

Consumption Quality: A New Perspective on the Welfare Implications of Business Cycle Fluctuations

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Two Long-standing Questions in Household Finance and Macroeconomics

- ① How do economic shocks affect household consumption?
- ② What are the welfare costs?

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- 2 What are the welfare costs?

State of Literature

- Examine effects on total consumption expenditures.
- Lucas calculation
 - Welfare gain of eliminating spending volatility $\approx 0.05\%$ of spending.
 - Incorporating additional features (incomplete markets, income insurance) yields $\approx 0.1\%$ of spending.

Instead of total consumption expenditures, we zoom into consumption bundles and composition:

- Households smooth consumption at both the **quality** and **quantity** margin when facing income shocks.
- Quality v. quantity margin of consumption reallocation has important welfare implications.

“Consumers choose the *quality* of their purchases, and unit values reflect this choice. Moreover, quality choice may itself reflect the influence of prices as consumers respond to price changes by altering both quantity and quality.”

— Angus Deaton (1988)

“The analysis of quality is an important topic in economics”

— Angus Deaton and John Muellbauer (1980)

- ① To what extent do households exploit quantity versus *quality* margins of adjustments in their consumption decision?
- ② How does this quantity-quality choice affect welfare implications of economic shocks?

- Empirical:
 - Quantify *quality* margin adjustment as a consumption smoothing mechanism, using granular panel data on household consumption.
 - Given negative income shock, 2/3 of expenditure reduction is due to adjustment at the quality margin.
 - Show considerable heterogeneity across income groups in their access to this consumption smoothing channel.
 - The low income groups are “*quality constrained*”.

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 - The low income groups are “*quality constrained*”.

- Theory:

- Non-homothetic preferences.
 - Households value both the quality and quantity margins of household consumption.
- Estimate welfare costs of income fluctuations for the full income distribution.
 - The low income groups bear a disproportionately greater share of the cost of business cycle fluctuations.

- 1 **Data and Quality Measures**
- 2 Empirical Analysis
- 3 Empirics \Rightarrow Theoretical Framework
- 4 Theory and Welfare Analysis

- Nielsen household panel dataset (2004-2013)
 - Transaction data: 4 million household purchase transactions
 - Scanned Universal Product Code (UPC) level data—
(Trip specific:) Date, store, quantity, price, deals;
(Product specific:) Brand, type, volume, packaging.
 - Product data
 - Food departments: dairy, deli, dry grocery, fresh produce, frozen food, packaged meat;
 - Non-food departments: alcohol, general merchandise, health, non-food grocery.
 - Demographic data: 98,547 households.
 - Yearly self-reported household-level data: income, age, sex, race, education, occupation, region of residency, employment status, family composition and household size.
- Nielsen Retailer Scanner Data (2006-2013)
 - Weekly point-of-sale information from 35,000 stores.
 - UPC level data—total sales in revenue and quantity

I. Data Dimensions on Product Groups

- Departments
 - Ex.: “Dairy”
- Product Modules
 - Ex.: “Dairy-Milk-Refrigerated”
- UPCs
 - Ex.: “Berkeley Farmers Organic 2% Milk”

Fundamental Assumptions

- When buying a product (fresh milk) households choose
 - *How much* milk to buy
 - *What quality* of milk to buy
- Extract quantity directly from the data
- Need measure for quality, reflecting
 - Perceived quality value

Price contains information about quality

① Demand theory

- Revealed preferences

If $A \succ B$ and $P_A \geq P_B$ then quality of A is higher than B.

② Supply theory

- Marginal costs

If MC is increasing in quality \Rightarrow Price is increasing in quality.

I. Quality Measures

Set 1: Baseline

- Average Price

Set 2: Quality Ranking

- UPC Price Ranking
- UPC Relative Price Ranking

Set 3: Quality Ladder

- Within-Store Quality Ladder
- Cross-Store Quality Ladder
- Overall Quality Ladder

*All measures are constructed within product module-size unit.

