

Prudential Policy in an Exuberant World

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Abstract

This paper studies financial and monetary policy in an economy where the financial sector can become excessively optimistic. I first decompose the welfare effects of bank capital regulation to demonstrate the effects of irrational exuberance. The right policy response depends not only on the extent, but also on whether the exuberance of banks focuses on neglected downside risk, as opposed to overstated upside opportunities. A central normative conclusion is that “leaning against the wind”, by tightening capital requirements in exuberant times, is not necessarily beneficial. I derive two sufficient statistics, describing the distortion in perceived upside and downside risk, that characterize the policy implications of exuberance, and can be quantified using recent empirical work on beliefs in financial markets. From a positive perspective, these results shed light on the diverse empirical evidence on the relationship between bank capital and risk-taking. I further show that monetary tightening is a useful substitute for financial regulation, because it affects exuberant incentives precisely in situations where capital capital regulation cannot do so. Finally, I investigate the sensitivity of these insights under different assumptions about government rationality and paternalism.

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1 Introduction

A large part of financial policy is motivated by the concern that banking systems might generate excessive levels of systematic risk during credit booms. The most common narrative is that financial institutions are aware of the risks they are taking, but decide to take them anyway because of bad incentives, that is, because they do not bear the full, society-wide downside of their actions. This can be because banks enjoy implicit government support (Farhi and Tirole, 2012), or because they fail to internalize the full macroeconomic costs of financial crises (Lorenzoni, 2008; Korinek and Simsek, 2016; Farhi and Werning, 2016). This insight underpins most of the literature which analyzes and calibrates optimal capital requirements and other macro-prudential policies.¹

In this paper, I propose a theory of prudential policy where, in addition to incentive problems, banks are subject to *irrational exuberance*. Some recent evidence suggests that, at the peak of a typical credit boom, financial investors might not be aware of the risks they are taking. Cheng et al. (2014) demonstrate that Wall Street insiders, even when trading on their personal account, did not act as if they knew of the risk of the 2008 housing crash. In credit markets more widely, Lopez-Salido et al. (2017) show that indicators of bond market sentiment predict subsequent increases in credit spreads. Greenwood and Hanson (2013) and Baron and Xiong (2017) show that indicators of credit booms can be used to predict significant *negative* returns on bank equity and corporate bonds. This evidence is consistent with models of exuberant beliefs during credit booms (e.g., Gennaioli et al., 2012; Bordalo et al., 2017).

In this context, the motivation for my analysis is twofold. First, while irrational exuberance during credit booms appears to be a viable hypothesis, there is little rigorous normative analysis of prudential policy in an exuberant world. Second, the available empirical evidence on the effectiveness of standard capital-based regulation is mixed. On one hand, better capitalized banks are less likely, on average, to take risk (e.g., Jiménez et al., 2014). On the other hand, Jorda et al. (2017) show that, historically, more bank capital has only a limited effect on the probability of severe credit crises.

How, in principle, should financial policy deal with exuberance? Which tools are effective in an exuberant credit boom? I consider a model that provides concrete insights on these questions. In particular, my analysis suggests that it is not always effective to “lean against the wind” by tightening capital requirements in exuberant booms. This result also provides a positive insight by shedding light on the mixed empirical track record of capital regulation.

¹See, for example: Van den Heuvel (2008); Corbae and D’Erasmus (2014); Bianchi and Mendoza (2017); Begenau (2019); Davila and Walther (2019); Bahaj and Malherbe (2019)

I then consider whether monetary policy can substitute for financial regulation. This has been advocated in situations where traditional financial regulation cannot reach the “shadow banking” sector, or is otherwise constrained (e.g., Stein, 2013; Caballero and Simsek, 2019). I show that, even in a model without such constraints, monetary policy is useful because it reins in exuberant credit booms, and is particularly effective at times when capital regulation is endogenously constrained by distorted beliefs.

The model features a single, large bank who is “too big to fail” because the social costs of its bankruptcy would be prohibitive. If the bank fails, the government raises distortionary taxes to bail it out. The bank borrows from households to invest in risky capital, anticipating its bailout subsidy. I allow the bank’s beliefs about the returns to investment to differ from the truth, which captures exuberance. Rather than using a specific definition of exuberance, I allow for arbitrary true and perceived distributions of investment returns. My model can therefore be used flexibly, to consider the consequences of any heuristic or bias that is supported by the data. For example, banks may overstate the expected value of investment returns, understate their variance, or downplay the likelihood of rare shocks.

This environment is designed to capture the twin problems of incentives and exuberance in the clearest manner. It reflects a very simple macroeconomic rationale for policy, because the health of the entire economy hinges on a single bank. The bank’s optimization problem in this paper builds upon the canonical “Tobin’s q ” theory of risky investment. This setup is modular and could, in principle, be integrated into richer models of banks’ incentives, with fire sales (Lorenzoni, 2008), nominal rigidities (Farhi and Werning, 2016), or heterogeneous banks (Davila and Walther, 2019). All of these environments would share the same key ingredient, namely, that banks do not internalize the social downside of their actions.

The government in my model is able to constrain banks’ leverage by imposing equity capital requirements. This is a second-best policy problem: While the government can require that some cents of every dollar invested must be the bank’s own money, it cannot dictate the scale of the bank’s risky investment.²

I consider various welfare functions: The economics are clearest in a paternalist mode of government, where the government knows the true distribution of investment returns and also knows the distortions in the private sector’s beliefs (see Dávila (2014) and Farhi and Gabaix (2017) for similar treatments). Paternalism obviously places a heavy burden

²Similar insights arise in a model where banks choose the composition of risk in a portfolio of fixed scale, and where the regulator cannot observe risk choices. Regulating the scale of banks is not a policy proposal that has been seriously considered in practice. Indeed, most regulatory tools in the Basel Accords – as in my model – constrain ratios and leave scale as a free variable. At a more formal level, “nationalization” policies that control every one of the bank’s decisions can be shown to be suboptimal in a world where private agents have real-time signals about investment opportunities that the government does not have (e.g., Walther, 2015).

of rationality on the public sector. For this reason, I also consider alternative setups where the government knows of potential exuberance but cannot spot it in real time (this may be where the psychological and econometric evidence leaves us at the moment), or where the government itself is exuberant.

To analyze optimal policy in this model, I take an approach inspired by the canonical treatment of second-best policy in public finance. If the government raises capital requirements, the effect on welfare can be decomposed into two terms, which reflect two common arguments for capital regulation in practice. The first is the mechanical effect of more capital, which creates a *buffer* that shields society from the costs of bank distress (as in optimal tax analysis, this term simplifies considerably due to an envelope condition). The second is the behavioral effect,³ which arises because the level of capital changes the bank’s *incentives* to engage in risky investments, by forcing the bank to have “skin in the game”. The behavioral effect, in turn, hinges on the *sensitivity* of the bank’s risky portfolio choices to its capital requirements.

I first derive this decomposition in a benchmark rational model, and then adjust the associated expressions for “wedges” that capture the gap between the bank’s beliefs and rational expectations. The main insights of this paper come from the comparative statics of this decomposition in an exuberant world. I derive four further sets of results.

First, I show that the sensitivity of portfolios to capital is muted when banks are exuberant. This is because an optimistic bank does not perceive a large probability of receiving a bailout in the first place and, therefore, makes investment decisions that are close to what an unlevered firm would choose. Speaking informally, giving somebody skin in the game does not change much if they believe that they have already won. This idea is key to the normative analysis that follows.

An additional, positive implication of this result is that strict capital requirements need not curb the most severe credit cycles. This goes some way towards reconciling the empirical evidence: Capital requirements are effective for incentives on average (e.g., Jiménez et al., 2014), but do not smooth out the largest booms and busts (e.g., Jorda et al., 2017).

Second, I study the welfare effects of raising capital requirements when the bank is exuberant. Formally, I ask whether the marginal welfare benefit of stricter capital regulation becomes larger or smaller, compared to a rational world, when banks are exuberant. An important intuition is that the answer is ambiguous. The type – not just the extent – of exuberance is crucial. This is because bailouts introduce an asymmetry into the bank’s incentives. For example, in a “neglected tail risk” scenario, where banks understate the

³Throughout this paper, I use the word “behavioral” as it is used in public finance: It means the response of agents’ optimizing behavior, as opposed to irrationality.

downside risk of making large losses relative to small ones, the sensitivity of the bank's portfolio choices to capital requirements may be too small to justify additional intervention. By contrast, in a "boom" scenario, where banks overstate the likelihood of large upside returns relative to normal ones,⁴ the case for tough capital regulation becomes stronger because it nudges banks towards rational levels of investment.

My formal results generalize these insights using a characterization of asymmetric optimism. In particular, I reduce the normative implications of exuberance to two sufficient statistics, which are (i) an *upside wedge* measuring the perceived overvaluation of the bank's equity tranche (or equivalently, the distortion to Tobin's q), and (ii) a *downside wedge* measuring the understatement of the probability of bank failure. Exuberance makes capital regulation more attractive if and only if the upside wedge is relatively large.

As a complementary exercise, I characterize the effect of exuberance in the sense of overstated returns (first-order stochastic dominance) and understated risk (second-order stochastic dominance). Consistent with the intuition above, capital regulation does not necessarily become more attractive when the bank overstate returns. Indeed, it can become less attractive in the neglected tail risk scenario, where optimism focuses on relatively bad states of the world. Perhaps surprisingly, the welfare implications are much clearer in the case of understated risk. This type of exuberance always weakens the case for capital regulation under realistic assumptions.

In summary, the results from this part of the analysis stand in contrast to the simple argument that banks should be regulated more stringently in boom times, or when they perceive the world to be safe. Once the mechanical and behavioral effects are taken into consideration, the welfare effects of raising capital requirements are much more nuanced.

The third set of results introduces monetary policy to the model. As in the baseline case, I show that banks become insensitive to capital regulation when they neglect downside risk. By contrast, they remain sensitive to monetary tightening (an increase in interest rates) because this policy raises the cost of leverage for a solvent bank. Crucially, the role of beliefs in the response to monetary policy is *opposite* from the response to capital regulation. The leverage cost increase is especially salient for banks who neglect the possibility of failure, because they expect the cost of leverage to come out of their own pocket, as opposed to the taxpayer's. Formally, I show that the marginal welfare benefit of monetary tightening can increase with exuberance, and does so precisely in situations where the benefit of capital regulation declines.

⁴Banks typically hold portfolios of fixed income securities. In this context, the upside of banks' investments refers to situations where the realized returns on these securities are large. This occurs when delinquency rates are low (e.g., nobody defaults in a portfolio of mortgages) or, in the case of securities that are traded in secondary markets, when bond yields decline.

Although the model is stylized, it is useful to provide a quantitative illustration of these effects. I show how, given some structural assumptions, one can measure the relevant wedges using the recent empirical literature on beliefs in financial markets (e.g., Greenwood and Hanson, 2013; Bordalo et al., 2017; Baron and Xiong, 2017). For standard parameters, the model suggests that the right policy response in terms of capital regulation depends on how much probability the (rational) government assigns to the event of a crash. If crash risk is elevated, then leaning against exuberance with stricter capital regulation may be counterproductive. However, the case for prudential monetary policy in my model becomes stronger for all reasonable calibrations of exuberance.

The final and fourth part of the paper relaxes the assumption of paternalism. When I assume that the government cannot measure banks' beliefs in real time, the effectiveness of capital regulation is weakened further. The welfare effect of raising capital requirements now contains the covariance between the government's *desire* to control banks' incentives and the *effectiveness* of the tools it has available (i.e., capital requirements). In an exuberant world, and in contrast to a model with rational expectations, this covariance tends to be negative: The government is keen to provide high powered incentives in exuberant booms. However, these are exactly the states of the world where the impact of capital on the bank's incentives is muted, because the bank does not consider failure a likely scenario.

The structure of the paper is as follows: Section 2 contains the model environment. Section 3 derives the central welfare effects and their decomposition in a benchmark model where the bank has rational expectations. Section 4 highlights the impact of exuberance on banks' portfolio choices, and Section 5 derives welfare effects with exuberance and a quantitative illustration. Section 6 considers monetary policy, Section 8 contains extensions, and Section 9 concludes.

