

# The Macroeconomic Impact of Money Market Disruptions\*

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## Abstract

We build a general equilibrium model featuring unsecured and secured interbank markets, and collateralized central bank funding. The model is calibrated and used to analyse the macroeconomic impact of four key developments observed in the European money markets since 2008: i) the reduced ability of banks to access the unsecured market since the onset of the financial crisis and the shift to secured market funding; ii) an impaired functioning of the secured market during the sovereign crisis; iii) increased fears of deposit runs in some peripheral countries; and iv) higher reliance of banks on central bank funding. In our model, disruptions in interbank markets, as observed during the financial and sovereign debt crises, generate a sizeable impact on real activity. We find that the central bank plays a key role in shielding the economy from this adverse impact.

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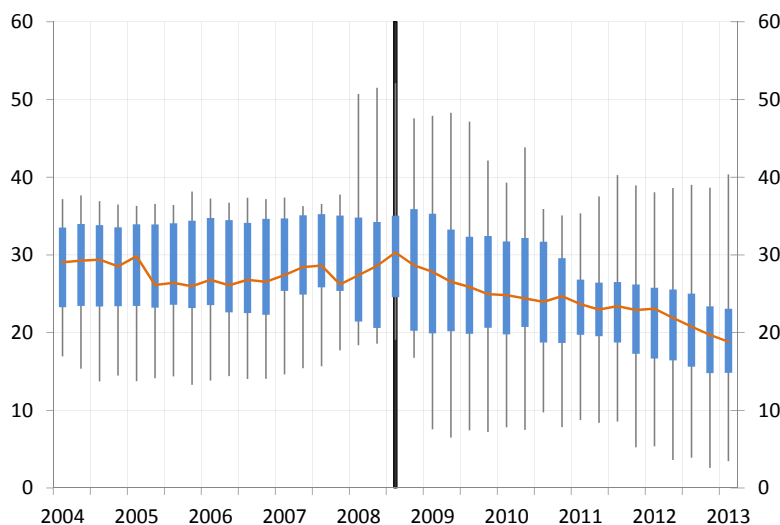
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# 1 Introduction

In this paper, we construct a dynamic general equilibrium model featuring heterogeneous banks, interbank money markets for both secured and unsecured credit, and a central bank providing funding against collateral. Interbank markets are essential to banks' liquidity management and play a key role in the transmission of monetary policy. These markets came under severe stress during the global financial crisis of 2007-2009 as well as during the euro area sovereign debt crisis of 2010-2012.<sup>1</sup> Our model can be used to assess the macroeconomic impact of the observed interbank market disruptions.

Our modelling approach is motivated by four stylized facts about euro area money markets. First, interbank money markets are an important funding source for banks in the euro area but their share in total bank funding has been diminishing since 2008 (see Figure 1). In 2008, the ratio of interbank liabilities to total assets was about 30%. This ratio started to decline with the onset of the global financial crisis, dipping below 20% by 2013. This downward trend reflects a decline in money market activity, with some market segments “freezing” or drying-up altogether.

Figure 1: Interbank liabilities in total assets



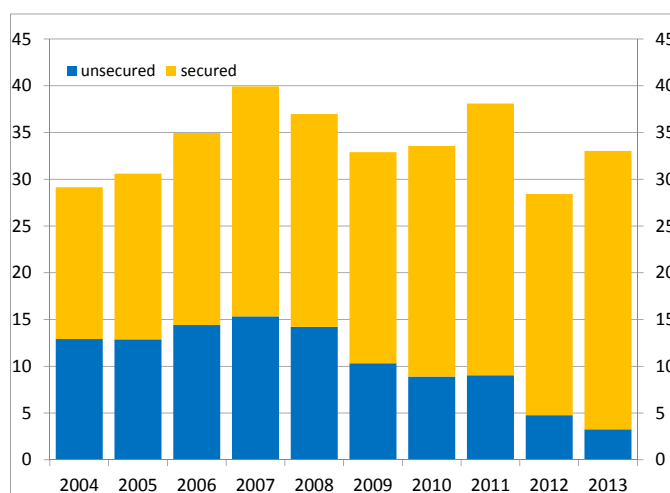
The grey vertical lines denote the range of observations across euro area countries. The blue vertical lines denote interquartile ranges. The red line denotes the unweighted average ratio. Source: ECB data.

