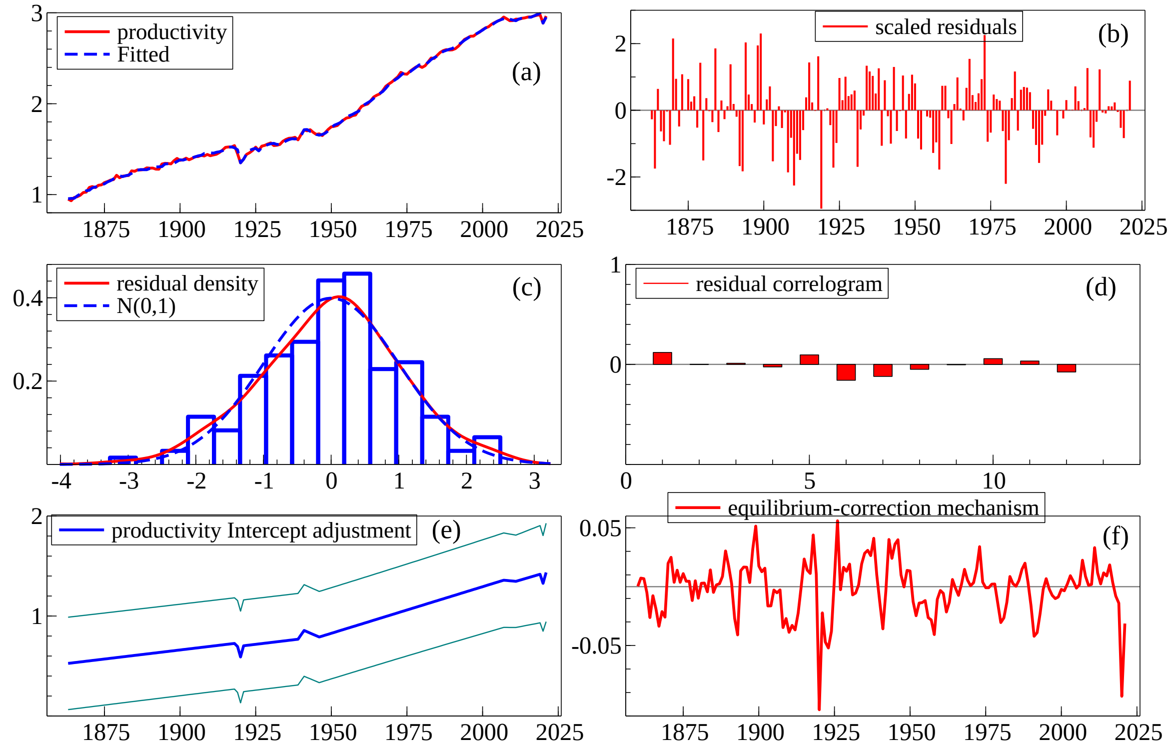

Figure 13: Graphical model statistics for (12) including (a) model fit; (b) residuals; (c) residual density; (d) residual autocorrelation function; (e) implied indicator adjustment; and (f) the long-run cointegrating relation after solving the dynamic model for the long-run solution.

The derived long-run coefficient of l was 0.59, similar to the aggregate labour share of about $2/3$:

$$\begin{aligned} \widetilde{(y-l)}_{LR} \;=\; & \underset{(0.044)}{0.41}\ (k-l) + \underset{(0.037)}{0.28}\ (e-k) + \underset{(0.003)}{0.017}\ t - \underset{(0.011)}{0.076}\ \tau_{1939} + \underset{(0.014)}{0.108}\ \tau_{1941} \\ & - \underset{(0.005)}{0.043}\ \tau_{1946} + \underset{(0.005)}{0.023}\ \tau_{2006} - \underset{(0.007)}{0.022}\ \tau_{2010} + \underset{(0.014)}{0.064}\ S_{1918} - \underset{(0.035)}{0.206}\ 1_{1920} \\ & - \underset{(0.037)}{0.19}\ 1_{2020} + \underset{(0.46)}{2.03} \end{aligned} \tag{13}$$

with $\mathsf{t}_{\mathsf{ur}}^{**} = -12.3$ strongly rejecting a unit root. Figure 13(f) records the equilibrium-correction values $q_{(\mathsf{y-l}),t} = (y-l) - \widetilde{(y-l)}_{\mathsf{LR}}$.