Average Price

$$P_{hgt} = \frac{X_{hgt}}{\sum_u c_{hut}} \quad (1)$$

- X_{hgt} = Total expenditure in product-module g by household h in month t : $\sum_u c_{hut} \cdot p_{hut}$
- c_{hut} : Volumes purchased of UPC u
- p_{hut} : *Unit* price of UPC u

Average UPC ranking of household purchases

$$R_{hgt} = \sum_u s_{hut} \cdot R_u \quad (2)$$

- s_{hut} : UPC budget share
- R_u proxies for a *quality index* : higher R_u means better quality
 - $R_u = \frac{\tilde{R}_u}{\tilde{R}^{max} + 1} \in (0, 1)$
 - $\tilde{R}_u = \omega_u \cdot \text{rank}(p_u | p_1, p_2, \dots, p_N)$
 - p_u : Yearly *unit* price
 - ω_u : UPC Scantrack marketshare
 - $\tilde{R}^{max} \equiv \max \tilde{R}_u$

Alternative UPC ranking of household purchases

$$R'_{hgt} = \sum_u s_{hut} \cdot \log(p_{ut}/p_{ut}^{med}). \quad (2')$$

- s_{hut} : UPC Budget share
- p_{ut} : *Unit* price
- p_{ut}^{med} : Median *unit* price

I. Quality Measure Set 3 – Quality Ladders

Within-Store Quality Ladder

$$x_{hc,g,t}^{UPC} = x_{hc,g,t} - \sum_{w \in t} \sum_s \sum_u c_{hw,u} \cdot \min_u \{p_{hw,s,u}\} \quad (3)$$

Cross-Store Quality Ladder

$$x_{hc,g,t}^{Store} = x_{hc,g,t} - \sum_{w \in t} \sum_u c_{hw,u} \cdot \min_s \{p_{hw,s,u}\} \quad (3')$$

Overall Quality Ladder

$$x_{hc,g,t}^{Extreme} = x_{hc,g,t} - \sum_{w \in t} \sum_u c_{hw,u} \cdot \min_{s,u} \{p_{hw,s,u}\} \quad (3'')$$

- w : Week
- s : Store
- c : Number of Items $\Rightarrow p$: *Actual price*
or Volume $\Rightarrow p$: *Unit price*

- 1 Defining and Calculating Quality
- 2 **Empirical Analysis**
- 3 Theory and Welfare Analysis

II. Empirics – Reduced Form Estimation

Regression Model

$$Y_{hc,g,t} = \beta_1 UE_{ct} + \beta_2 UE_{ct} \times I_{h,l,t} + \theta' X_{h,t} + \zeta_{h,g} + \eta_t + \varepsilon_{hc,g,t} \quad (4)$$

- $Y_{hc,g,t}$: Consumption expenditure, quantity and quality for household h in county c of product module g at month t .
- UE_{ct} : Monthly county-level unemployment rate.
- $I_{h,l,t}$: Lagged (year) income quintile.
- X_{it} : Controls: (yearly) employment status, lagged (monthly) shopping trips, lagged (monthly) unique stores visited.
- $\zeta_{h,g}$: Household-Product-Module fixed-effect.
- η_t : Time dummies.
- Standard errors clustered at the county level.

II. Summary Statistics – Demographics

| Variable | Income Quintile | | | | | All |
|--------------------------------|-----------------|-------|-------|-------|--------|--------|
| | 1st | 2nd | 3rd | 4th | 5th | |
| | Yearly Averages | | | | | |
| Income, \$1,000 | 15.28 | 33.88 | 53.11 | 78.56 | 133.04 | 65.23 |
| Age | 50.5 | 48.4 | 47.3 | 48.1 | 48.6 | 48.5 |
| HH size | 2.1 | 2.4 | 2.8 | 2.8 | 2.9 | 2.6 |
| All Adults Not Employed | 0.4 | 0.1 | 0.1 | 0.1 | 0 | 0.1 |
| One Not Employed | 0.6 | 0.4 | 0.4 | 0.3 | 0.3 | 0.4 |
| Monthly Unempl. (Sea. Adj.), % | 8.49 | 8.25 | 8.12 | 7.99 | 7.88 | 8.13 |
| No. Observations | 7,939 | 8,360 | 8,817 | 8,852 | 9,555 | 43,523 |