2 Model

There are two dates $t \in \{0, 1\}$, two consumption goods (dollars and capital), and three types of agents: A single bank, a population of identical households, and a benevolent government.

Preferences. Everybody is risk-neutral. Households' lifetime utility is the sum $u = c_0 + c_1$ of their consumption at date 0 and 1. The bank is less patient, discounts the future at rate $\rho > 0$, and has utility $\hat{u} = \hat{c}_0 + \frac{1}{1+\rho}\hat{c}_1$. This generates gains from trade: It is better for households to finance up front investments because they are more patient. The government wishes to maximize the utilitarian social welfare function $W = u + \hat{u}$.

Endowments and taxation. The bank and households have endowments of consumption at date 0. Households have a further endowment at date 1, which can be subjected to taxation. The government can raise fiscal revenue t at date 1 by levying a tax of $(1 + \kappa)t$ units of consumption on households, where $\kappa > 0$ is the deadweight cost of taxation. I assume that households' endowments are large enough so that their consumption never becomes negative.

Investment technology. The bank can make investments at date 0 to create $i \geq 0$ units of productive capital. This capital can be used in production at date 1 and yields θi dollars at that time. The return on investment $\theta \geq 0$ is a random variable, with cumulative distribution $F(\theta)$, density $f(\theta)$, and full support on the interval $[0, \theta_{max}]$. As in canonical “Tobin’s q ” models of optimal investment, I assume that investment at date 0 costs $pi + c(i)$ dollars, where p is the replacement cost of capital, and $c(i)$ is a strictly convex adjustment cost.

Beliefs and exuberance. I allow for misperceptions of the distribution of investment returns. In particular, the bank evaluates the distribution to of returns as $\hat{F}(\theta)$, which can be different from the true distribution $F(\theta)$. This formulation can capture situations where the bank is irrationally exuberant. Rather than specify a single definition of exuberance, I take a more flexible approach that characterizes welfare for any true distribution $F(\theta)$ and any perceived distribution $\hat{F}(\theta)$ of returns. For example, the bank could be exuberant if $\hat{F}(\theta)$ is either more optimistic than $F(\theta)$ in the sense of first-order stochastic dominance, or less risky than $F(\theta)$ in the sense of second-order stochastic dominance. However, no such ranking is required for my analysis below, which boils the differences between \hat{F} and F down to two sufficient statistics. The advantage of this flexible approach is that my model can be used, in principle, to analyze the consequences of the many different biases that have been studied in behavioral economics.

Financial contracts. The bank finances itself by issuing bonds with face value b per unit of investment (i.e., the total stock of debt issued is bi , and the bank’s leverage ratio is simply b). Any remaining financing is obtained with an equity contribution from its own endowment. The fact that the bank is impatient implies that bond finance is cheaper than equity, and the rate ρ of time preference captures the (private and social) costs of equity issuance. There are readily available micro-foundations that can generate the cost ρ of equity issuance from first principles, for example, moral hazard among shareholders, a demand for “money-like” claims (Gorton and Pennacchi, 1990; Stein, 2012; DeAngelo and Stulz, 2015), or bank runs and market discipline (Diamond and Rajan, 2001). I choose a reduced form approach to

focus the analysis. None of my results depend on the social cost ρ of equity being large, as long as it is not zero.⁵

“Too big to fail” problem. The bank is too big to fail: The bank is unable to repay its debt bi , and faces default, whenever the returns to investment $\theta \leq b$. In this situation, the government always steps in and provides a bailout $t = \max\{b - \theta, 0\}$ per unit of capital to save the bank. This bailout policy captures a situation where it is prohibitively costly to allow the financial sector to close down.

The social costs of letting banks fail are the subject of a long literature, which traces them to the social cost of credit crunches or bank runs from date 1 onwards (e.g., Holmstrom and Tirole, 1997), the danger of lost output or harmful fire sales if bank assets are liquidated by non-expert agents (e.g., Gromb and Vayanos, 2002; Lorenzoni, 2008; Shleifer and Vishny, 2010), or demand-driven recessions when prices are sticky (e.g., Korinek and Simsek, 2016; Farhi and Werning, 2016). A related literature studies the issue that government are not generally able to make credible commitments that prevent bailouts (e.g., Freixas, 1999; Acharya and Yorulmazer, 2007; Keister, 2016). For the sake of clarity, I do not introduce additional notation to replicate these insights. I focus on the clearest case where bailouts are comprehensive, and where the government has no commitment.

In anticipation of this bailout, households know that their debt is safe. This implies that the bank can issue bonds at par at date 0, raising bi dollars for investment. However, at date 1, bailouts impose a total fiscal burden of $(1 + \kappa)(b - \theta)i$ on households, whenever $\theta < b$. I define the expected fiscal burden on households as:

$$\phi(b) = (1 + \kappa) \int_{\theta < b} (b - \theta) dF(\theta) \tag{1}$$

Financial policy. To combat the distortion in incentives that arises from bailouts, the government is able to impose a standard capital ratio requirement on the bank at date 1. This requirement constrains the bank to set $b \leq \bar{b}$, where $1 - \bar{b}$ is the minimal permitted ratio of bank equity to risky investment. This constraint imposes a debt limit per unit of risky investment, or equivalently, a minimal equity contribution. However, I assume that the government cannot impose direct controls on the magnitude i of risky investment. This restriction means that bank regulation is a *second-best* policy problem.

It is possible to micro-found the assumption that the government cannot control the scale of investment. For example, “nationalization” policies that control every one of the bank’s decisions are not optimal in a world where private agents have real-time information about

⁵If $\rho = 0$, the best capital regulation is (trivially) to forbid any leverage in the banking sector.

investment opportunities that the government does not have (e.g., Walther, 2015). Perhaps for this reason, all relevant regulatory constraints in practice (e.g., capital requirements, leverage, liquidity coverage, and net stable funding requirements in Basel III) focus on ratios of bank assets to liabilities. All of these instruments leave the dollar amount of banks' investments as a free variable.

As an alternative, one could consider a model where banks can engage in asset substitution (or "risk shifting"), which is typically modeled as a situation where banks can increase the riskiness of a portfolio of fixed scale, but where the regulator cannot observe this choice (e.g., Allen and Gale, 2000; Repullo, 2004). This alternative second-best problem is likely to yield similar insights: The key market failure in my model the fact that banks do not internalize the fiscal cost of bailouts. This raises the private benefit of risky investment above the social benefit, especially when beliefs about risky returns are exuberant. This mechanism is at the core of my results, and would remain important in a world where the margin of adjustment is the composition, instead of the scale, of the bank's portfolio.

Equilibrium. Given a regulatory debt limit \bar{b} , a (Subgame Perfect Nash) *equilibrium* in this economy is defined by an investment scale $i \geq 0$ and a leverage choice $b \leq \bar{b}$ that maximize the bank's expected utility, anticipating that the government will provide a bailout $t = \max\{b - \theta, 0\}$ per unit of investment at date 1.

3 The rational benchmarks

To set a benchmark, I analyze the equilibrium of an economy where the bank has rational beliefs $F(\theta)$ about investment returns. It is easy to see that, in order to maximize its lifetime utility, a rational bank would make its choices to maximize the expected present value of its profits:

$$\max_{b \leq \bar{b}, i \geq 0} \pi(i, b) \equiv \frac{1}{1 + \rho} \int_{\theta \geq b} (\theta - b) i dF(\theta) - \rho i - c(i) + b i \quad (2)$$

3.1 Optimal leverage

A simple consequence of the bank's maximization problem in (2) is that the regulatory capital constraint $b \leq \bar{b}$ always binds:

Lemma 1. *If there is a capital requirement, then the bank's privately optimal choice of debt b is always the largest permitted value $b = \bar{b}$. If there is no capital requirement, then the privately optimal choice is $b = \infty$.*

Proof. We have

$$\frac{\partial \pi}{\partial b} = i \left[1 - \frac{1}{1 + \rho} (1 - F(b)) \right] > 0. \quad (3)$$

□

Intuitively, it is optimal for the bank to maximize the value of its bailout-based subsidy, and it does not bear any costs of financial distress. Hence, there is no reason to choose leverage below the permitted limit.

Since the capital requirement is always binding, I will now treat bank leverage b as an effective policy choice variable for the government. In other words, I will evaluate the welfare effects of the government choosing different levels of bank leverage b directly, since this is equivalent to choosing different levels of the binding debt limit \bar{b} (and it is easier to write b without the bar).

Two features of the model are worth discussing in brief: First, capital requirements in the model are always binding, while in reality, banks tend to voluntarily operate above the legal minimum capital ratio. This discrepancy arises because, in this simple model, the bank's problem is static and bailouts are comprehensive. In a dynamic world, banks would clearly have an incentive to keep higher-than-required capital so as to avoid violating future constraints. Another special property of this simple model is that the bank's objective function is convex in b . This implies that, unlike in many macroeconomic models with leverage, Pigouvian taxes would not work in this economy. If the government imposed a linear tax $t_b \cdot b$ on leverage, the bank's objective would remain convex, and the solution would be either $b = 0$ or $b = \infty$. However, one should not view this as a robust prediction of the theory. In a richer model where, for example, the bank bears some of the costs of bankruptcy with some positive probability, its problem would have an interior solution given enough regularity (e.g., Davila and Walther (2019)).

3.2 Optimal investment

Next, I consider the bank's optimal investment problem. The bank's optimal investment $i(b)$, for a given value of leverage b (i.e., a given level of leverage that has been imposed by the government) solves the first-order condition

$$c'(i) = q(b) - p + b \quad (4)$$

where we define

$$q(b) = \frac{1}{1 + \rho} \int_{\theta \geq b} (\theta - b) dF(\theta) \quad (5)$$

This is a levered version of Tobin’s marginal q : It measures the private value to equity-holders of owning an additional unit of productive capital at date 1, holding constant their leverage b . The optimality condition differs from standard investment theory because of b on the right-hand side. When some leverage is permitted, debt presents a subsidized form of finance, which in turn lowers the bank’s weighted average cost of capital, and makes investment more attractive. Hence, investment is positive whenever Tobin’s q is above the replacement cost p , adjusted for the leverage subsidy b .

3.3 Social welfare

Social welfare differs from the bank’s objective due to the fiscal burden on households. Let $i(b)$ be the bank’s optimal investment choice, which solves (4). Then the bank’s utility is proportional to its profits $\pi(i, b)$, which are defined in (2). Households do not extract any surplus from their financial contracts with the bank, since their debt is safe and the interest rate equals their intertemporal rate of substitution. However, they suffer the fiscal burden of $\phi(b)$ per unit of investment, defined as in (1), when the government provides bailouts to the bank at date 1.

Hence, if the government imposes that the bank’s leverage is b (or, equivalently, if it imposes a binding leverage requirement $b = \bar{b}$) and the bank invests i , then utilitarian social welfare function in this economy is

$$\begin{aligned} W(i, b) &\equiv \pi(i, b) - \phi(b)i \\ &= \pi(i, b) - (1 + \kappa) \int_{\theta < b} (b - \theta)idF(\theta) \end{aligned} \tag{6}$$

3.4 Decomposition of local welfare effects

In the rest of the paper, I will focus on the *local welfare effect* $\frac{dW}{db}$ of raising permitted bank leverage by $db > 0$. Of course, one can read each result in two directions: Either in terms raising the maximal permitted leverage by db , or in terms of reducing minimum capital ratios by db . I will use both interpretations, depending on which one is more intuitive, when discussing the results.

In principle, one can take this analysis further by studying under what conditions the welfare function is quasiconcave in b ; this is not necessarily the case, neither in this problem nor in most other second-best analyses with financial frictions (e.g., Lorenzoni, 2008). Under such conditions, one would be able to translate all of my results into explicit comparative statics on the optimal policy b^* , which would be the solution to $\frac{dW}{db} = 0$.