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<sup>1</sup>The failure of the interbank market to redistribute liquidity was highlighted in a number of accounts of the recent crisis (see, for example, Allen and Carletti (2008) and Brunnermeier (2009))

Second, there was a dramatic shift away from unsecured and towards secured money market funding since 2008 (see Figure 2). The secured money market segment was nearly double that of the unsecured segment in 2008 in terms of transaction volumes. During the financial crisis, the share of secured funding grew, as some banks became unable to borrow in the unsecured markets due to perceptions of increased counterparty risk and shifted to secured borrowing instead. By 2013, the secured segment was ten times bigger compared to the unsecured segment.

Figure 2: Unsecured vs secured money market segments



Cumulative quarterly turnover in euro area money markets (EUR trillion). Source: Euro Area Money Market Survey. The panel comprises 98 euro area credit institutions.

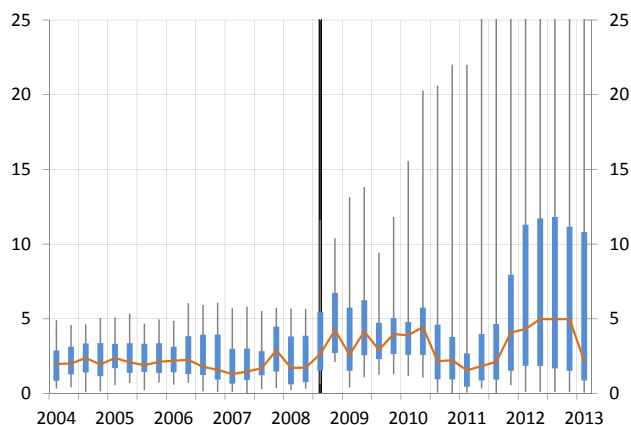
Third, in some peripheral countries, banks highly exposed to domestic government bonds experienced increases in solvency risk during the sovereign debt crisis. The ensuing fears of deposit runs generated high liquidity needs for those banks.<sup>2</sup>

Fourth, with private money markets malfunctioning, banks increasingly turned to the central bank for refinancing (see Figure 3). Reliance on central bank funding gradually rose in the euro area, particularly with the onset of the sovereign debt crisis. Banks located in euro area countries with vulnerable sovereigns started borrowing larger amounts from the ECB

<sup>2</sup>The Wall Street Journal report that on May 15, 2012, at the peak of the sovereign crisis, Greek depositors withdrew 700 million euros (amounting to 0.4 percent of total deposits) from the country's banks on a single day, fueling fears of a bank run. See also "Worrying about a Greek bank run," Reuters, April 15, 2010, <http://blogs.reuters.com/felix-salmon/2010/04/15/worrying-about-a-greek-bank-run/>.

and pledging riskier collateral, taking advantage of the more favorable haircuts on risky assets imposed by the ECB relative to the secured market (see Table 1).

Figure 3: Eurosystem funding in total deposit liabilities



The grey vertical lines denote the range of observations across euro area countries. The blue vertical lines denote interquartile ranges. The red line denotes the unweighted average ratio. Source: ECB data.

In our model, banks with a liquidity need can obtain funding in the secured or unsecured market, or at the central bank. Banks face an exogenous probability of being “connected,” defined as the ability to borrow in the unsecured market. Those banks that are unable to borrow in the unsecured market, the “unconnected” banks, can access the secured market or central bank funding. To access the secured market or central bank funding banks need to hold government bonds which can be pledged as collateral. At the beginning of each period, after knowing whether they are connected or unconnected, banks choose their liabilities (how much deposit and central bank funding to raise) and their assets (choosing between loans, bonds and cash). After making their asset-liability choices, banks face idiosyncratic deposit withdrawal shocks. Banks experiencing low withdrawals can lend funds in the secured or unsecured market. Banks experiencing high withdrawals can cover them with unsecured borrowing (connected banks), or the combination of collateralized borrowing and cash buffers (unconnected banks).