II. Summary Statistics – Monthly Shopping

| Variable | Income Quintile | | | | | All |
|------------------------------------|------------------|--------|--------|--------|--------|--------|
| | 1st | 2nd | 3rd | 4th | 5th | |
| | Monthly Averages | | | | | |
| Monthly Expenditures, \$ | 286.89 | 324.06 | 355 | 371.62 | 402.87 | 350.53 |
| No. Unique PMs Bought | 39.41 | 43.46 | 46.07 | 46.72 | 45.51 | 44.36 |
| No. UPCs Bought | 100.98 | 109.02 | 115.14 | 114.53 | 112.15 | 110.6 |
| Avg. Price per UPC, \$ | 3.14 | 3.25 | 3.44 | 3.59 | 3.99 | 3.5 |
| Quality | | | | | | |
| UPC Rank, % | 19.3 | 20.27 | 20.91 | 21.46 | 22.08 | 20.86 |
| Relative UPC price, % | -3.85 | -2.01 | -0.4 | 1.89 | 5.33 | 0.39 |
| UPC Arbitrage (Actual Price), \$ | 49.68 | 60.34 | 67.8 | 72.01 | 85.67 | 67.85 |
| Store Arbitrage (Actual Price), \$ | 85.31 | 101.57 | 117.63 | 129.54 | 155.74 | 119.45 |
| Extreme (Actual Price), \$ | 216.21 | 249.16 | 279.86 | 301.5 | 339.41 | 279.84 |

II. Reduced Form Results – Monthly Expenditures

| Variable | Dependent Variables (Estimates Multiplied by 100) | | |
|--|---|-----------------------------|------------------------------|
| | (1) Log Expenditure | (2) Log Volume | (3) Log Average Price |
| Local Unemployment, β_1 (Seasonally Adj.) | -0.674*** (0.125) | -0.204*** (0.061) | -0.468*** (0.0686) |
| Income Quintile \times Unemployment | | | |
| 1st | 0.568*** (0.201) | -0.0462 (0.0798) | 0.462*** (0.115) |
| 2nd | 0.0344 (0.161) | -0.102 (0.0699) | 0.0562 (0.0975) |
| 3rd | | —Base Level— | |
| 4th | 0.0306 (0.125) | 0.0781 (0.0673) | -0.0243 (0.0791) |
| 5th | -0.035 (0.134) | 0.126* (0.0756) | -0.0786 (0.0771) |
| Controls | Yes | Yes | Yes |
| HH-Product Module FE | Yes | Yes | Yes |
| Time dummies | Yes | Yes | Yes |
| Million Observations | 61.95 | 62.23 | 61.95 |
| Thousand Clusters | 77.2 | 77.2 | 77.2 |

II. Reduced Form Results – Interpretation

- A one percentage point increase in the local unemployment level is associated with 0.6 percent reduction in monthly expenditures on average.
 - 2/3 of this reduction comes from the quality margin, and 1/3 from the quantity margin.

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| | (1) Log Expenditure | (2) Log Volume | (3) Log Average Price |
| Local Unemployment, β_1 (Seasonally Adj.) | -0.674*** (0.125) | -0.204*** (0.061) | -0.409*** (0.0686) |
| Income Quintile \times Unemployment | | | |
| 1st | 0.568*** (0.201) | -0.0462 (0.0798) | 0.402*** (0.115) |
| 2nd | 0.0344 (0.161) | -0.102 (0.0699) | 0.0562 (0.0975) |
| 3rd | | —Base Level— | |
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II. Reduced Form Results – Interpretation

- A one percentage point increase in the local unemployment level is correlated with 0.6 percent reduction in monthly expenditures on average.
 - 2/3 of the this reduction comes from the quality margin, and 1/3 from the quantity margin.
- When facing negative income shock, low income households do not appear to lower consumption quality.

II. Reduced Form Results – UPC Ranking

| Variable | Dependent Variables (Estimates Multiplied by 100) | |
|---|---|----------------------------------|
| | (1) UPC Rank | (2) Rel. UPC Price |
| Local unemployment (Seasonally Adj.) | -0.034** (0.0159) | -0.153*** (0.0317) |
| Income Quintile × Unemployment | | |
| 1st | 0.0573*** (0.0192) | 0.198*** (0.04) |
| 2nd | 0.0263 (0.0166) | 0.0725** (0.0349) |
| 3rd | —Base Level— | |
| 4th | -0.0368** (0.0183) | -0.0428 (0.0369) |
| 5th | -0.0643*** (0.0202) | -0.0397 (0.0386) |
| Controls | Yes | Yes |
| HH-Product Module FE | Yes | Yes |
| Time Dummies | Yes | Yes |
| Million Observations | 61.4 | 61.4 |
| Thousand Clusters | 77.4 | 77.4 |