I choose to focus on local effects for three reasons. First, local effects contain all of the relevant economic effects. Second, it is highly unlikely in reality that regulators calculate and impose truly optimal capital requirements, both due to political constraints and due to the limits of computational feasibility. Capital reform over the past three decades has been decidedly incremental. Therefore, my view is that the most useful quantity to measure is the value of a small change to current policy. Third, local effects allow me to take steps towards isolate the key sufficient statistics that one would need to observe to conduct this measurement (see Chetty, 2009).

As in second-best tax theory, the marginal impact of permitting more leverage db is the sum of two terms:

Proposition 1. *When the bank is rational, the welfare effect of permitting more leverage satisfies*

$$\begin{aligned} \frac{dW}{db} &= \underbrace{\left[\frac{\rho}{1+\rho}(1-F(b)) - \kappa F(b) \right]}_{\text{mechanical effect ("buffer")}} i(b) \\ &\quad - \underbrace{\phi(b) \frac{\partial i}{\partial b}}_{\text{behavioral effect ("incentives")}} \end{aligned} \tag{7}$$

where $\phi(b)$ is the fiscal burden imposed on households, as defined in (1).

Proof. Noting that the rational bank chooses the optimal investment $i(b)$ (defined by the first-order condition (4)), we get

$$\begin{aligned} \frac{dW(i(b), b)}{db} &= \frac{\partial W}{\partial b} + \frac{\partial W}{\partial i} \frac{\partial i}{\partial b} \\ &= \frac{\partial \pi}{\partial b} - \frac{\partial \phi}{\partial b} i(b) + \left[\frac{\partial \pi}{\partial i} - \phi(b) \right] \frac{\partial i}{\partial b} \end{aligned}$$

where all derivatives are evaluated at $i = i(b)$. The second equality follows from the characterization of welfare in (6). The envelope condition is that, at the bank's optimal choice, we have $\frac{\partial \pi}{\partial i} = 0$. Moreover, note from (1) that

$$\frac{\partial \phi}{\partial b} = (1 + \kappa)F(b)i(b)$$

Substituting this expression and the expression for $\frac{\partial \pi}{\partial b}$ in (3), and simplifying, we obtain (7). \square

The first, *mechanical* effect of leverage on welfare in (7) consists of two terms. The first

term captures excess costs of equity finance, or equivalently, the gains from trade that are realized when patient agents (households) rather than impatient ones (the bank) finance investments. When the impatience parameter $\rho = 0$, then there are no gains from trade and equity finance is socially free. The second term captures the costs of financial distress. In this model, distress manifests itself through the deadweight cost κ of fiscal support. A useful intuition is to think of the mechanical effect, which trades off the costs of equity finance against the social costs of distress, as a society-wide instance of the “trade off” theory in classical corporate finance (Kraus and Litzenberger, 1973; Myers, 1984).

The second, *behavioral*⁶ effect comes from the fact that leverage changes the bank’s optimal investment. As we will see, leverage encourages more investment, with $\frac{\partial i}{\partial b} > 0$. An envelope argument implies that the effect of this change on bank profits is second-order. The first order change in welfare arises due to the expected social costs of bailouts $\phi(b)$, which scale with i .

These two effects not only mirror the standard decomposition in tax theory, but also reflect two common practical rationales for bank capital. On one hand, the mechanical effect captures the view that bank capital is a “buffer”: More leverage mechanically increases the likelihood of failure and, in turn, the social costs of bank distress. On the other hand, the behavioral effect gives an “incentives” or “skin in the game” rationale for bank capital: Due to bailouts, the social costs of investment exceed the private, and an increase in leverage only widens the wedge. Conversely, a stricter capital requirement (i.e., a decrease in b) aligns incentives because it encourages the bank to internalize the downside of its actions.

4 Exuberance

This section develops some key insights about the level and sensitivity of the bank’s risky investments when its beliefs are exuberant. A bank who has distorted beliefs $\hat{F}(\theta)$ about investment returns perceives its expected profits to be

$$\hat{\pi}(i, b) \equiv \frac{1}{1 + \rho} \int_{\theta \geq b} (\theta - b) i d\hat{F}(\theta) - pi - c(i) + bi$$

4.1 The level of exuberant investment: Upside wedges

The exuberance optimal investment, denoted $\hat{i}(b)$, therefore solves the first-order condition

$$c'(i) = \hat{q}(b) - p + b \tag{8}$$

⁶I use the word behavioral as it is used in public finance: It means the response of agents’ optimizing behavior, as opposed to irrationality.

where

$$\hat{q}(b) = \frac{1}{1 + \rho} \int_{\theta \geq b} (\theta - b) d\hat{F}(\theta) \quad (9)$$

is the exuberant version of Tobin's q . As in Farhi and Gabaix (2017), we can therefore define the “wedge” that exuberance introduces to investment incentives as

$$\tau(b) \equiv \hat{q}(b) - q(b) \quad (10)$$

This wedge measures the distortion to Tobin's q that is brought about by exuberance. Equivalently, it is the overvaluation that the bank attaches to its equity tranche, per unit of investment. Integrating by parts, we can express

$$\begin{aligned} \tau(b) &= \int_{\theta \geq b} (\theta - b) (\hat{f}(\theta) - f(\theta)) d\theta \\ &= \int_{\theta \geq b} (F(\theta) - \hat{F}(\theta)) d\theta \end{aligned} \quad (11)$$

This makes clear that $\tau(b)$ is related to the difference $F(\theta) - \hat{F}(\theta)$ of probability assessments. This difference grows, for example, whenever the bank's beliefs become more optimistic in the sense of first-order stochastic dominance. Moreover, the difference in assessments matters only in relatively good states of the world where the bank is solvent, with $\theta \geq b$. This is because decisions by the private sector (i.e., the bank and its creditors) are affected only by assessments about *upside* risk, since all downside risk is borne by the public sector. I refer to $\tau(b)$ as the *upside wedge* in banks' beliefs.

4.2 The sensitivity of exuberant investment: Downside wedges

Rearranging (8), the exuberant bank's optimal investment can also be written as

$$\hat{i}(b) = A(\hat{q}(b) - p + b) \quad (12)$$

where the function $A(\cdot)$ denotes the inverse of the marginal investment cost $c'(\cdot)$. I write $a = A' = \frac{1}{c''} > 0$ for its first derivative (I sometimes omit the dependence of a on its argument to reduce notation, but this dependence remains implicit).

The sensitivity of investment with respect to permitted leverage b is therefore

$$\frac{\partial \hat{i}}{\partial b} = a \times \left[\frac{\partial \hat{q}}{\partial b} + 1 \right] = a \times \frac{\hat{F}(b) + \rho}{1 + \rho} \quad (13)$$

It follows that:

Proposition 2. *The sensitivity of risky investment to leverage is an increasing function of the perceived probability $\hat{F}(b)$ of receiving a bailout at date 1. Therefore, the sensitivity of risky investment to leverage is smaller when banks are exuberant than in a rational model if and only if $\hat{F}(b) < F(b)$.*

Leverage, combined with limited liability, implies that the bank ignores the downside of its risky investments. A capital requirement has bite and affects behavior because it forces banks to internalize some of this downside. Indeed, this is the classic “skin in the game” motive for capital regulation. However, if an exuberant bank believes that the tail risk $\hat{F}(b)$ of failure is small, then it considers itself to have plenty of skin in the game already. As a result, a marginal increase in capital has a muted effect on its choice of risky investment. Notice that, in the limit $\hat{F}(b) \rightarrow 0$, the sensitivity becomes smaller but does not go to zero. This is because, even for an exuberant bank who does not expect to fail or receive a bailout, debt is a cheaper source of finance than equity, so that an increase in permitted leverage lowers the cost of capital and encourages investment.

An important point to note is that only exuberance about *downside risk* blunts the impact of capital requirements in this manner. For example, one can imagine a “bubble” scenario, where exuberant agents overstate the possibility of abnormally large positive returns relative to average-sized positive returns. The distribution $\hat{F}(\theta)$ in this case differs from the true distribution $F(\theta)$ only in its right tail, and as long as bank default is a left-tail event, the probability of default $\hat{F}(b)$ and, hence, the sensitivity $\frac{\partial i}{\partial b}$ of investment to leverage, are unaffected by exuberance.

This discussion motivates the definition of an additional wedge in beliefs, which captures the extent to which the bank’s beliefs understate the downside risk, as measured by the probability of receiving a bailout. I denote this understatement as

$$\delta(b) \equiv F(b) - \hat{F}(b) \tag{14}$$

which we will refer to as the *downside wedge* in the bank’s beliefs.

I will argue below that this statistic, along with the upside wedge $\tau(b)$ are crucial for optimal policy. Figure 1 visualizes these wedges for a few interesting cases. The blue curve (in each panel) is the true cumulative distribution $F(\theta)$ of investment returns, while the red curve is the perceived one $\hat{F}(\theta)$. In panel (a), the bank is exuberant in the sense of overstated returns (first-order stochastic dominance). Using Equation (11), the upside wedge $\tau(b)$ is the *area* between the true and perceived distribution to the right of the default boundary where $\theta = b$. The downside wedge $\delta(b)$ is the *distance* between the two curves at the default

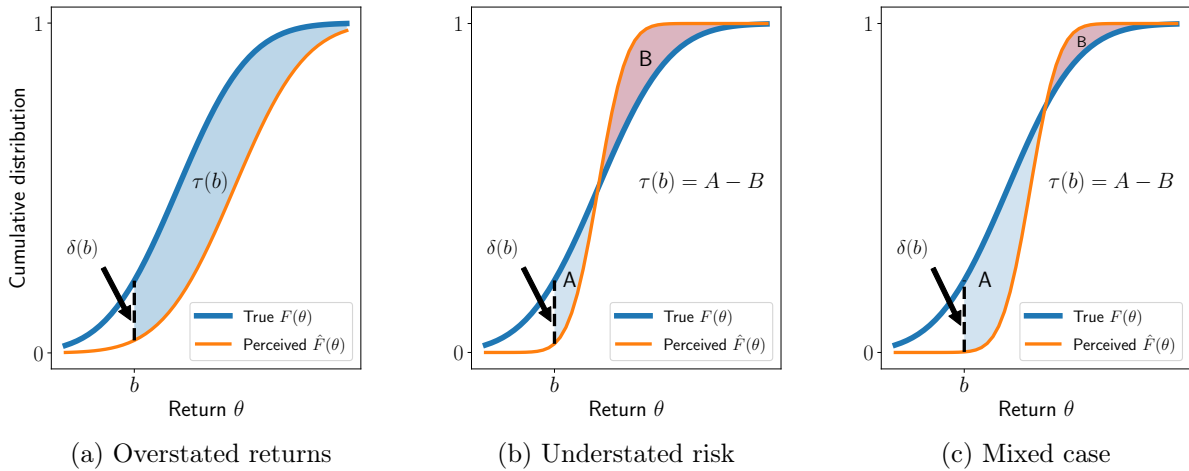


Figure 1: Illustration of Upside and Downside Wedges

boundary. Both wedges are clearly positive; this means that the bank will invest too much but also be less sensitive to capital regulation. In panel (b), the bank is exuberant in the sense of understated risk (second-order stochastic dominance). Now, the upside wedge is negative – as is well known, a decrease in perceived risk *decreases* the value of the convex equity claim – but the downside wedge remains positive. Panel (c) shows a hybrid case where the bank overstates the precision of a favorable Gaussian signal, leading it to both overstate mean returns and understate variance. In this case, the sign of the upside wedge $\tau(b)$ is generally ambiguous (I have drawn a particular case where it is positive). An advantage of boiling the relevant economics down to two wedges is that we do not need a clear stochastic ranking between true and perceived distributions to make useful predictions.

5 Optimal regulation with paternalism

Assume that the government knows the true distribution $F(\theta)$ of investment returns. Assume further that the government is aware that the bank is exuberant and perceives the wrong distribution $\hat{F}(\theta)$. We can now consider an optimal paternalist policy, which takes into account the fact that the bank makes decisions given wrong beliefs, but evaluates the consequences of these decisions using correct ones. This environment clearly places a high burden of foresight on the government, but has the advantage of bringing out the underlying economic effects most cleanly. I consider alternative assumptions below.