Expressed as a production function, where A_t collects all the deterministic functions:

$$\widetilde{Y}_{LR,t} = A_t L_t^{0.59} K_t^{-0.13} E_t^{0.28} \tag{14}$$

The role of energy seems overly large, although increases in energy use were as crucial to the industrial revolution as machinery to utilise it, and $(k-l)$ and $(e-k)$ are highly negatively correlated. Transforming to an equilibrium-correction form yields:

$$
\begin{aligned}
\widehat{\Delta(y-l)}_t \;=\; & \underset{(0.0015)}{0.012} + \underset{(0.06)}{0.26}\,\Delta(k-l)_t + \underset{(0.02)}{0.15}\,\Delta(e-k)_t - \underset{(0.056)}{0.47}\,q_{t-1} \\
& - \underset{(0.016)}{0.053}\,1_{1919} - \underset{(0.015)}{0.11}\,1_{1920} - \underset{(0.015)}{0.05}\,1_{1940} - \underset{(0.015)}{0.11}\,1_{2020} \qquad (15) \\
& \widehat{\sigma} = 1.50\% \;\; \mathsf{R}^2 = 0.65 \;\; \mathsf{F}_{\mathsf{ar}}(2,149) = 3.54^{*} \;\; \chi^2_{\mathsf{nd}}(2) = 0.22 \\
& \mathsf{F}_{\mathsf{arch}}(1,157) = 1.65 \;\; \mathsf{F}_{\mathsf{Het}}(6,148) = 1.47 \;\; \mathsf{F}_{\mathsf{Reset}}(2,149) = 0.08
\end{aligned}
$$

The fitted and actual values and residual graphs of the equilibrium-correction productivity model (15) are shown in Figure 14. The model is well-specified, passing diagnostic tests at the 1% significance level. The slowdown in productivity is mostly driven by the trend shift in 2006 in levels, but there is no corresponding step shift in the differenced model. The intercept adjustment recorded in Figure 14(e) shows very few adjustments are needed for the model of the change in output per worker.

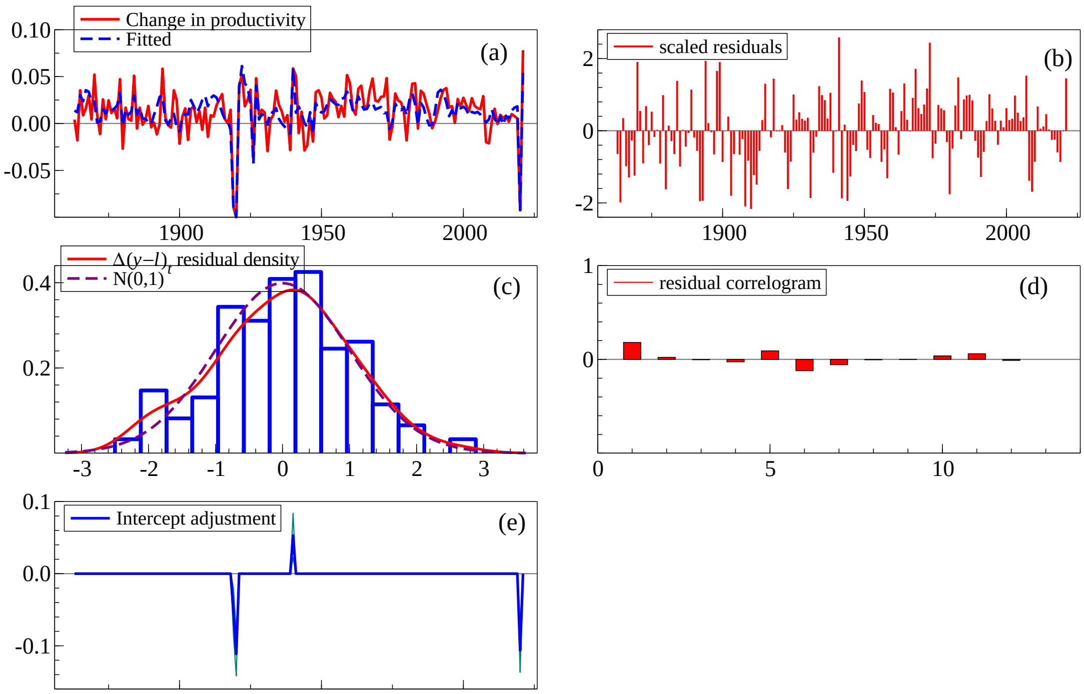

Figure 14: (a) actual and fitted values of $\Delta(y-l)$; (b) scaled residuals; (c) residual density; (d) residual correlogram; and (e) intercept adjustment.