II. Reduced Form Results – Unexploited “Arbitrage”

| Variable | Dependent Variables, Actual Price (Estimates Multiplied by 100) | | |
|---|--|--------------------------------|---------------------------------|
| | (1) UPC | (2) Store | (3) Extreme Shopping |
| Local Unemployment (Seasonally Adj.) | -0.866*** (0.236) | -3.1*** (0.576) | -5.28*** (0.743) |
| Income Quintile × Unemployment | | | |
| 1st | 0.886*** (0.289) | 1.49* (0.766) | 2.7*** (0.955) |
| 2nd | 0.403 (0.254) | 0.838 (0.612) | 0.931 (0.814) |
| 3rd | | —Base Level— | |
| 4th | 0.164 (0.221) | 0.711 (0.664) | -0.671 (0.801) |
| 5th | -0.0764 (0.255) | 1.55* (0.846) | 0.491 (0.886) |
| Controls | Yes | Yes | Yes |
| HH-Product Module Fixed-Effects | Yes | Yes | Yes |
| Time dummies | Yes | Yes | Yes |
| Million Observations | 62.23 | 62.23 | 62.23 |
| Thousand Clusters | 77.2 | 77.2 | 77.2 |

II. Reduced Form Results – Yearly Income

| (Estimates Multiplied by 100) | Income | | HH-PM-FE & Controls | Mio. Obs. | 1,000 Clusters |
|-------------------------------|----------|---------|------------------------|--------------|-------------------|
| | Estimate | Std.Err | | | |
| Log Expenditure | 17.1*** | (2.11) | Yes | 28.6 | 76.1 |
| Log UPCs Bought | 13.5*** | (2.05) | Yes | 28.7 | 76.1 |
| Log Avg. Price | 3.53*** | (0.891) | Yes | 28.6 | 76.1 |
| Quality | | | | | |
| UPC Rank | 0.494** | (0.252) | Yes | 28.7 | 76.1 |
| Relative UPC price | 2.5*** | (0.713) | Yes | 28.7 | 76.1 |
| UPC (Actual Price), \$ | 6.26** | (2.56) | Yes | 28.8 | 76.1 |
| Store (Actual Price), \$ | 38.9*** | (7.83) | Yes | 28.8 | 76.1 |
| Extreme (Actual Price), \$ | 69.9*** | (9.27) | Yes | 28.8 | 76.1 |

II. Reduced Form Results – -Product Module Substitution

Estimated Partial Effects of Local Unemployment Rate Shock, by Department

| Department | (-All Estimates Multiplied by 100-) | | | | | |
|----------------------|-------------------------------------|---------------------|---------------------|--------------------|-----------------------|----------------------|
| | Log Expenditures | | Log Products Bought | | Log Avg. Price Paid | |
| | Avg. | 1st Quantile | Avg. | 1st Quantile | Avg. | 1st Quantile |
| Alcoholic Beverages | 1.4 (1.3) | 7.17*** (1.3155) | 1.44 (1.05) | 2.81* (1.4956) | -0.354 (0.692) | 1.87* (1.0694) |
| Dairy | -1.15*** (0.164) | -1.02*** (0.276) | -0.0974 (0.15) | 0.05 (0.2754) | -1.23*** (0.0786) | -1.18*** (0.1398) |
| Deli | -0.925** (0.383) | 0.15 (0.7336) | -0.769** (0.324) | -0.01 (0.6486) | -0.615** (0.242) | -0.2 (0.4827) |
| Dry Grocery | -0.604*** (0.123) | 0.05 (0.2237) | -0.223* (0.116) | -0.02 (0.1935) | -0.238*** (0.0727) | 0.22* (0.1293) |
| Fresh Produce | 0.135 (0.302) | 0.88 (0.5762) | 0.126 (0.226) | 0.33 (0.4159) | -0.314 (0.205) | 0.23 (0.4675) |
| Frozen Foods | -0.168 (0.231) | 0.23 (0.3797) | 0.447** (0.221) | 0.7** (0.3527) | -0.157 (0.148) | 0.6** (0.2625) |
| General Merchandise | -0.264 (0.902) | 2.66** (1.3175) | 0.0662 (0.609) | 2.27** (0.9646) | -0.189 (1) | 1.71 (1.4269) |
| Health & Beauty Care | -0.649 (0.402) | -0.3 (0.6387) | -0.229 (0.271) | -0.24 (0.4943) | -0.498 (0.36) | -0.15 (0.5889) |
| Non-food Grocery | -0.371 (0.234) | -0.45 (0.619) | -0.0854 (0.193) | -0.33 (0.3996) | -0.281* (0.165) | -0.38 (0.2991) |
| Packaged Meat | -0.268 (0.294) | -0.12 (0.4736) | -0.145 (0.254) | -0.16 (0.4526) | -0.619*** (0.229) | -0.1 (0.3491) |

II. Stylized Facts

- ① When facing negative income shock, households lower their monthly consumption expenditure by reducing both the *quality* and *quantity* of the products purchased.
 - Households engage in both intra-product module switching and inter-product module switching.
- ② Low-income households *do not* downgrade the quality of the products purchased, while they still reduce their consumption quantities.
 - They face *quality constraints*.