Welfare $W(i, b)$ continues to be determined by (6), but the paternalist government realizes that the bank will choose the exuberant investment $\hat{i}(b)$. This changes the social trade-offs

associated with bank leverage. In particular, the description of the welfare effects of bank leverage in Proposition 1 relies on the envelope theorem. If the bank chooses the rationally optimal investment $i(b)$, then its true expected profits satisfy $\frac{\partial \pi}{\partial i} = 0$. With a behavioral bank who chooses $\hat{i}(b)$, however, the envelope argument breaks down and we have

$$\frac{\partial \pi(\hat{i}(b), b)}{\partial i} = -\tau(b) \quad (15)$$

where $\tau(b)$ is the upside wedge defined in the previous section.

Repeating the arguments of Proposition 1, I find that the relevant welfare effect now contains an additional term:

Proposition 3. *When the bank is exuberant, the effect on welfare of permitting more leverage satisfies*

$$\begin{aligned} \frac{dW}{db} = & \underbrace{\left[\frac{\rho}{1+\rho}(1-F(b)) - \kappa F(b) \right] \hat{i}(b)}_{\text{mechanical effect ("buffer")}} \\ & - \underbrace{[\tau(b) + \phi(b)] \frac{\partial \hat{i}}{\partial b}}_{\text{behavioral effect ("biased incentives")}} \end{aligned} \quad (16)$$

where $\phi(b) = (1 + \kappa) \max \{b - \theta, 0\}$ is the fiscal burden imposed on households, per unit of investment, due to government bailouts at date 1.

Proposition 3 permits a clearer assessment of how, and why, optimal capital regulation changes when the bank becomes exuberant. There are three terms in the welfare decomposition in (16) that depend on banks' beliefs:

First, the mechanical effect of capital regulation (the first term in (16)) scales with the level $\hat{i}(b)$ of the bank's investment. By Equation (12), this level increases whenever the bank's exuberant valuation $\hat{q}(b)$ of its equity tranche increases.

Second, the behavioral effect of capital (the second term in (16)) depends on the upside wedge $\tau(b)$. The upside wedge $\tau(b)$ strengthens the case for providing the bank with incentives to scale down its investment. This is in contrast to the rational case in Proposition 1, where the only reason to incentivize lower investment was the expected fiscal cost $\phi(b)$ of bailouts. In the paternalist mode of government, there is a new case for strengthening these incentives, namely, to "nudge" the bank towards more rational behavior.

Third, the behavioral effect depends further on the sensitivity $\frac{\partial \hat{i}}{\partial b}$ of optimal investment to leverage. Recall (from Equation (15)) that this sensitivity depends only on the perceived

probability $\hat{F}(b)$ of receiving a bailout. As I state more explicitly below, the sensitivity component of the behavioral effect therefore hinges on the downside wedge $\delta(b)$. The sensitivity term also depends on beliefs indirectly through the inverse curvature $a = \frac{1}{c''(\hat{i}(b))}$ of the cost function, which is evaluated at the bank's optimal investment. In the propositions that follow, I abstract from this dependence by focusing on quadratic cost functions, for which a is a constant.

This proposition further highlights how the twin problems of incentives and exuberance *interact* in the model. Consider the decomposition of the welfare effects of leverage in Propositions 1 and 3. The incentive-based buffer term $-\kappa F(b)$ is multiplied by the scale $\hat{i}(b)$ of investment, which grows when an exuberant bank has an inflated perception of Tobin's q . In this sense, incentives become more important. However, the fiscal burden term $-\phi(b)$ is multiplied by the sensitivity of investment to leverage, which shrinks when banks are optimistic about the downside of investment returns. Hence, it is unclear whether incentive effects become, on balance, more or less important when banks are exuberant. This ambiguity is at the core of the results in the next section, which paint a mixed picture of the optimal policy responses to exuberance.

5.1 Sufficient statistics for exuberance

Combining the insights above, it is possible to completely characterize the effect of exuberance on the effectiveness of capital regulation:

Proposition 4. *Suppose that adjustment costs are quadratic with $c(i) = \frac{i^2}{2a}$. Then capital regulation is more desirable with exuberance (i.e., the marginal welfare benefit $\frac{dW}{db}$ of permitting more leverage is smaller with exuberance than in the rational benchmark) if and only if*

$$(1 + \kappa)F(b)\tau(b) \geq [\phi(b) + \tau(b)] \frac{\delta(b)}{1 + \rho} \quad (17)$$

This proposition shows the circumstances under which exuberance renders capital regulation more desirable, up to a quadratic approximation of investment costs.

This characterization points out two *sufficient statistics* for the normative implication of exuberance. These statistics are the upside wedge $\tau(b)$ and the downside wedge $\delta(b)$ defined in the previous section. They are sufficient for exuberance in the following sense: Suppose we hold fixed all parameters of the model except for the bank's perceived distribution $\hat{F}(\theta)$ of investment returns. Then, condition (17) shows that the distribution $\hat{F}(\theta)$ enters into welfare considerations only via the two statistics $\delta(b)$ and $\tau(b)$. No other properties or moments of the distorted distribution are relevant for local welfare effects.

The left-hand side of (17) is the additional welfare benefit of lowering leverage in a hypothetical scenario where banks become exuberant but do not change the sensitivity $\frac{\partial i}{\partial b}$ of their investment. This benefit is positive and scales with exuberance $\tau(b)$, due to the government's desire to nudge banks to more rational investment. The proof of the proposition demonstrates that this effect dominates another one, namely, that optimism increases the scale of investment and, hence, the cost-of-capital benefits of leverage.

The right-hand side of (17) measures the strength of a countervailing force, which arises because the bank's sensitivity to capital requirements is diminished. As Equation (13) suggests, the reduction in sensitivity is proportional to the downside wedge $\delta(b)$, discounted by the bank's rate of time preference ρ . The importance of this effect scales with the total size of the externality, namely, the distance $\phi(b) + \tau(b)$ between the private and social marginal value of investment.

Proposition 4 delineates two kinds of optimism that are important for financial policy. It is easy to see that Condition (17) implies that exuberance is likely to increase the marginal benefit of capital regulation when the upside wedge $\tau(b)$ is large.⁷ Intuitively, optimism about upside risk (i.e., an overstatement of the possibility of large returns) increases the social case for having capital requirements. This is because upside optimism leads the bank to overvalue its equity tranche, which drives the perceived Tobin's q further away from its true value, leading to overinvestment.

By contrast, Condition (17) shows that, as long as the total externality $\phi(b) + \tau(b)$ is positive, the marginal benefit of capital regulation is greater under exuberance when the downside wedge $\delta(b)$ is small. Optimism about downside risk (i.e., an understatement of the likelihood of catastrophic states of the world compared to merely bad ones) weakens the effectiveness of capital requirements. The reason is that the sensitivity $\frac{\partial i}{\partial b}$ of investments to leverage falls with downside optimism. Downside-optimistic banks do not consider default or bailouts to be salient, and therefore do not respond to leverage-based incentives in the usual way.

For an alternative intuition, consider what happens when condition (17) fails. Suppose that a regulator thinks that banks are rational, and picks the associated optimal capital requirement (where $\frac{dW}{db} = 0$). Suppose now that banks become exuberant, and that condition (17) is not satisfied. Proposition 4 implies that the regulator can locally improve welfare by *relaxing* capital regulation. The intuition is as follows: When banks become exuberant, they will invest too much, which lowers the level of welfare. However, the regulator realizes that she cannot undo this welfare loss by tightening capital requirements, because banks are no

⁷When $\tau(b)$ increases by $d\tau$, the left-hand side increases by $(1 + \kappa)F(b)d\tau > F(b)d\tau$, while the right-hand side increases by $\frac{1}{1+\rho}\delta(b)d\tau < F(b)d\tau$.

longer sensitive enough to this policy. Moreover, the same lack of sensitivity generates a case for relaxing capital requirements. Under rationality, a relaxation of capital regulation was an unattractive policy because it would have worsened the bank's incentives. Under exuberance, by contrast, incentives are insensitive to policy, so that it is worthwhile to allow slightly more leverage in order to exploit gains from trade.

5.2 Overstated returns versus understated risk

The sufficient statistics in Proposition 4 permit a clean analysis of how particular types of optimism affect welfare. One can extract further economic insight by exploring exuberance in the sense of overstated returns (first-order stochastic dominance) and understated risk (second-order stochastic dominance).

The case of overstated returns follows as a corollary of Proposition 4:

Proposition 5. *Suppose that the perceived distribution $\hat{F}(\theta)$ is more optimistic than $F(\theta)$ in the sense of first-order stochastic dominance.⁸ Then capital regulation is more desirable with exuberance if and only if the optimism in $\hat{F}(\theta)$ is sufficiently concentrated on the upside (i.e., if $\tau(b)$ is sufficiently large, holding $\delta(b)$ constant).*

As is anticipated by the discussion above, optimism about returns does not create a case for capital regulation per se. The policy implications of exuberance in a first-order sense depend on its type (downside vs. upside) as well as the extent of exuberance.

Second, I consider the consequences of understated risk. The policy implications are much clearer in this case:

Proposition 6. *Suppose that the distribution $\hat{F}(\theta)$ is less risky than $F(\theta)$ in the sense of second-order stochastic dominance.⁹ Then, for an interval of leverage levels $b \in [0, \hat{b}]$, for some upper bound \hat{b} , capital regulation always becomes less desirable with exuberance.*

If the bank perceives less risk in investment returns, then it generally affects both behavioral wedges in the same direction. On one hand, the upside wedge $\tau(b)$, which represents the overvaluation of the bank's equity, is negative because of the convexity of the equity claim. This lowers the marginal social benefit of giving the banks incentives to reduce its risky investment. On the other hand, the perceived tail probability $\hat{F}(b)$ with which the bank defaults also decreases when perceived risk is low, as long as b is not too large, which reduces the sensitivity of the bank's investment to leverage. Both effects weaken the rationale for stricter capital requirements.

⁸Technically: $\hat{F}(\theta) \leq F(\theta)$ for all θ , with strict inequality for some θ .

⁹Technically: $F(\theta)$ is a mean-preserving spread of $\hat{F}(\theta)$.

The upper bound on b in this proposition a weak requirement: For example, in the case of symmetric distributions with a single-crossing property (e.g., Gaussian), the default probability $\hat{F}(b)$ is guaranteed to increase with a mean-preserving spread as long as the default boundary $\theta = b$ is a left-tail event, i.e., as long as the probability of a bailout is below 50%. This is likely to be the empirically relevant region: See, for example, the historical frequency of financial crises and fiscal support reported by Reinhart and Rogoff (2009) or Laeven and Valencia (2013), which suggests a crisis about once every 20 years in developed economies.

The results in this section paint a nuanced picture about the normative implications of exuberance. On one hand, I have shown that these effects are complex, and that the type of optimism (i.e., optimism about upside and downside risk) is crucial. On the other hand, I have shown that the reasoning can be simplified by boiling the relevant welfare effects down to two sufficient statistics. Although ambiguity abounds in these results, Propositions 5 and 6 deliver a consistent intuition: Capital regulation becomes less attractive in situations where banks neglect downside risk, either in the sense of understating the likelihood of failure states (Proposition 5) or in the sense of understating the overall risk in investment returns (Proposition 6).

6 Monetary policy in an exuberant financial market

A somewhat negative lesson that emerges from my analysis so far is that capital regulation can be ineffective in exuberant booms. If banks chiefly understate downside risk, then the effect of capital regulation on banks' incentives is muted, and leverage-based regulation becomes a blunt tool. This section investigates whether contractionary monetary policy can be a substitute for prudential regulation.