5 Price inflation

Hendry (2001),(2015) derived a model for UK inflation that included excess demand for output, money, and national debt; unemployment, exchange rate, unit labour costs, interest rates, wages, world and energy prices. The selected model found significant roles for excess demand for goods and services, world and energy prices, M4 growth and short and long-term interest rates, and an equilibrium correction markup of prices over unit labour costs. There was little inertia via lagged inflation, but a small direct impact of wages via unit labour costs. Neither unemployment nor inflation expectations were found to be relevant but many impulse and step indicators were needed, explaining events outside of the economic variables included.

Here we build a model of price inflation as measured by the implicit GDP deflator updated to 2021, although we commence the sample in 1965 given the many structural changes and non-constancies in the earlier period. The model selection methodology described in the Appendix §8 is applied and the resulting model for 1965–2021 is:[3]

$$\begin{aligned}
\widehat{\Delta p_t} = \; & \underset{(0.040)}{0.29}\ \Delta p_{t-1} + \underset{(0.032)}{0.12}\ \Delta m_{t-1} + \underset{(0.095)}{0.17}\ \Delta R_{s,t} + \underset{(0.023)}{0.06}\ \Delta p_{w,t} \\
& - \underset{(0.091)}{0.45}\ (R_s - R_l - \mu_R)_{t-1} + \underset{(0.005)}{0.007}\ \Delta p_{o,t-1} + \underset{(0.051)}{0.45}\ \Delta ulc_t \\
& + \underset{(0.004)}{0.03}\ I_{agg} - \underset{(0.002)}{0.005}\ \text{ChinaEffect} \\
& \widehat{\sigma} = 0.90\% \quad \mathsf{R}^2 = 0.98 \quad \mathsf{F_{ar}}(2,46) = 0.09 \quad \chi^2_{\mathsf{nd}}(2) = 0.25 \\
& \mathsf{F_{arch}}(1,55) = 0.70 \quad \mathsf{F_{Het}}(17,39) = 1.08 \quad \mathsf{F_{Reset}}(2,46) = 0.62
\end{aligned} \tag{16}$$

The supplementary data file details the data and sources but for convenience we summarise the relevant regressors here. Δm is the growth rate of broad money, R_s the short-term interest rate and $R_{s,t} - R_{l,t} - \mu_R$ is the difference between the short and long interest rates corrected for a zero mean over the full sample such that $\mu_R = \overline{R_s - R_l}$. Δp_o is the growth rate of a commodity price index linked to oil post 1997, measured in £, Δulc is a measure of the change in unit labour costs, and $\Delta p_{w,t}$ is a measure of world inflation based on a trade-weighted world price index measured in £. Finally, I_{agg} is an aggregated index of retained indicator variables based on Hendry (2001), weighting small, medium and large outliers into an index mostly covering wars and the Great Depression, with the final non-zero value of the data series in 1980, and ChinaEffect is a step shift taking the value 0 to 1993 and 1 from 1994 onwards to represent the downward pressure from Chinese price competition.

Figure 15 records the model fit in panel (a), scaled residuals in panel (b), residual density in panel (c) and residual autocorrelation function in panel (d). The model passes all diagnostics and is well-specified with an equation standard error of 0.9% despite inflation ranging from upwards of 24% to 0%. There is some inflation inertia, along with effects from broad money growth and interest rates, both the change in the short-run and the short-long spread. Energy prices, which are proxied by commodity prices, do not play a significant role, but we note the possibility of testing for non-constant parameters once we have data for 2022 using multiplicative step indicator saturation (MSIS, see Castle, Doornik, and Hendry, 2022). There is a sizable coefficient on unit labour costs which, linking back to §2.1, shows the important role for a wage price spiral effect in the price inflation equation. We do not find a role for excess demand for goods and services or the mark-up of prices over unit labour costs unlike the earlier studies. Given the congruent specification we next investigate the implications for current inflation under alternative scenarios for energy prices in 2022.

6 Wage-price spirals and projections for 2022 inflation

Having developed models for price and wage inflation, we can undertake scenario analysis to investigate what the likely impact of the energy price rises seen in 2022 will be on UK price inflation. A full system model of wages, prices, productivity and unemployment is left for future research, but combining the single equation models is informative.

The change in unit labour costs is $\Delta ulc_t = (\Delta w_t + \Delta l_t - \Delta y_t)$ and $(y + p - w - l) = \pi$ denotes the markup of nominal output over the wage bill, so letting $D_{\Delta p_t}$ summarise the other drivers in (16) and

[3] Similar results are obtained if we estimate using instrumental variables for Δulc_t as it implicitly uses Δw_t. The instruments included the lagged wage share, lagged change in unit labour costs, lagged output gap, the change in the working population and the change in the unemployment rate.