II. Stylized Facts \Rightarrow Theoretical Framework – Key Features

- ① Non-homotheticities of quality and quantity.
 - Allow substitution *between* UPCs within product module.
 - Allow substitution *across* product modules.
- ② Quality constraints.

II. Theoretical Framework – Limitations in Existing Literature

- Standard framework assumes UPC choice is binary choice:
 - ① Quality-quantity choice is *homothetic*
 - No substitution *between* UPCs within Product Module: agent picks (one and only) $u^* = \arg_u \max\{Q_u \cdot M/p_u\}$
 - From above: u^* is *independent* of M
 - Budget shares across Product Module are constant wrt income.
 - ② No constraints on quality
- Common approach: Assume preference for quality changes with income. (Handbury 2015; Redding and Weinstein 2016, Faber and Fally 2016)
 - ⇒ Agent picks $u^* = \arg_u \max\{Q_u(I) \cdot M/p_u\}$
 - UPC choice now varies with income (and budget).
 - Challenge: Cannot address welfare costs of income fluctuations since stable preference over time and across income groups is required.

- ① Defining and Calculating Quality
- ② Empirical Analysis
- ③ **Theory and Welfare Analysis**

IV. Theory – Standard Utility Aggregation

Utility Function: Agent maximizes utility by choosing the consumption bundle of quantity, C_j , and quality, Q_j , across all product modules, j .

$$U(\mathbb{C}, \mathbb{Q}) = \left(\sum_j v_j u_j(C_j, Q_j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad v_j \geq 0, \quad (5)$$

- v_j : product module share.
- $\sigma (> 1)$: elasticity of substitution between product modules.

Budget Constraint

$$\sum_j C_j \cdot P(Q_j) \leq M. \quad (6)$$

- M : total expenditure/budget .

IV. Theory – Power-Log Sub Utility

$$u_j(C_j, Q_j) = \ln(C_j)^{\alpha_j} + \ln(Q_j)^{\beta_j}. \quad (7)$$

- α : preference for quantity of pm j .
- β : preference for quality of pm j .
- *Non-homothetic* for (almost) all values of $\alpha \neq \beta$.
- With non-homothetic utility and if the elasticity of the price is non-unit value, $\varepsilon_{P|Q} \neq 1$, then the demand for quality (and quantity) is a non-linear function of income.

Price Relation

$$P(Q) = \eta Q^{\gamma_j}. \quad (8)$$

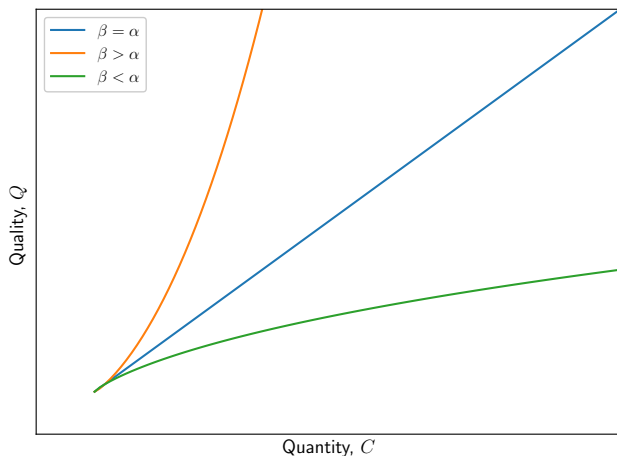
- η : normalizing factor
- γ_j : Price Elasticity wrt. Quality = $\varepsilon_{P|Q} \equiv \frac{P(Q)}{Q} \frac{\partial P(Q)}{\partial Q}$

IV. Theory – Optimal Relation between Quantity and Quality

$$\ln(Q_j)^{1-\beta_j} = \frac{\beta_j}{\alpha_j \gamma_j} \ln(C_j)^{1-\alpha_j}. \quad (9)$$

- For $\alpha \neq \beta$ and $\gamma \neq 1$ then the optimal relationship between quantity, C , and quality, Q , is non-linear
- For $\beta > \alpha$: Quality increase *faster* than quantity
- For $\beta = \alpha$ and $\gamma < 1$: Quality increase *faster* than quantity
- For $\beta = \alpha$ and $\gamma > 1$: Quality increase *slower* than quantity

IV. Theory – Quantity Quality Trade Off



Relationship between optimal choice of quantity and quality for hypothetical preferences and $\gamma = 1$, based on the optimality condition (9).