I introduce a reduced-form treatment of monetary policy, which is similar to the one used by Farhi and Tirole (2012), to the baseline model. In addition to the bank's investment technology, there is a storage technology which transforms one unit of consumption at date 0 into one unit at date 1. The natural interest rate, which prevails in equilibrium without government intervention, is therefore $r = 0$. The government can subsidize the storage technology in order to induce a positive equilibrium interest rate $r > 0$. The subsidy is paid for via lump-sum taxation at date 1, and introduces a deadweight loss $\mathcal{L}(r)$ into households' utility, where $\mathcal{L}(0) = 0$.¹⁰ I assume that the government never raises interest rates above the

¹⁰For clarity of exposition, I treat the government's budgets for subsidizing storage on one hand, and for bailing out the bank on the other hand, as separate. I also assume that the deadweight loss from interest rate distortions is independent of the magnitude of distortive taxation for the purpose of bailouts. These assumptions can easily be relaxed without changing the qualitative insights.

required return ρ on bank equity, so that $r \leq \rho$.¹¹ In Appendix B, I present a full formal treatment of welfare in this model.

A key new feature is that a positive interest rate affects the bank's average cost of capital independently of its beliefs. In particular, if an exuberant bank borrows b per unit of investment, then the equivalent of Tobin's q is now

$$\hat{q}(b, r) = \frac{1}{1 + \rho} \int_{\theta \geq (1+r)b} (\theta - (1+r)b) d\hat{F}(\theta)$$

As before, this represents the bank's own valuation of its equity tranche, but now includes the fact that a higher interest repayment rb at date 1 will diminish this value.

Hence, the bank's optimal investment decision is now sensitive to monetary policy as well as capital regulation.

Lemma 2. *Let $\hat{i}(b, r)$ denote the bank's optimal investment under exuberant beliefs when interest rates are r and permitted leverage is b . Then the sensitivity of the bank's optimal investment $\hat{i}(b, r)$ to policy is given by*

$$\frac{\partial \hat{i}}{\partial b} = a \left[\frac{\rho - r + (1+r)\hat{F}((1+r)b)}{1 + \rho} \right] \quad (18)$$

for a change in permitted leverage b , and by

$$\frac{\partial \hat{i}}{\partial r} = -a \frac{1}{1 + \rho} b (1 - \hat{F}((1+r)b)) \quad (19)$$

for a change in monetary policy r .

Equation (18) simply generalizes the sensitivity of investment to permitted leverage (i.e., Equation (13) above). In common with the baseline model, this depends on the bank's assessment of downside risk, particularly the probability $\hat{F}((1+r)b)$ of receiving a bailout. As before, bank investments become insensitive to capital regulation when the bank neglects downside risk.

Equation (19) conveys an important positive implication of the model with monetary policy. It characterizes how the bank responds to contractionary monetary policy. Not surprisingly, an increase in the interest rate r lowers investment, because it increases the cost of leverage. Crucially, the role of beliefs in the response to monetary policy is *opposite*

¹¹This assumption simplifies the analysis because it implies that the bank continues to discount future cash flows at rate ρ . If $r > \rho$, by contrast, then the bank would not consume at date 0 and instead invest in storage. The effective discount rate in the bank's objective function would then be the return r on storage.

from the response to capital regulation. Indeed, the response to monetary policy scales with the perceived probability $1 - \hat{F}((1+r)b)$ of remaining solvent. Intuitively, this follows because the cost of leverage only affects the bank's utility in states of the world where it repays its debt from its own pocket, as opposed to receiving a bailout. Consequently, monetary policy is most effective for curbing exuberant investment when the bank neglects downside risk.

Lemma 2 therefore underlines that capital regulation becomes ineffective in times of neglected downside risk, and points to monetary policy as a particularly useful substitute. I now confirm this intuition in a more rigorous welfare analysis. I define the fiscal burden $\phi(b, r)$, the upside wedge $\tau(b, r)$ and the downside wedge $\delta(b, r)$ as before, allowing for non-zero interest rates:

$$\begin{aligned}\tau(b, r) &= \hat{q}(b, r) - q(b, r) \\ \delta(b, r) &= F((1+r)b) - \hat{F}((1+r)b) \\ \phi(b, r) &= (1 + \kappa) \int_{\theta < (1+r)b} ((1+r)b - \theta) dF(\theta)\end{aligned}$$

With this notation, I obtain the following result:

Proposition 7. *Capital regulation is more desirable with exuberance (i.e., the marginal welfare benefit $\frac{dW}{db}$ of permitting more leverage is smaller with exuberance than in the rational benchmark) if and only if*

$$\left[(1 + \kappa)F((1+r)b) + \frac{r}{1+r} \right] \tau(b) \geq [\phi(b, r) + \tau(b, r) - rb] \frac{\delta(b)}{1 + \rho} \quad (20)$$

Moreover, suppose that $\tau(b, r) + \phi(b, r) - br > 0$, and that the true probability of failure is $F(b) < \frac{1}{1+\kappa}$. Then contractionary monetary policy is more desirable with exuberance whenever $\tau(b, r) \geq 0$ and $\delta(b, r) \geq 0$, with at least one strict inequality.

The first part of the proposition generalizes Proposition (4) to the case of positive interest rates. The intuition is the same as before: Stricter capital regulation is beneficial if and only if upside optimism is large relative to downside optimism. By contrast, the second part emphasizes that monetary policy is valuable even when capital regulation is not. Under two mild sufficient conditions, overly optimistic beliefs (with at least one strictly positive wedge) *always* strengthen the case for contractionary monetary policy, regardless of the type of optimism.

The first sufficient condition for this result is $\tau(b, r) + \phi(b, r) - br > 0$. This is equivalent to saying that the bank has a stronger incentive to make risky investments than the government. Indeed, the first part, $\tau(b, r) + \phi(b, r)$, denotes the standard wedge between private and social

incentives to invest (see, for example, Proposition 3). In a model with monetary policy, there is an additional term: When the government raises interest rates, it forces banks to pass on a fraction of the surplus from investment to households. This fraction is equal to $br \times i$. Hence, there is a marginal surplus of br per unit of investment that the bank does not internalize, leading it to understate the social value of investment. Consequently, the incentive wedge has to be adjusted downwards by the surplus effect br . The condition in the proposition focuses our attention on the case where r is relatively small, so that the surplus effect does not outweigh the distortions introduced by bailouts and exuberance. The second sufficient condition is that the true probability of failure $F(b)$ is not too large. This is quite a weak requirement: For instance, standard estimates put the deadweight loss from taxation between 10% and 20% (e.g., Dahlby, 2008). In this region, the condition merely says that the true probability of bank failure is less than about 80%.

In summary, if downside optimism is dominant, we know that the government wishes to curb bank investment, but cannot do so using capital regulation when the bank neglects the possibility of failure. Monetary policy remains effective in these situations, precisely because it renders investment costly for the bank in states of the world in which it does not fail. In this reduced-form treatment, I have not considered other benefits of monetary policy, such as the stabilization of demand in an economy with sticky prices. In recent work, Caballero and Simsek (2019) show that such concerns do not mitigate the case for monetary policy, if required, to assume a prudential role. Indeed, if monetary policy is set optimally from the perspective of aggregate demand management, then an envelope argument implies that the social costs of small interest rate increases are second-order. By contrast, the social benefits of raising interest rates to take excessive risky investment are first-order. In their model, prudential monetary policy therefore becomes desirable if (and only if) there are exogenous limits to the ability to conduct traditional macroprudential policy.

My analysis in this section strengthens this point considerably. Without any exogenous limits to capital regulation, Proposition 7 shows that monetary policy is particularly effective at times when traditional prudential regulation is not. This conclusion arises because capital regulation becomes *endogenously* ineffective when banks neglect downside risk.

7 Quantitative Illustration

I use recent empirical estimates to gauge the plausible regions for the sufficient statistics for exuberance, namely, the upside and downside wedges $\tau(b)$ and $\delta(b)$. Two strands of the literature on beliefs in behavioral economics are useful here, and provide somewhat independent quantifications: Empirical work on expected returns (e.g., Greenwood and Hanson,

2013; Baron and Xiong, 2017) and more structural work on non-Bayesian belief formation (e.g., Bordalo et al., 2017).

7.1 Measuring wedges

I first describe how to map the model to data on expected returns. Under some weak assumptions, I can relate the upside wedge $\tau(b)$ to measurable quantities from the analysis of bank equity returns in Baron and Xiong (2017). In particular, assume that the measured market value of bank equity in the data is $p_0 \leq \hat{q}(b)i$. In other words, the bank is being valued by exuberant equity investors, and its market value weakly below the exuberant expectation of future cashflows to equity-holders (where the difference can account for any positive risk premium). The true (rational) expected payoff to equity, by contrast, is $\bar{p}_1 = q(b)i$. The true expected return on equity, which an econometrician would measure, is therefore $\bar{R}_e = \frac{\bar{p}_1 - p_0}{p_0}$, and it is easy to see that it satisfies

$$\tau(b) \geq -\bar{R}_e \cdot \frac{p_0}{i} \quad (21)$$

Thus, the upside wedge can be bounded below by the negative of true expected returns on bank equity times the bank's market-to-book ratio $\frac{p_0}{i}$. This is not very informative when the expected return \bar{R}_e on bank stocks is positive. However, Baron and Xiong (2017) argue that \bar{R}_e is *negative* during significant credit expansions. For instance, conditional on bank credit expansions above the 95th percentile, they estimate a value of \bar{R}_e between -9% and -12% .¹² Minton et al. (2017) report that the market-to-book ratio for banks is fairly stable over time and across different types of bank, with an average value of about 1.7. Substituting into (21), these estimates imply a lower bound for $\tau(b)$ in credit expansions that is between 0.153 ($= 0.09 \cdot 1.7$) and 0.2.

One can further use the analysis of bond returns in Greenwood and Hanson (2013) to bound the magnitude of the downside wedge $\delta(b)$. This is more difficult and requires additional structural assumptions. Indeed, if investors expect bailouts (as they do in my model), then the prices of bank bonds will not contain much information about their perception of downside risk. One approach to this problem is to measure the wedge between perceived and true default risk for a wider benchmark portfolio of bonds, issued by firms without a bailout subsidy, and then to assume that this is a valid proxy for the associated wedge in the banking sector. Suppose that there is a portfolio of bonds with the same true proba-

¹²The measured value of excess returns following credit booms tends to depend on the horizon. The 2-year conditional mean return reported in Baron and Xiong (2017) is about -18% , and the 3-year return is about -37% .

bility $F(b)$ of default, and the same perceived probability $\hat{F}(b)$, as in the banking sector. Assume that the measured market value of this portfolio in the data, per unit of face value, is $v_0 \leq 1 - \hat{F}(b) \cdot \lambda$. That is, bonds are priced at par minus the probability of default times the loss given default, which I denote by λ , with the inequality again allowing for any risk premium. The true (rational) expected payoff is $\bar{v}_1 = 1 - F(b) \cdot \lambda$. The true expected return on bonds is therefore $\bar{R}_b = \frac{\bar{v}_1 - v_0}{v_0}$, and we can rearrange this to get

$$\begin{aligned} \delta(b) &\geq -\bar{R}_b \cdot \frac{v_0}{\lambda} \\ &= -\bar{R}_b \cdot \frac{1}{(1+Y)\lambda}, \end{aligned} \tag{22}$$

where Y is the bond’s yield to maturity. As with the bound on the upside wedge in (21), this inequality is useful only when expected returns \bar{R}_b are negative. Greenwood and Hanson (2013) show that this is the case when the share of high-yield bonds in new issuances is high. In particular, their estimates imply that, conditional on the high-yield share reaching its 95th percentile, the expected return is about -6.3% on high yield bonds, -1.3% on lower-investment grade (BBB/Baa) bonds, and -0.49% on AAA-rated bonds.¹³ Together with standard values of bond yields and loss given default,¹⁴ this implies a lower bound for $\delta(b)$ in credit booms that is between 0.02 (for the BBB benchmark group) and 0.07 (for high yield).¹⁵

As an alternative approach, I use an explicit model of non-rational, “diagnostic” beliefs, which Bordalo et al. (2017) argue is consistent with survey data on forecasters’ beliefs about credit conditions. Suppose that banks’ return on assets θ is an AR(1) process over time, with $\theta_t = \mu + \rho\theta_{t-1} + u_t$. The rational one-period-ahead forecast of θ_{t+1} is $E_t\theta_{t+1} = \rho\theta_t$. The diagnostic forecast is $\hat{E}_t\theta_{t+1} = E_t\theta_{t+1} + \eta n_t$, where $n_t = E_t\theta_{t+1} - E_{t-1}\theta_{t+1}$ is the rational update in beliefs (or “news”) that arrives at date t . This formulation leads agents to extrapolate from good news in proportion to the diagnosticity parameter η . Estimates that are consistent with various datasets imply $\eta \in [0.5, 1]$ (e.g., Bordalo et al., 2017; D’Arienzo, 2019). Assuming that shocks u_t are Gaussian, Bordalo et al. (2017) show that rational and

¹³This is based on a value of the high-yield share 1.64 standard deviations above its mean (see Greenwood and Hanson (2013), Table 1) along with their estimates of predictive regressions of excess bond returns on the high-yield share between 1983 and 2008 (Table 2).