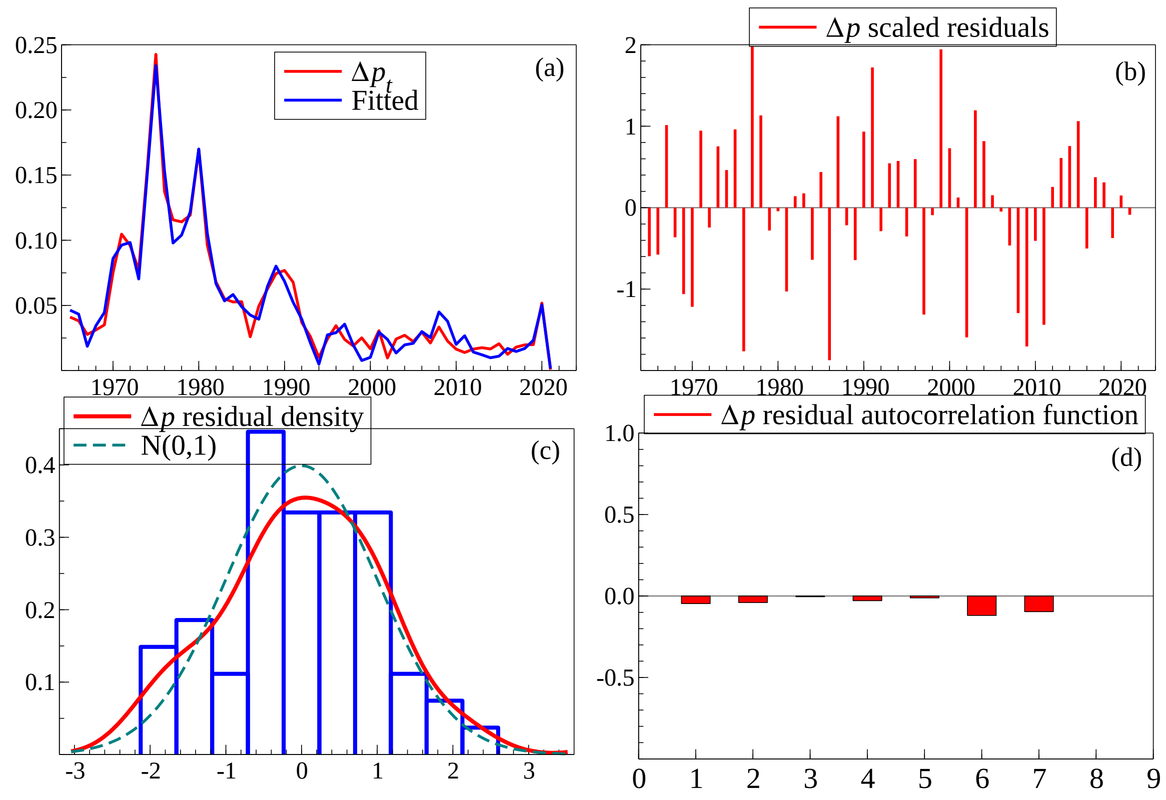

Figure 15: Graphical statistics for the price inflation model in (16).

$D_{\Delta w_t}$ summarise the other drivers in (5), we can simplify the the estimated equations to:

$$\Delta p_t = \lambda \Delta w_t + \rho \Delta p_{t-1} - \lambda \Delta (y-l)_t + D_{\Delta p_t} \quad (17)$$

$$\Delta w_t = (1+\gamma f_t) \Delta p_t + D_{\Delta w_t} \quad (18)$$

where f_t is defined by equation (4). Solving out for price inflation results in:

$$\begin{aligned} \Delta p_t &= \lambda (1+\gamma f_t) \Delta p_t + \rho \Delta p_{t-1} - \lambda \Delta (y-l)_t + (D_{\Delta p_t} + \lambda D_{\Delta w_t}) \\ &\approx \frac{D_{\Delta p_t} + \lambda D_{\Delta w_t} - \lambda \Delta (y-l)_t}{(1 - \lambda (1+\gamma f_t) - \rho)}. \end{aligned} \quad (19)$$

In the limit when workers demand 100% inflation compensation, so $f_t = 0$ given $\lambda = 0.5$ and $\rho = 0.25$ then:

$$\Delta p_t \approx \frac{1}{(0.25)} (D_{\Delta p_t} + \lambda D_{\Delta w_t} - \lambda \Delta (y-l)_t). \quad (20)$$