IV. Theory – Structural Estimation

Estimating Utility Parameters

$$\ln \tilde{Q}_j = \theta_0^j + \theta_1^j \ln \tilde{C}_j + \varepsilon_j. \quad (10)$$

where $\tilde{Q}_j = \ln Q_j$ and $\tilde{C}_j = \ln C_j$.

Estimating Price Parameters

$$\ln P(Q_j) = \ln \eta_j + \ln \eta_t + \gamma_j \ln Q_j + \epsilon_j. \quad (11)$$

Dealing With Endogeneity

- Instrumenting Q_j with lagged total expenditure and lagged income.
- Household fixed-effects.
- Interior solution: Use only HHs with income $>$ median.

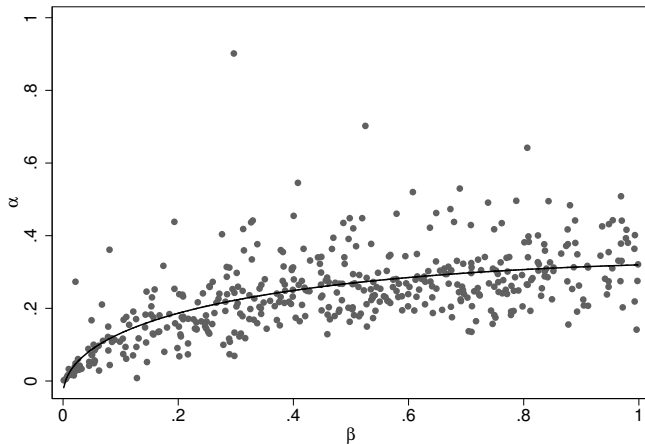
Estimating Preference Parameters

- 1 Obtain estimates for θ_0, θ_1 and γ from regression models (10) and (11) above.
- 2 Then solve for α and β in the equation system below.
- 3 In general, there can be multiple solutions, we pick the solutions obeying fundamental utility assumptions.

$$\hat{\theta}_0 = \frac{1}{1 - \beta} (\ln(\beta/\alpha) - \ln(\hat{\gamma})), \quad (12)$$

$$\hat{\theta}_1 = \frac{1 - \alpha}{1 - \beta}. \quad (13)$$

IV. Theory – Plotting Utility Parameters



Note: Solid line shows polynomial fit.

Scatter of Estimated Quantity Parameters, α , and Quality Parameters, β .

$$\begin{aligned}\ln(X) &= \ln(C) + \ln(P(Q)) \\ &= \left(\frac{\alpha\gamma}{\beta}\right)^{\frac{1}{1-\alpha}} \left(\ln(Q)\right)^{\frac{1-\beta}{1-\alpha}} + \eta + \gamma \cdot \ln(Q)\end{aligned}\quad (14)$$

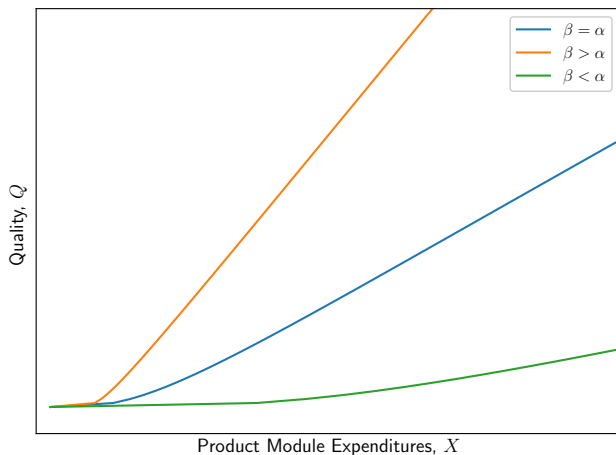
- Non-linear relationship between quality, Q , and PM expenditure, X .
- No closed form solution; perform numerical search.