¹⁴Historical average yields are 5.7% for AAA, 7.7% for BBB and 8.8% for high yield (see data published by the Federal Reserve at <https://fred.stlouisfed.org>; series identifiers AAA, BAA, and BAMLH0A0HYM2EY). Standard industry assumptions for recovery rates $(1 - \lambda)$ are around 80% for AAA, 40% for BBB and 20% for high yield (see, for example: <https://www.moody.com/sites/products/defaultresearch/2003000000439818.pdf>).

¹⁵The bound implied for AAA bonds is in between at $\delta(b) \geq 2.32\%$. This is due to the high recovery rate (low λ) in this class.

diagnostic beliefs about future θ_{t+1} are also Gaussian, with the means described above and variance $Var(u_t)$. I estimate an AR(1) process for banks' annual return on assets from the FDIC's call reports between and simulate one-year-ahead rational and diagnostic beliefs about θ_{t+1} over a sample between 1990 and 2013, with parameter $\eta = 0.75$.¹⁶ I assume that the value of debt-to-assets is $b = 0.9$, consistent with the call reports, to calculate $\delta(b)$ and $\tau(b)$ at each date.¹⁷ I then calculate average values of $\delta(b)$ and $\tau(b)$ conditional on news n_t being above its 95th percentile. This yields estimated values of $\tau(b) = 0.03$, which is slightly below the range implied by expected returns. The same exercise yields $\delta(b) = 0.001$, which is much lower than implied by the Greenwood and Hanson (2013) estimates. One reason for this difference may be that the call reports imply very little volatility in returns on assets. Indeed, recent papers seeking realistic values for the probability of bank failure (e.g., Begenau, 2019; Davila and Walther, 2019) use alternative calibrations for risky returns which imply much more volatility. In this sense, the estimates obtained by this calibration are likely to be a lower bound.

7.2 Exuberance, Capital Regulation and Monetary Policy

In order to see what these measured wedges imply for capital regulation, it is useful to make the following observation:

Corollary 1. *Capital regulation is always more desirable with exuberance (i.e., Conditions (17) in the baseline model, and (20) in the model with monetary policy, are guaranteed to hold) if the upside wedge $\tau(b)$ satisfies:*

$$\tau(b) \geq \frac{\phi(b)}{(1 + \rho)(1 + \kappa) - 1} \equiv \tau^*$$

In other words, if upside exuberance exceeds a critical level τ^* , then regulation should lean against the wind, even if banks completely ignore downside risk. This follows directly

¹⁶I define return on assets θ_{it} as net income in year t divided by assets in the first quarter of year t , for all banks i in the call reports. Estimates are: $\hat{\mu} = 0.003$, $\hat{\rho} = 0.792$ and $st.\hat{dev.}(u_t) = 0.026$.

¹⁷With Gaussian returns, the downside wedge is

$$\delta(b) = \Phi\left(\frac{b - E_t\theta_{t+1}}{\sigma}\right) - \Phi\left(\frac{b - \hat{E}_t\theta_{t+1}}{\sigma}\right)$$

Using the properties of truncated Gaussian variables, the equity valuation is

$$q(b) = P[\theta \geq b]E[\theta|\theta \geq b] = \left(1 - \Phi\left(\frac{b - \mu}{\sigma}\right)\right)\mu + \phi\left(\frac{b - \mu}{\sigma}\right)\sigma$$

with an analogous expression for $\hat{q}(b)$. This yields $\tau(b) \equiv \hat{q}(b) - q(b)$.

from Propositions 4 and 4, and by noting that the downside wedge $\delta(b) = F(b) - \hat{F}(b)$ cannot be larger than the true probability $F(b)$ of bank failure.

Figure 2, panel (a), shows values of τ^* as a function of parameters. On the horizontal axis is the social cost ρ of bank equity. On the vertical axis is the expected fiscal burden $\phi(b)$. I hold the marginal cost of public funds constant at $\kappa = 0.13$, which is a standard estimate in public finance (e.g., Dahlby, 2008), and set the interest rate to $r = 0$ as in the baseline model. The labels on the contours correspond to values of τ^* . The figure suggests that the measurements of $\tau(b)$ derived above, which are between 0.05 and 0.2, lie above the critical value τ^* unless the expected fiscal burden is roughly larger than one percent ($\phi = 0.01$) of bank assets. This is quite a large value for the fiscal burden. Indeed, in developed countries, the data suggests that there is a financial crisis once every 20 years on average (e.g., Reinhart and Rogoff, 2009), and that the average magnitude of bailouts conditional on a crisis is around 4% of bank assets (Laeven and Valencia, 2013), which would imply $\phi \simeq 0.002$. However, a number of recent studies have shown that, during credit booms, the risk of financial crises becomes significantly larger than these average values.¹⁸

Figure 2, panel (b), considers a scenario where the government believes that there is heightened crash risk. This calibration has an expected fiscal burden of 2.5 percent ($\phi = 0.025$) of bank assets, and true probability of bank failure of 10 percent.¹⁹ We have the upside wedge $\tau(b)$ on the horizontal axis, and the downside wedge $\delta(b)$ is on the vertical. The heatmap in the background of the figure measures the welfare benefit of increasing bank capital (i.e., decreasing leverage b) as a function of these two wedges. In the red region (below the thick curve), capital regulation is more desirable with exuberance relative to a rational model. In the blue region (above the thick curve), it is less desirable. The thick curve between the regions is the boundary where exuberance does not affect the relevant welfare benefit $\frac{dW}{db}$. For the lower end of the above estimates $\tau(b) \in [0.03, 0.2]$, along with the upper range of estimates $\delta(b) \in [0.001, 0.07]$, this scenario implies that leaning against the wind can be counterproductive in this scenario. However, for a large part of the empirically relevant range, it appears that capital regulation should be stricter when banks are exuberant.

Importantly, even if the estimates leave it unclear which side of the boundary we are on, all of the above estimates imply that $\tau(b) > 0$ and $\delta(b) > 0$ in a typical credit boom. Hence, Proposition 7 implies that exuberance creates a rationale for contractionary monetary policy

¹⁸See, for example: Reinhart and Rogoff (2009); Borio and Drehmann (2009); Mendoza and Terrones (2012); Schularick and Taylor (2012). Brunnermeier et al. (2019) also find significantly elevated levels of systemic risk during financial booms.

¹⁹As before, I set $\kappa = 0.13$ and $r = 0$. I fix the social cost of equity at $\rho = 0.1$ in line with average estimates of equity costs in the literature (see, e.g., Davila and Walther, 2019)). The patterns in the figure are not sensitive to this choice.

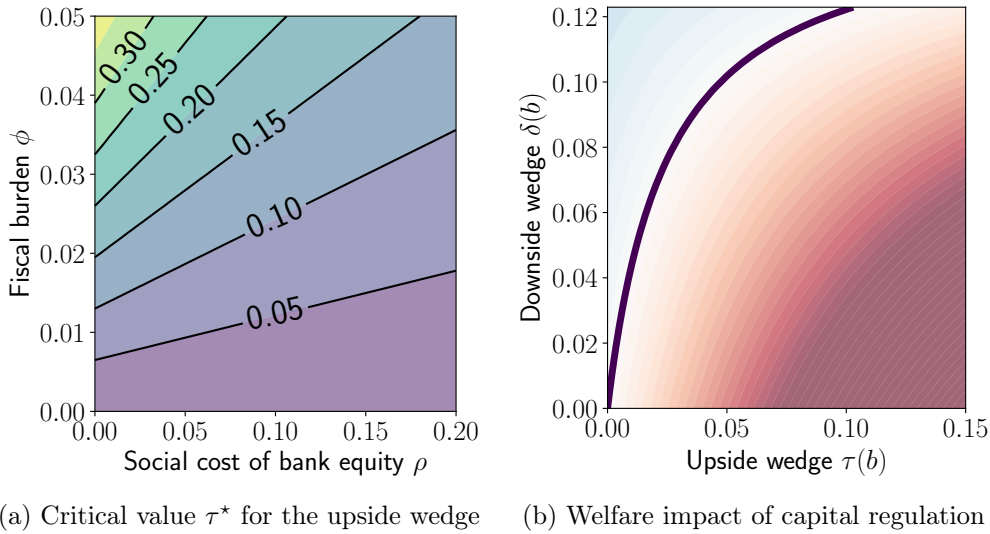


Figure 2: Quantitative Illustration

in credit booms for all reasonable calibrations of the model.

8 Optimal regulation without paternalism

This section considers two new variants of the model. First, in a setup that I call “semi-paternalism”, the government understands the true joint distribution of investment returns and banks’ sentiments, and anticipates the distortion in the banks’ beliefs due to sentiment, but still cannot make capital requirements contingent on the *realization* of sentiment. Second, in a model with a “behavioral government”, the government itself evaluates investment returns according to a distorted distribution.

8.1 Semi-paternalism

To reflect the idea that the government cannot always detect banks’ exuberance in real time, I study an extension of the model with a random variable $s \in [\underline{s}, \bar{s}]$, which indexes the bank’s sentiment about investment returns. Sentiment s is observed only by the bank. Consider a version of the model above where the timing of events is as follows: First, the government imposes a capital requirement without observing sentiment. Second, the bank observes s and makes its investment decision. Thereafter, the game unfolds as in the previous sections.

With sentiment s , the bank perceives investment returns to have the distribution $\hat{F}(\theta|s)$, while the true conditional distribution is $F(\theta|s)$. For example, in the case where the true distribution $F(\theta|s)$ does not depend on s , sentiment contains no real information. Otherwise,

s is a genuinely informative signal of returns, although banks' reactions to it may be too strong or too weak.

For concreteness, I assume that an increase in sentiment s induces a perceived distribution, as well as a true distribution, that are more optimistic about θ in the sense of first-order stochastic dominance: $\hat{F}(\theta|s)$ and $F(\theta|s)$ are both decreasing in s . I write $G(s)$ for the true marginal distribution of sentiment, and $F(\theta) = \int F(\theta|s)dG(s)$ and $\hat{F}(\theta) = \int \hat{F}(\theta|s)dG(s)$ for the average probabilities *ex ante* (by the law of iterated expectation, $F(\theta)$ is also the true unconditional distribution of θ).

The goal of this exercise is to isolate the effect of the government's uncertainty about sentiment, which distinguishes this case from the paternalist one I have already analyzed. For this reason, I compare the expected welfare effect of allowing more leverage under semi-paternalism to a benchmark scenario where the bank holds the average exuberant beliefs $\hat{F}(\theta)$ with probability one. Let $W(b)$ continue to denote welfare in this benchmark without uncertainty, and with a small abuse of notation, let $W(b|s)$ be realized welfare in case the bank is permitted leverage b and has sentiment s . Similarly, I write $\hat{i}(b)$, $\hat{q}(b)$ and $\tau(b)$ for investment, Tobin's q and the behavioral wedge in the benchmark, and their counterparts conditional on sentiments are denoted $\hat{i}(b|s)$, $\hat{q}(b|s)$ and $\tau(b|s)$. The expected fiscal burden conditional on s is denoted $\phi(b|s)$ (with formal definitions in the appendix).