Substituting back in the other drivers for wage and price inflation, since:

$$\Delta (y-l)_t = 0.44\Delta(k-l) + 0.28\Delta(e-k) + 0.005$$

this results in:

$$\begin{aligned} \Delta p_t &\approx 4 (0.1\Delta m_t - 0.3R_{s,t} + 0.4R_{l,t} + 0.01\Delta p_{ot} + 0.06\Delta p_{we,t} - 0.5\Delta (y-l)_t) \\ &\quad +4 (0.25\Delta (y-l)_t - 0.1U_{r,t} + 0.1\pi_{t-1}) \\ &= 0.4\Delta m_t - 1.2R_{s,t} + 1.6R_{l,t} + 0.04\Delta p_{ot} + 0.24\Delta p_{we,t} + 0.4\pi_{t-1} \\ &\quad -0.44\Delta(k-l)_t - 0.28\Delta(e-k)_t - 0.4U_{r,t}. \end{aligned} \quad (21)$$

Thus, increases in the growth rate of M4, energy prices and world prices, and in the markup and long-term interest rates all raise inflation, whereas increases in capital and energy, short-tern interest rates, and unemployment reduce it. Also, a reduction in energy availability of 10% (say) would simultaneously reduce output by 2.8% and exacerbate inflation to the same extent.

Table 4 shows conditional projections of inflation in 2022 from (21), where $\Delta p_{o,t}$ in the top 3 rows uses an equally-weighted average of a 50% increase for oil and 250% increase for natural gas, resulting in a 150% increase in commodity prices, whereas the 'bad scenario' has a 300% increase. Despite a small and insignificant direct effect of commodity prices on inflation in (16), the impact of the energy price increase contributes almost half of the projected contribution to price inflation in the first scenario and $2/3$rds in the second. These projections assume constant parameters in 2022, but if there was a direct location shift which increased the coefficient on commodity prices in the price inflation equation the impact could be even larger, and would be testable using MSIS as soon as the data for 2022 is available, see Castle, Doornik, and Hendry (2022). Given the contributions to inflation from the model, short term interest rates would need to rise to 5% to offset the direct contributions of energy price rises in the first scenario, and as during the 1970s oil crises, interest rates would need to rise towards 20% in the second scenario to dampen inflation down to the government set 2% target for inflation, *ceteris paribus*, exceeding any levels seen since the 1970s (see Hendry, 2001).

	$U_{r,t}$	π_t	$\Delta p_{w,t}$	Δp_{ot}	Δm_t	$R_{s,t}$	$R_{l,t}$	$\Delta(k-l)_t$	$\Delta(e-k)_t$	$\widehat{\dot{p}}$	$\dot{p}$
Coefficient	-.4	0.4	0.24	0.04	0.4	-1.2	1.6	-0.44	-0.28	-	-
$\approx$ % change	1	5	6	150	4	3	2	1	-10	-	11
$\approx$ % impact	-.4	2.0	1.4	6	1.6	-3.6	3.2	-.44	2.8	12.6	11
high % change	2	-5	10	300	8	6	4	1	-10	-	11
bad scenario	-.8	-2.0	2.4	12	3.2	-7.2	6.4	-.44	2.8	16.4	-

Table 4: Contributions to 2022 Inflation. $\widehat{\dot{p}}$ denotes the projected inflation rate under alternative scenarios and $\dot{p}$ denotes the current inflation rate (August 2022).

7 Conclusions

The recent rise in UK price inflation was unanticipated, leading to a flurry of activity rethinking inflation models.[4] But high inflation rates are not new and history can shed light on the current inflationary climate. We use a long-run time-series dataset to model price inflation along with real wages, unemployment and productivity to gain insight into the current implications of inflationary pressures. The advantage of a long time-series of data is that there is a lot of variation which helps to identify explanatory factors. The disadvantage is that history is fraught with outliers, structural breaks, and distributional shifts (as is the future). The present is not at all like the past. In order to use the historical evidence we rely on econometric methods that can handle non-stationarities in the form of distributional shifts, resulting in congruent econometric models despite the vast change that the UK economy has experienced.

The paper highlights the importance of joint modelling of dynamics, location shifts, relevant variables and non-linearities. The automatic model selection approach implemented in our empirical models can handle many more variables than observations and this approach led to the detection of non-linearities which are fundamental in explaining the relationship between wages and prices. There is strong empirical evidence for non-linear adjustments of real wages to inflation where a wage-price spiral essentially adds a unit root to the wage-price process when inflation exceeds about 8%. Given that inflation was 8.8% in July 2022, this non-linearity is essential to understanding how inflation could potentially take

[4]The Bank of England Monetary Policy Report in November 2021 included a central projection of inflation to rise to almost 5% in mid-2022, declining back to 2% over the two year horizon, but the widest confidence bands indicating a 90% confidence interval were at a maximum of 7% in mid-2022. The outturn was 8.8% in July 2022.

off. We also found an additional non-linearity in unemployment which is consistent with involuntary unemployment.