Optimal Quality as a Function of PM Expenditures

$$\ln(Q) = \Gamma(X), \quad (15)$$

where $\Gamma(\cdot)$ represents the solution to $\ln(Q)$ in (14).

IV. Theory – Optimal Quality and Expenditure (cont.)



Relationship between optimal choice of quality as a function of product module expenditures, based on equation (14).

Sub-Utility as a Function of PM expenditures, X

$$\begin{aligned}u(X) &= \left(\frac{\alpha\gamma}{\beta}\right)^{\frac{\alpha}{1-\alpha}} \ln(Q(X))^{\alpha\frac{1-\beta}{1-\alpha}} + \ln(Q(X))^{\beta} \\ &= \Omega\Gamma(X)^{\psi} + \Gamma(X)^{\beta}.\end{aligned}\tag{16}$$

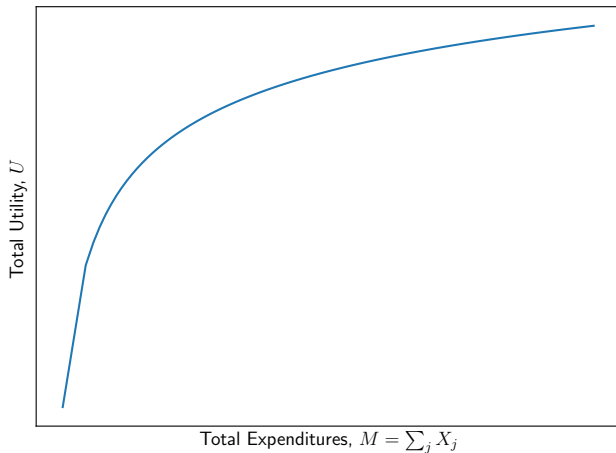
IV. Theory - Aggregating Utility

- Applying the standard sub-utility aggregation approach shows the well-known *super-utility* as a direct function of total expenditures, $M = \sum_j X_j$.

$$\tilde{U}(M) = \left(\sum_j \tilde{v}_j u(X_j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \quad (17)$$

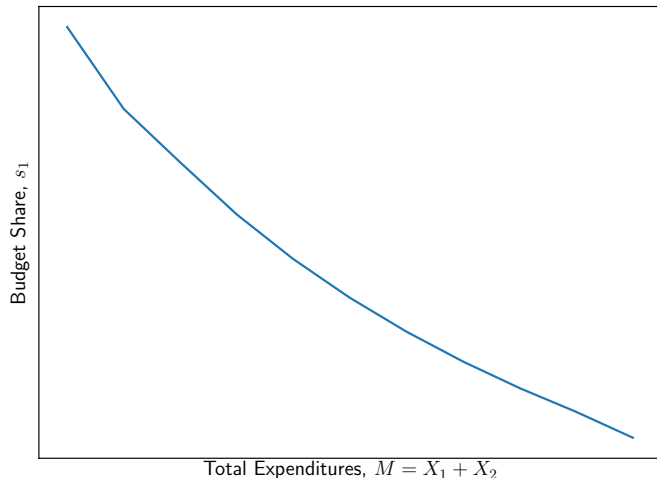
- Utility is a *concave* function of total expenditures.
- Recall that, for almost any α_j and β_j , this utility function is non-homothetic in its inputs, C and Q , and therefore in total expenditures M .
- Thus, budget shares are *non-linear*, leading to non-linear Engel curves.

IV. Theory – Graphing the Utility Function



Relationship between utility and total budget based on equation (17).

IV. Theory - Illustrating Budget Shares, Two-PM Case

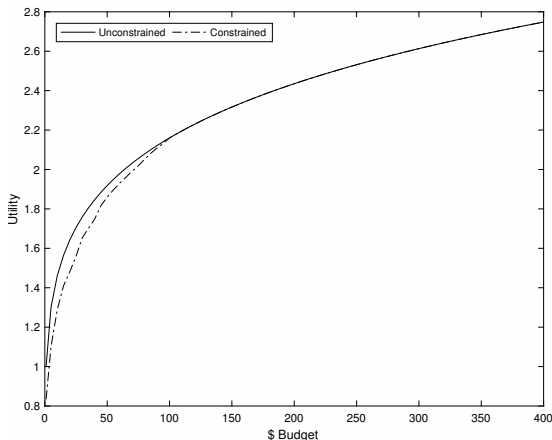


Budget share, s_1 calculated as X_1/M , where $\alpha_1 < \alpha_2$, $\beta_1 < \beta_2$, and all other parameters are identical.