The welfare effect of permitting more leverage under semi-paternalism is distinguished from the benchmark by two covariance terms:

Lemma 3. *Compared to a benchmark where the bank holds the average beliefs $\hat{F}(\theta)$ with certainty (and which has welfare function $W(b)$), the effect on expected welfare of permitting more leverage under semi-paternalism satisfies*

$$\begin{aligned}
E \left[\frac{dW(b|s)}{db} \right] &= \frac{dW(b)}{db} \\
&+ Cov \left[\frac{\rho}{1+\rho}(1 - F(b|s)) - \kappa F(b|s), \hat{i}(b|s) \right] \\
&- Cov \left[\tau(b|s) + \phi(b|s), \frac{\partial \hat{i}(b|s)}{\partial b} \right] + \xi,
\end{aligned} \tag{23}$$

where ξ is an approximation error, and is proportional to the deviation of investment costs from a quadratic function.

This has an intuitive interpretation. The first covariance is between the marginal benefit of leverage, which trades off gains from trade against a buffer stock of bank equity, and the scale of the bank's investment. If this is positive, then the social benefit of leverage per

unit of investment are particularly large in states of the world where investment is booming. This weakens the case for capital requirements. The second covariance is between the total incentive wedge $\tau(b|s)+\phi(b|s)$, which measures the deviation of the bank’s incentives from the planner’s, and the sensitivity of the bank’s investment to leverage. Intuitively, it measures the co-movements between the government’s *desire* to control incentives, and the *effectiveness* of the tool (i.e., capital requirements) at its disposal. If this is negative, then in states of the world where it would be good to discourage investment, the bank does not respond strongly. This further weakens the case for capital regulation.

Next, I derive conditions under which I can determine the sign of these covariances:

Proposition 8. *Consider the semi-paternalist model where the government cannot make capital requirement contingent on the bank’s sentiment. Let*

$$\eta(s, \theta) = \frac{\partial \hat{F}(\theta|s)/\partial s}{\partial F(\theta|s)/\partial s}$$

denote the responsiveness the bank’s beliefs to sentiment, relative to the true conditional distribution of investment returns θ . If this sensitivity is bounded from below by $\eta(s, \theta) \geq \eta_0 > 1$, and if η_0 is large enough, then the two welfare effects in Lemma 3 satisfy:

$$E \left[\frac{dW(b|s)}{db} \right] > \frac{dW(b)}{db},$$

so that permitting more leverage is more beneficial when the regulator is uncertain about sentiment.

The economic interpretation is as follows: In an exuberant world, where we cannot fine-tune capital requirements in real time, it is ineffective to impose high capital requirements ex ante. There are two reasons for this, which correspond to the two covariances in (23), and once again reflect the decomposition of welfare effects into the “buffer” and “incentive” rationales. First, the social cost of capital regulation is amplified by uncertainty, because bank leverage is particularly socially valuable in states of the world where the bank operates at a large scale. Second, in states where it is socially beneficial to control incentives, i.e. when exuberance is strong (large s), the bank’s investments choices are actually insensitive to capital requirements. The government’s desire to regulate is negatively correlated with the effectiveness of its tools. This effect partially defeats the point of capital regulation.

In addition to this simple argument, the proposition needs to impose a bound on the sensitivity of beliefs to sentiment. To see this, it is useful to discuss the mathematics of the proof. The first (easier) half of the proof shows that the first covariance term in (23) is always

positive (with strict inequality whenever sentiment s contains some true information). This is because the “buffer” argument for preventing leverage is less relevant in good times (i.e., high s), which is exactly when the bank chooses larger investment scales and hence creates larger gains from trade. The second half shows that the second covariance in (23) is negative as long as the bank’s beliefs are sufficiently responsive – relative to the true ones – to sentiment. There are two competing forces. On one hand, the bank’s behavioral wedge $\tau(b|s)$, which measures the deviation of its equity valuation from the truth, is larger in good times as long as $\eta(s, \theta) > 1$. Its slope with respect to s is proportional to the relative responsiveness of the bank’s beliefs to sentiment. On the other hand, the expected bailout $\phi(b|s)$ is larger in bad times, but its slope with respect to s is proportional to the responsiveness of *true* beliefs to sentiment. If the bank’s relative responsiveness is large enough, the former effect dominates, which implies that the first term in the covariance is increasing in s and, hence, correlates negatively with the sensitivity $\frac{\partial i(b|s)}{\partial b}$ of optimal investments to leverage.

Another interesting feature to note is that the sufficient conditions in the proposition have no bite in a rational world. The proposition focuses on cases where the bank’s beliefs are more responsive to sentiment than the true ones, i.e., $\eta(s, \theta) > 1$. In a rational world, by contrast, we have $\eta(s, \theta) \equiv 1$. In other words, the bank reacts to the true information contained in sentiment s , then the behavioral wedge $\tau(b|s) = 0$, and the government’s desire to control incentives is dominated by the expected bailout $\phi(b|s)$. This term is large in bad states of the world, when the sensitivity $\frac{\partial i(b|s)}{\partial b}$ is also large. Hence, in a rational world, uncertainty can generate a *stronger* case for capital regulation, because there is no trade-off between desire to regulate and policy effectiveness. The new result, in the context of this paper, is that concerns about exuberance can reverse this argument force and, hence, make capital requirements less attractive.

8.2 Exuberant government

Now I return to the baseline model of exuberance from Section 4, where banks hold fixed and distorted beliefs $\hat{F}(\theta)$,²⁰ but assume that the government itself agrees with these beliefs and calculates welfare accordingly. Also assume that both the government and the bank are exuberant, that is, that $\hat{F}(\theta)$ is more optimistic than the true distribution $F(\theta)$ in the sense of first-order stochastic dominance.

Write $\hat{W}(b)$ for the behavioral government’s perception of welfare. By a similar argument

²⁰It is also easy to analyze the case with uncertain sentiments s , as in the last subsection, with an exuberant government. This analysis is available on request, but not many new insights emerge from it.

to Proposition 1, the expected welfare effect of permitting more leverage in this world is

$$\frac{d\hat{W}}{db} = \left[\frac{\rho}{1+\rho}(1 - \hat{F}(b)) - \kappa\hat{F}(b) \right] \hat{i}(b) - \hat{\phi}(b) \frac{\partial \hat{i}}{\partial b}, \quad (24)$$

Comparing this to the welfare effect perceived by a rational government in (16), there are three main differences. First, the government's beliefs $\hat{F}(b)$ affect the perceived mechanical welfare effect of more leverage (the first term in (24)). Second, the incentive effect of more leverage (the second term in (24)) no longer contains the behavioral wedge $\tau(b)$. Since the government agrees with the bank's probability assessment, the only reason to push the bank towards lower investment is to reduce the expected bailout. Third, the expected bailout itself is evaluated according to the government's distorted beliefs, written here as $\hat{\phi}(b)$.

Since the behavioral government is more optimistic than the rational one, it is easy to see that all three effects go in the same direction:

Proposition 9. *If both the bank and the government are exuberant, the perceived welfare effect of increasing leverage is always smaller than in the paternalist model where the government is rational.*

The intuition is straightforward: All the costs of permitting leverage arise in bad states of the world. Hence, an exuberant government who discounts these states has greater incentives to permit leverage than a more cautious government who evaluates welfare according to the true distribution of investment returns.

To summarize, the lessons from this section are twofold: First, in the semi-paternalist world where the government is rational but cannot spot exuberance in real time, capital regulation is less effective, on average, than in the fully paternalist benchmark. This is because capital requirements are especially ineffective in states of the world where steep incentives are needed. Second, as would be expected, the case for capital requirements is weaker when the government itself becomes exuberant.

9 Conclusion

To conclude, this paper provides a formal analysis of financial and monetary policy in a world where private financial institutions are subject to bouts of irrational exuberance. Based on recent evidence, this may be a relevant case to consider alongside the standard, incentive-based rationale for financial regulation. I have taken a price-theoretic approach to give insights into the welfare effects of bank capital regulation in an exuberant world. At a

high level, my results yield two sets of conclusions – one normative and one positive – and associated directions for future research.

On the normative side, it is not always clear that “leaning against the wind” with countercyclical capital requirements is the optimal policy when the private financial sector is exuberant and optimistic about the future returns to investment. Indeed, the rationale for this policy is very nuanced and depends on the *nature* as well as the *extent* of optimism. If optimism focuses on neglected downside risk, or if banks understate the variability of returns, capital requirements actually become less desirable in exuberant times. Monetary policy, on the other hand, remains an effective tool in exactly this situation.

The positive implication of my analysis is that we should expect capital requirements to reduce bank risk taking, and to smooth out credit cycles at the margin. However, we should not expect them to be effective in terms of curbing the most exuberant credit booms, or in terms of preventing the crises that tend to follow such booms. This testable prediction could be used to compare and reconcile different results in the existing empirical literature.

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A Proofs

A.1 Proof of Proposition 4

Define $\Delta(b)$ as the difference in the welfare effects of permitting more leverage when banks are exuberant and when they are rational:

$$\Delta(b) \equiv \frac{dW(\hat{i}(b), b)}{db} - \frac{dW(i(b), b)}{db}$$

From Propositions 1 and 3, we have

$$\begin{aligned} \Delta(b) = & \left[\frac{\rho}{1+\rho}(1-F(b)) - \kappa F(b) \right] [\hat{i}(b) - i(b)] \\ & - \phi(b) \left[\frac{\partial \hat{i}}{\partial b} - \frac{\partial i}{\partial b} \right] - \tau(b) \frac{\partial \hat{i}}{\partial b} \end{aligned}$$

With quadratic adjustment costs,

$$\hat{i}(b) - i(b) = a(\hat{q}(b) - q(b)) = a\tau(b)$$

and

$$\frac{\partial \hat{i}}{\partial b} = a \frac{\hat{F}(b) + \rho}{1 + \rho}, \quad \frac{\partial i}{\partial b} = a \frac{F(b) + \rho}{1 + \rho}$$

Simplifying yields

$$\Delta(b) = a \left\{ \frac{\phi(b)}{1+\rho} \delta(b) - \left[(1+\kappa)F(b) - \frac{1}{1+\rho} \delta(b) \right] \tau(b) \right\} \quad (25)$$

and, hence, $\Delta(b) \leq 0$ if and only if (17) holds, as required.

A.2 Proof of Proposition 5

With first-order stochastic dominance, we have $\delta(b) \geq 0$ and $\tau(b) \geq 0$. Holding constant all parameters other than $\tau(b)$, we see that Condition (17) (which is linear in $\tau(b)$) fails when $\tau(b) = 0$ but holds when $\tau(b) \rightarrow \infty$. This establishes the required claim.

A.3 Proof of Proposition 6

With second order stochastic dominance, we have $\delta(b) \geq 0$ for small enough b ; let $\bar{b} = \inf \{b : \delta(b) < 0\}$. Moreover, noting that $\hat{q}(b)$ is the expectation of the convex function $(\theta - b)^+$ under $\hat{F}(\theta)$, we have $\tau(b) \leq 0$. Hence, the left-hand side of condition (17) is positive, while the right-hand side is negative, so the condition holds. Combining with Lemma 4 establishes the claim in the proposition.