Although the models are single equation, we use automatic tests for super-exogeneity to justify the modelling procedure. An area for future research is to combine the four models into a system complete with non-linearities and distributional shifts. However, we show that the price and wage equations can be combined along with the fundamental non-linearities to obtain projections for the contributions to current inflation. By imposing a 150% increase on energy prices (made up of 50% price rise in oil and 250% in natural gas in equal parts, a conservative estimate at the time of writing) we show that inflation is projected to rise to 12.6%, very close to the 11% current inflation level. Energy costs along with unit labour costs are fundamental to explaining past inflation episodes, and hence understanding current inflationary pressures.

References

Bernstein, D. H. and A. B. Martinez (2021). Jointly modeling male and female labor participation and unemployment. *Econometrics 9*, https://doi.org/10.3390/econometrics9040046.

Castle, J. L., M. P. Clements, and D. F. Hendry (2016). An overview of forecasting facing breaks. *Journal of Business Cycle Research 12*, 3–23. https://doi.org/10.1007/s41549-016-0005-2.

Castle, J. L., J. A. Doornik, and D. F. Hendry (2020). Multiplicative-indicator saturation. Working paper, Nuffield College, Oxford University.

Castle, J. L., J. A. Doornik, and D. F. Hendry (2021). The value of robust statistical forecasts in the Covid-19 pandemic. *National Institute Economic Review 256*, 19–43. https://doi.org/10.1017/nie.2021.9.

Castle, J. L., J. A. Doornik, and D. F. Hendry (2022). Discriminating direct from induced equilibrium mean shifts. Working paper, Nuffield College, Oxford University.

Castle, J. L., J. A. Doornik, D. F. Hendry, and F. Pretis (2015). Detecting location shifts during model selection by step-indicator saturation. *Econometrics 3(2)*, 240–264. https://doi.org/10.3390/econometrics3020240.

Castle, J. L., J. A. Doornik, D. F. Hendry, and F. Pretis (2019). Trend-indicator saturation. Working paper, Nuffield College, Oxford University.

Castle, J. L. and D. F. Hendry (2009). The long-run determinants of UK wages, 1860–2004. *Journal of Macroeconomics 31*, 5–28. https://doi.org/10.1016/j.jmacro.2007.08.018.

Castle, J. L. and D. F. Hendry (2010). A low-dimension portmanteau test for non-linearity. *Journal of Econometrics 158*, 231–245. https://doi.org/10.1016/j.jeconom.2010.01.006.

Castle, J. L. and D. F. Hendry (2011). Automatic selection of non-linear models. In L. Wang, H. Garnier, and T. Jackman (Eds.), *System Identification, Environmental Modelling and Control*, pp. 229–250. New York: Springer.

Castle, J. L. and D. F. Hendry (2014). Semi-automatic non-linear model selection. In N. Haldrup, M. Meitz, and P. Saikkonen (Eds.), *Essays in Nonlinear Time Series Econometrics*, pp. 163–197. Oxford: Oxford University Press.

Castle, J. L. and D. F. Hendry (2019). *Modelling our Changing World*. Palgrave Mcmillan: London. https://link.springer.com/book/10.1007%2F978-3-030-21432-6.

Castle, J. L. and D. F. Hendry (2022). Econometrics for modelling climate change. In J. Hamilton (Ed.), *Oxford Research Encyclopedia of Economics and Finance*. Oxford: Oxford University Press. https://doi.org/10.1093/acrefore/9780190625979.013.675.

Castle, J. L. and N. Shephard (Eds.) (2009). *The Methodology and Practice of Econometrics*. Oxford: Oxford University Press.

Chow, G. C. (1960). Tests of equality between sets of coefficients in two linear regressions. *Econometrica 28*, 591–605. https://doi.org/10.2307/1910133.

Doornik, J. A. (2009). Autometrics. See Castle and Shephard (2009), pp. 88–121.

Doornik, J. A. and H. Hansen (2008). An omnibus test for univariate and multivariate normality. *Oxford Bulletin of Economics and Statistics 70*, 927–939. https://doi.org/10.1111/j.1468-0084.2008.00537.x.