VI. Welfare Analysis

- Quality constraints
 - Utility maximization problem is now a constrained optimization problem
 - Impose lower bound on quality, Q
 - We solve the agent's utility maximizing problem numerically
- Welfare calculations
 - Equivalent variation: \$ value of difference between utility in constrained vs. unconstrained environment.
 - Compensating variation: \$ compensation for not being quality constrained.

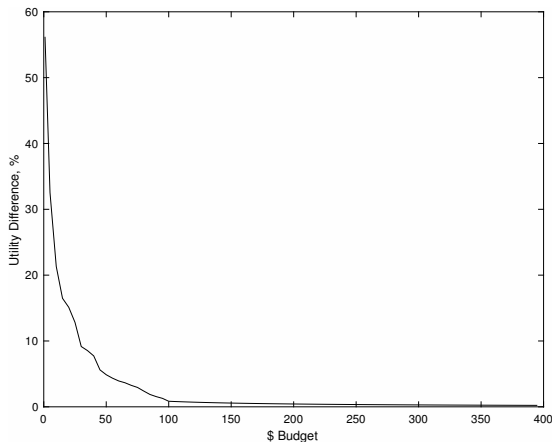
Unconstrained vs. Constrained Optimal Utility



Unconstrained versus Constrained Optimal Utility as a Function of Monthly Budget.

Notes: Let U and V be unconstrained and constrained optimal utility, respectively, then the lines plots $U(M)/U(M = 1)$ and $V(M)/U(M = 1)$.

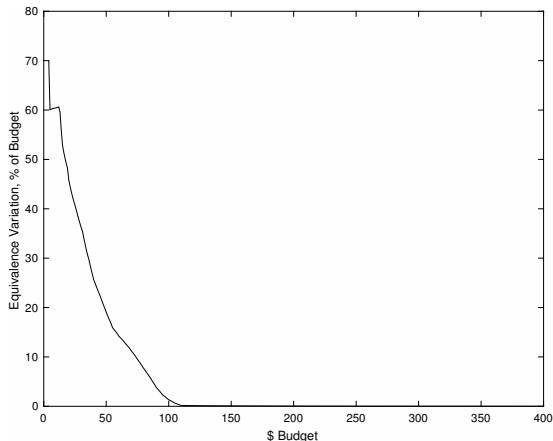
Utility Gains from Being Unconstrained



Relative Difference (%) in Unconstrained and Constrained Optimal Utility as a Function of Monthly Budget.

Notes: Let U and V be unconstrained and constrained optimal utility, respectively, then the line plots $(U - V)/V * 100$.

Equivalent Variation



% dollar compensation required to balance constrained and unconstrained optimal utility.

VI. (Lucas-style) Welfare Calculations (work in progress)

- To address welfare costs from income fluctuations, we calculate the *Lucas* welfare gain, λ^* , of eliminating consumption fluctuations

$$\lambda^* \equiv \arg_{\lambda} : \sum_{t=0}^T \beta^t E \left[\tilde{U}((1 + \lambda)M_t) \right] = \sum_{t=0}^T \beta^t \tilde{U}(Ae^{\mu t}), \quad (18)$$
$$M_t = Ae^{\mu t} e^{-(1/2)\sigma^2 \varepsilon_t}.$$

Where

- $\tilde{U}(M_t)$ is the period utility function from expenditures in equation (17),
- β is the (constant) inter-temporal discount factor,
- $\log(\varepsilon_t) \sim i.i.N(0, \sigma^2)$,
- $E[e^{-(1/2)\sigma^2 \varepsilon_t}] = 1$,
- $E[M_t] = Ae^{\mu t}$.

- Quantified the *quality* channel for consumption smoothing in response to income shocks.
 - ① Households decrease their monthly consumption expenditure by reducing both the *quantity* and *quality* of the products purchased on average.
 - ② Low-income households do not downgrade the quality of the products purchased, while they still reduce their consumption quantities.
 - ⇒ Lower income households are facing *quality constraints*
- Developed a tractable model
 - Non-homotheticities of the quantity-quality choice: agents value both the quality and quantity margins of household consumption.
 - Study consequences of income shocks on household welfare through both margin of adjustments.

Thank you!