A.4 Proof of Proposition 7

Define the difference in the welfare effects of permitting more leverage when banks are exuberant and when they are rational:

$$\Delta_b(b, r) \equiv \frac{dW(\hat{i}(b, r), b, r)}{db} - \frac{dW(i(b, r), b, r)}{db}$$

Using Lemma 5 (stated in Appendix B below), and repeating the arguments of this lemma for rational beliefs, we get

$$\begin{aligned} \Delta_b(b, r) &= (1+r) \left[\frac{\rho}{1+\rho} (1 - F((1+r)b)) - \kappa F((1+r)b) \right] [\hat{i}(b, r) - i(b, r)] \\ &\quad - [\phi(b, r) - rb] \left[\frac{\partial \hat{i}}{\partial b} - \frac{\partial i}{\partial b} \right] - \tau(b, r) \frac{\partial \hat{i}}{\partial b} \end{aligned} \quad (26)$$

With quadratic adjustment costs, we have

$$\hat{i}(b, r) - i(b, r) = a\tau(b, r)$$

and, from Equation (18),

$$\frac{\partial \hat{i}}{\partial b} - \frac{\partial i}{\partial b} = -a \frac{1+r}{1+\rho} \delta(b, r)$$

Substituting into (26) and simplifying now gives

$$\Delta_b(b, r) = a(1+r) \left\{ [\phi(b, r) + \tau(b, r) - rb] \frac{\delta(b, r)}{1+\rho} - \left[(1+\kappa)F((1+r)b) + \frac{r}{1+r} \right] \tau(b, r) \right\}$$

This establishes the first claim. Similarly, for the effect of interest rates, define

$$\Delta_r(b, r) \equiv \frac{dW(\hat{i}(b, r), b, r)}{dr} - \frac{dW(i(b, r), b, r)}{dr}$$

From Lemma 5 we get

$$\begin{aligned} \Delta_r(b, r) &= b \left[\frac{\rho}{1+\rho} (1 - F((1+r)b)) - \kappa F((1+r)b) \right] [\hat{i}(b, r) - i(b, r)] \\ &\quad - [\phi(b, r) - rb] \left[\frac{\partial \hat{i}}{\partial r} - \frac{\partial i}{\partial r} \right] - \tau(b, r) \frac{\partial \hat{i}}{\partial r} \end{aligned} \quad (27)$$

We can again use the fact that, with quadratic adjustment costs, $\hat{i}(b, r) - i(b, r) = a\tau(b, r)$. From Equation (19),

$$\frac{\partial \hat{i}}{\partial r} - \frac{\partial i}{\partial r} = -a \frac{b}{1 + \rho} \delta(b, r)$$

Substituting into (27) and simplifying now gives

$$\Delta_r(b, r) = ab \left\{ \tau(b, r) [1 - (1 + \kappa)F((1 + r)b)] + [\phi(b, r) + \tau(b, r) - rb] \frac{\delta(b, r)}{1 + \rho} \right\}$$

This immediately leads to the second claim.

A.5 Proof of Lemma 3

Proof. In the benchmark where the bank has beliefs $\hat{F}(\theta)$ with probability 1, the welfare effect $\frac{dW}{db}$ is given by (16). Conditional on sentiment s , by a parallel argument, we get

$$\begin{aligned} \frac{dW(b|s)}{db} &= \left[\frac{\rho}{1 + \rho} (1 - F(b|s)) - \kappa F(b|s) \right] \hat{i}(b|s) \\ &\quad - [\tau(b|s) + \phi(b|s)] \frac{\partial \hat{i}(b|s)}{\partial b} \end{aligned}$$

where

$$\hat{i}(b|s) = A(\hat{q}(b|s) - p + b)$$

and

$$\tau(b|s) = \int_{\theta \geq b} (F(\theta|s) - \hat{F}(\theta|s)) d\theta$$

and

$$\phi(b|s) = (1 + \kappa) \int_{\theta < b} (b - \theta) dF(\theta|s)$$

Taking expectations across $s \in [\underline{s}, \bar{s}]$,

$$\begin{aligned} E \left[\frac{dW(b|s)}{db} \right] &= E \left[\frac{\rho}{1 + \rho} (1 - F(b|s)) - \kappa F(b|s) \right] E [i(b|s)] \\ &\quad + Cov \left[\frac{\rho}{1 + \rho} (1 - F(b|s)) - \kappa F(b|s), i(b|s) \right] \\ &\quad - E [\tau(b|s) + \phi(b|s)] E \left[\frac{\partial i(b|s)}{\partial b} \right] \\ &\quad - Cov \left[\tau(b|s) + \phi(b|s), \frac{\partial i(b|s)}{\partial b} \right] \end{aligned}$$

By the law of iterated expectations,

$$E \left[\frac{\rho}{1+\rho}(1 - F(b|s)) - \kappa F(b|s) \right] = \frac{\rho}{1+\rho}(1 - F(b)) - \kappa F(b)$$

and

$$\begin{aligned} E[\tau(b|s) + E[\phi|s]] &= \int_s \int_{\theta \geq b} \left(F(\theta|s) - \hat{F}(\theta|s) \right) d\theta dG(s) + \phi(b) \\ &= \int_{\theta \geq b} \left[\int_s F(\theta|s) dG(s) - \int_s \hat{F}(\theta|s) dG(s) \right] d\theta + \phi(b) \\ &= \tau(b) + \phi(b) \end{aligned}$$

With quadratic costs, $A(x) = ax$, and

$$\begin{aligned} E[i(b|s)] &= a \\ E \left[\frac{\partial i(b|s)}{\partial s} \right] &= aE \left[\frac{\hat{F}(b|s) + \rho}{1 + \rho} \right] = a \frac{\hat{F}(b) + \rho}{1 + \rho} \\ &= E \left[\frac{\partial i(b)}{\partial s} \right] \end{aligned}$$

Combining the above, we obtain Equation (23). □

A.6 Proof of Proposition 8

Proof. By Lemma 3, it is sufficient to show that, under the proposed condition we have:

$$Cov \left[\frac{\rho}{1+\rho}(1 - F(b|s)) - \kappa F(b|s), i(b|s) \right] > 0 \quad (28)$$

$$Cov \left[\tau(b|s) + \phi(b|s), \frac{\partial i(b|s)}{\partial b} \right] \leq 0 \quad (29)$$

By first-order stochastic dominance, both variables in the first covariance in (28) are strictly increasing in s and, hence, the covariance is strictly positive.

For the covariance in (29), the second argument $\frac{\partial i(b|s)}{\partial b}$ is decreasing in s , using first-order stochastic dominance and (13). Recalling the definition of ϕ , we can write

$$\begin{aligned} \phi(b|s) &= (1 + \kappa) \int_{\theta \leq b} (b - \theta) f(\theta|s) d\theta \\ &= (1 + \kappa) \int_{\theta \leq b} F(\theta|s) d\theta \end{aligned}$$

We can therefore write the first argument of the covariance as

$$\xi(s) \equiv \tau(b|s) + E[\phi|s] = \int_{\theta \geq b} \left(F(\theta|s) - \hat{F}(\theta|s) \right) d\theta + (1 + \kappa) \int_{\theta \leq b} F(\theta|s) d\theta$$

Differentiating, and imposing the proposed condition on $\eta(s, \theta)$, we get

$$\begin{aligned} \xi'(s) &= \int_{\theta \geq b} (\eta(s, \theta) - 1) \left| \frac{\partial F(\theta|s)}{\partial s} \right| d\theta - (1 + \kappa) \int_{\theta \leq b} \left| \frac{\partial F(\theta|s)}{\partial s} \right| d\theta \\ &\geq (\eta_0 - 1) \int_{\theta \geq b} \left| \frac{\partial F(\theta|s)}{\partial s} \right| d\theta - (1 + \kappa) \int_{\theta \leq b} \left| \frac{\partial F(\theta|s)}{\partial s} \right| d\theta \end{aligned}$$

For any given family of true conditional distributions $F(\theta|s)$, it follows that $\xi'(s)$ is strictly positive for large enough η_0 , which completes the proof. \square

B Analysis with Monetary Policy

Consider the model with monetary policy defined in Section 6. I write s_0 and s_1 for households' endowments (savings) at date 0 and 1, respectively, and \hat{s}_0 for the bank's endowment at date 0. The following result characterizes welfare in this model:

Lemma 4. *In any equilibrium where banks invest i , borrow b per unit of investment, and where the interest rate is $r > 0$, expected welfare is a constant plus*

$$W(i, b, r) = \pi(i, b, r) + rbi - \phi(b, r)i - \mathcal{L}(r)$$

where the bank's profits are

$$\pi(i, b, r) = \frac{1}{1 + \rho} \int_{\theta \geq (1+r)b} (\theta - (1+r)b)idF(\theta) + bi - pi - c(i)$$

Proof. Whenever $r > 0$, it is optimal for households to invest all their endowment in either the bank or storage. Thus households' consumption plan is $c_0 = 0$ and

$$c_1 = s_1 + (1+r)(s_0 - bi) + (1+r)bi - T - (1+\kappa)((1+r)b - \theta)^+ i - \mathcal{L}(r)$$

The government's budget for monetary policy dictates that $T = r(s_0 - bi)$. Substituting and taking expectations, we find that households' expected lifetime utility is

$$U = c_0 + E[c_1] = s_0 + s_1 + rbi - \phi(b, r)i$$

Given the assumption that $r \leq \rho$, it is optimal for the bank to consume its endowment, net of any investments, at date 0. Hence the bank's consumption plan satisfies

$$\hat{c}_0 = \hat{s}_0 + bi - pi - c(i)$$

and

$$\hat{c}_1 = (\theta - (1 + r)b)^+ i$$

Taking expectations, the bank's lifetime utility becomes

$$\hat{U} = \hat{c}_0 + \frac{1}{1 + \rho} E[\hat{c}_1] = \hat{s}_0 + \pi(i, b, r)$$

Noting that aggregate welfare is $U + \hat{U}$, we obtain the required expression. \square

The next result extends the decomposition of welfare effects to the model with monetary policy:

Lemma 5. *The respective welfare effects of permitting more leverage, and of raising the interest rate, satisfy:*

$$\begin{aligned} \frac{dW}{db} &= (1 + r) \left[\frac{\rho}{1 + \rho} (1 - F((1 + r)b)) - \kappa F((1 + r)b) \right] \hat{i}(b, r) \\ &\quad - [\tau(b, r) + \phi(b, r) - rb] \frac{\partial \hat{i}}{\partial b} \end{aligned}$$

and

$$\begin{aligned} \frac{dW}{dr} &= b \left[\frac{\rho}{1 + \rho} (1 - F((1 + r)b)) - \kappa F((1 + r)b) \right] \hat{i}(b, r) \\ &\quad - [\tau(b, r) + \phi(b, r) - rb] \frac{\partial \hat{i}}{\partial r} \end{aligned}$$

Proof. The welfare effect of leverage policy in equilibrium is equal to

$$\frac{dW(\hat{i}(b, r), b, r)}{db} = \frac{\partial W}{\partial b} + \frac{\partial W}{\partial i} \frac{\partial \hat{i}}{\partial b}$$

Substituting the expression for welfare from Lemma 4, we have

$$\frac{dW(\hat{i}(b, r), b, r)}{db} = \frac{\partial \pi}{\partial b} + r\hat{i}(b, r) - \frac{\partial \phi}{\partial b} \hat{i}(b, r) + \left[\frac{\partial \pi}{\partial i} + rb - \phi(b, r) \right] \frac{\partial \hat{i}}{\partial b} \quad (30)$$

Evaluating the required derivatives, we have

$$\frac{\partial \pi}{\partial b} = \hat{i}(b, r) \left[1 - \frac{1+r}{1+\rho} (1 - F((1+r)b)) \right]$$

$$\frac{\partial \phi}{\partial b} = (1 + \kappa)(1 + r)F((1 + r)b)$$

and

$$\begin{aligned} \frac{\partial \pi}{\partial i} &= q(b, r) - (p + c'(i) - b) = \hat{q}(b, r) - (p + c'(i) - b) - [\hat{q}(b, r) - q(b, r)] \\ &= -[\hat{q}(b, r) - q(b, r)] \equiv -\tau(b, r) \end{aligned}$$

where the last equality follows from the bank's first-order condition. Substituting into (30) and simplifying, we get the required expression.

Similarly, the welfare effect of raising the interest rate is

$$\begin{aligned} \frac{dW}{dr} &= \frac{\partial W}{\partial r} + \frac{\partial W}{\partial i} \frac{\partial i}{\partial r} \\ &= \frac{\partial \pi}{\partial r} + b\hat{i}(b, r) - \frac{\partial \phi}{\partial r} \hat{i}(b, r) + \left[\frac{\partial \pi}{\partial i} + rb - \phi(b, r) \right] \frac{\partial \hat{i}}{\partial r} \end{aligned} \quad (31)$$

Evaluating the additional required derivatives, we have

$$\frac{\partial \pi}{\partial r} = -\hat{i}(b, r) \frac{b}{1+\rho} (1 - F((1+r)b))$$

$$\frac{\partial \phi}{\partial r} = (1 + \kappa)bF((1 + r)b)$$

Substituting into (31) and simplifying, we get the required expression. □