Doornik, J. A. and D. F. Hendry (2021). *Empirical Econometric Modelling using PcGive: Volume I.* (9th ed.). London: Timberlake Consultants Press.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity, with estimates of the variance of United Kingdom inflation. *Econometrica 50*, 987–1007. https://doi.org/10.2307/1912773.

Engle, R. F. and D. F. Hendry (1993). Testing super exogeneity and invariance in regression models. *Journal of Econometrics 56*, 119–139. https://doi.org/10.1016/0304-4076(93)90103-C.

Ericsson, N. R. and E. L. Reisman (2012). Evaluating a global vector autoregression for forecasting. *International Advances in Economic Research 18*, 247–258. https://doi.org/10.1007/s11294-012-9357-0.

Galí, J. and M. Gertler (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics 44*, 195–222. https://doi.org/10.1016/S0304-3932(99)00023-9.

Galí, J., M. Gertler, and J. D. Lopez-Salido (2001). European inflation dynamics. *European Economic Review 45*, 1237–1270. https://doi.org/10.1016/S0014-2921(00)00105-7.

Godfrey, L. G. (1978). Testing for higher order serial correlation in regression equations when the regressors include lagged dependent variables. *Econometrica 46*, 1303–1313. https://doi.org/10.2307/1913830.

Hendry, D. F. (1995). *Dynamic Econometrics*. Oxford: Oxford University Press.

Hendry, D. F. (2001). Modelling UK inflation, 1875–1991. *Journal of Applied Econometrics 16*, 255–275. https://doi.org/10.1002/jae.615.

Hendry, D. F. (2015). *Introductory Macro-econometrics: A New Approach*. London: Timberlake Consultants. http://www.timberlake.co.uk/macroeconometrics.html.

Hendry, D. F. (2022). Does an empirical economic relation have a life? A review essay. *History of Political Economy*, 163–179. https://doi.org/10.1215/00182702-9699096.

Hendry, D. F. and J. A. Doornik (2014). *Empirical Model Discovery and Theory Evaluation*. Cambridge, Mass.: MIT Press.

Hendry, D. F. and S. Johansen (2015). Model discovery and Trygve Haavelmo's legacy. *Econometric Theory 31*, 93–114. https://doi.org/10.1017/S0266466614000218.

Hendry, D. F., S. Johansen, and C. Santos (2008). Automatic selection of indicators in a fully saturated regression. *Computational Statistics 23*, 317–335, Erratum, 337–339. https://doi.org/10.1007/s00180-007-0054-z.

Hendry, D. F. and C. Santos (2010). An automatic test of super exogeneity. In M. W. Watson, T. Bollerslev, and J. Russell (Eds.), *Volatility and Time Series Econometrics*, pp. 164–193. Oxford: Oxford University Press.

Johansen, S. and B. Nielsen (2009). An analysis of the indicator saturation estimator as a robust regression estimator. See Castle and Shephard (2009), pp. 1–36.

Pretis, F., J. J. Reade, and G. Sucarrat (2018). Automated general-to-specific (GETS) regression modeling and indicator saturation for outliers and structural breaks. *Journal of Statistical Software 86, 3*, https://www.jstatsoft.org/article/view/v086i03.

Pretis, F., L. Schneider, J. E. Smerdon, and D. F. Hendry (2016). Detecting volcanic eruptions in temperature reconstructions by designed break-indicator saturation. *Journal of Economic Surveys 30*, 403–429. https://doi.org/10.1111/joes.12148.

Ramsey, J. B. (1969). Tests for specification errors in classical linear least squares regression analysis. *Journal of the Royal Statistical Society B, 31*, 350–371. https://www.jstor.org/stable/2984219.

Reis, R. (2006). Inattentive producers. *Review of Economic Studies 73*, 793–821. https://doi.org/10.1111/j.1467-937X.2006.00396.x.

Walker, A., F. Pretis, A. Powell-Smith, and B. Goldacre (2019). Variation in responsiveness to warranted behaviour change among NHS clinicians: a novel implementation of change-detection methods in longitudinal prescribing data. *British Medical Journal 367, l5205*. https://www.bmj.com/content/367/bmj.l5205.

White, H. (1980). A heteroskedastic-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica 48*, 817–838. https://doi.org/10.2307/1912934.

8 Appendix: Econometric tools for modelling non-stationary time series

Given the manifest evidence of changing changes in all the variables associated with energy and economic outputs and their prices, *a priori* specification of a complete and correct model of the DGP is infeasible. Instead, to make the economic analysis empirically useful, model selection allowing for any number, magnitude, type, sign and timing of shifts is needed. Indicator saturation estimators (ISEs) offer a possible approach, available in software like *Autometrics PcGive* (see Doornik, 2009, and Doornik and Hendry, 2021), in EViews and as *gets* in R (see Pretis, Reade, and Sucarrat, 2018) based on a variant of machine learning for time series that uses block multi-path expanding and contracting searches. The main ISEs are impulse (IIS) for detecting outliers: see Hendry, Johansen, and Santos (2008), analyzed by Johansen and Nielsen (2009); step (SIS) for modelling location shifts (see Castle, Doornik, Hendry, and Pretis, 2015); trend (TIS) for trend shifts (applied in Walker, Pretis, Powell-Smith, and Goldacre, 2019 to health care management); multiplicative (MIS) for parameter changes (see Castle, Doornik, and Hendry, 2020); and 'designed' (DIS) for modelling repeating shift patterns (e.g., the impacts of volcanic eruptions on temperatures as in Pretis, Schneider, Smerdon, and Hendry, 2016); combinations of these, called super-saturation, are proposed in Ericsson and Reisman (2012). Impulse indicators are the first difference of step indicators which are the first difference of trend indicators, so SIS and TIS can capture outliers and the latter also steps, and while not parsimonious, that can be adjusted manually. Although stringent significance levels like $\alpha = 0.01\%$ are required to avoid excess numbers of irrelevant indicators being selected, these only apply to the indicators as all other regressors can be retained at that stage and only need selecting at more conventional significance levels like 1% later (see Hendry and Johansen, 2015).

The general approach to modelling non-stationary economic times series in §2–§5 commences with a very general model specification that allows for all possible explanatory variables, unknown functional forms of non-linearity, general dynamics, distributional shifts and outliers. Such generality necessarily implies more variables than observations at the outset, but it enables a congruent, well-specified model which nests the Data Generating Process (DGP) via the Theory of Reduction, see Hendry (1995, ch.9). Model selection using a tree search algorithm reduces the candidate set of regressors while allowing for complex correlations, ensuring that congruency is retained at every reduction stage, see Hendry and Doornik (2014).

General non-linear functions are used at the initial specification stage if agnostic on the specific functional form of possible non-linearities, and linearity in the parameters is maintained to ensure an efficient reduction although this is not required as reduction is done via maximum likelihood. Weierstass's approximation theorem suggests using polynomials as the general non-linear functional form, and we use encompassing tests to identify specific functional forms against this general alternative. §2 demonstrates this approach by testing a general polynomial model against a logistic smooth transition wage-price spiral to obtain identification of the non-linearity inherent in the model.

Selection with more variables than observations inevitably means that the initial general model specification cannot be estimated. An iterative approach is needed with expanding as well as contracting searches to allow for correlations between variables that aren't jointly included in each block search. Backtesting ensures that any reduced model encompasses the general model so there is no significant loss of information by eliminating regressors. Diagnostic checking also ensures the selected models are well-specified such that the model is a close approximation to the data generating process. Finally, if a range of models are retained, denoted terminal models, then encompassing tests or information criteria can be used to select the final preferred model.

Having arrived at a model that is congruent with relevant explanatory variables and any breaks, outliers and non-linearities have been explicitly modelled, tests of exogeneity on contemporaneous regressors can be undertaken, see Engle and Hendry (1993). As a final stage, forecasts can be computed by extending the data set or having held back a subset of data to ensure *ex ante* forecasts. Evaluating the forecasts does not validate the model as the forecast performance will depend on the out-of-sample data and need not indicate a poor model even if the forecasts are poor. However, poor forecasts could highlight *ex post* parameter non-constancy.

Despite searching over many candidate variables, Hendry and Johansen (2015) show that under the null of N possible candidate regressors that are all irrelevant, αN will be retained by chance even when $N > T$, where α is the significance level used to select the candidate regressors. Furthermore, if a theory model is retained without selection, if all other variables included in selection are orthogonalised with respect to the theory variables, then the resulting parameter estimates will be exactly the same as if the theory model was directly estimated. But as the data from §1 shows, any theory model that doesn't allow for change cannot be empirically relevant. Selection enables us to learn about the non-constancy in the data.

Throughout the modelling process we emphasize the joint nature of all modelling decisions. All aspects must be selected jointly for a coherent economic model, including all substantively relevant variables, their dynamics, outliers and location shifts, and non-linearities. Testing for each aspect individually and sequentially will result in a well-specified model. For example, location shifts and non-linearities can be observationally equivalent and yet have very different economic interpretations and forecast implications, and not removing large outliers or shifts could hide the presence of other relevant variables or non-linearities.