All collateralized borrowing is subject to a haircut, which can be different in the private secured market and at the central bank. If a bank loses access to the unsecured market but its government bond holdings are sufficiently valuable, it can replace unsecured funding with

secured funding. Our model can therefore capture the shift from the unsecured to secured funding observed in the recent years. However, if private counterparties become reluctant to accept a particular government bond as collateral (due to, e.g., concerns about that sovereign's health), access to the secured market will become impaired as well. The possibility that banks face different haircuts on private and central bank funding allows us to capture the fact that haircuts set by the central bank can be more favorable in crisis times compared to the private market haircuts for low-quality collateral. In such circumstances, banks holding such collateral may replace their lost unsecured and secured funding with borrowing from the central bank. In our model, the choice between secured market funding and central bank funding is driven by the comparison of the respective interest rates and haircuts charged in the secured market and at the central bank.

We calibrate the model on euro area data and use it to quantify the macroeconomic impact of money market disruptions. We also evaluate the role of the central bank in shielding the economy from the adverse consequences of such disruptions.

We analyze the macroeconomic impact of three alternative scenarios: 1) increased haircuts in the secured market; 2) reduced access to the unsecured money market; and 3) fears of increased deposit withdrawals. Our model suggests that the effects of these market disruptions on investment and output are sizeable.

In the first scenario - when haircuts in the secured market increase - the model economy moves between two regions. When private haircuts are low and collateral valuable, banks have access to the private funding markets (either secured or unsecured), and there is no recourse to central bank funding. When private haircuts increase beyond a certain threshold, banks relying on secured markets for funding become unable to cover their liquidity needs there as the value of their collateral drops. They access central bank funding instead. Specifically, as private haircuts increase, the value of bonds as collateral in the secured market decreases. Unconnected banks react by holding fewer bonds, raising fewer deposits, and reducing their investment in capital. Although connected banks maintain their investment in capital broadly unchanged, the overall effect is a decline in aggregate investment and output. As private haircuts decline further, the economy moves to the second region where unconnected banks access central bank funding. As the central bank haircut is stable and more favorable compared to the private market haircut, banks receive more funding for the same amount of pledged bonds and they can stabilize their deposit intake and investment. Therefore, the availability of central

bank funding puts a floor to the decline in deposits and capital. We find that an increase in private haircuts from 3 to 45 percent generates an output contraction of 0.65 percent. The contractionary impact would be much more severe (2 percent) in the absence of central bank funding.

In the second scenario - when access to the unsecured money market is reduced - a higher proportion of banks becomes unconnected and needs to satisfy possible deposit withdrawals by holding bonds and/or money. These banks can therefore invest less in the productive asset, i.e., capital. It is this substitution from investing in productive capital when connected to investing in unproductive assets when unconnected that generates output effects of unsecured market disruptions in our model. In our benchmark calibration with moderate liquidity outflows in the afternoon, unconnected banks are not too much constrained and shifting from unsecured to secured funding is only moderately costly. We find that an increase in the share of unconnected banks from 0.58 (corresponding to the average pre-2008 share of secured transactions in total volumes) to 0.76 (corresponding to the same average share during the crisis) generates a decline in output of around 0.5 percent. The adverse impact on real activity changes substantially if expected liquidity outflows in the afternoon increase with the disruptions experienced in the unsecured market. If those outflows are expected to double, the contraction in output is about 4 percent.

In the third scenario - when banks face increased fears of deposit withdrawals - unconnected banks react similarly by reducing their deposit, bond and cash holdings. As a result, the aggregate amount of deposits falls which in turn limits productive investment and reduces the capital stock. We find that doubling the expected maximum share of deposit withdrawals (from 0.1 to 0.2) generates an output reduction of 3 percent.

In the last part of the numerical analysis, we use the model to evaluate alternative monetary policies. In particular, we compare policies that stabilize the amount of bonds purchased by the central bank to policies that actively use bond purchases to stabilize inflation or the price of sovereign bonds. We show that central bank policies interact in a complex way with banks' behaviour. For instance, when banks cannot raise secured funding due to high private haircuts, a policy that keeps inflation or the bond price constant by changing the stock of bonds purchased is effective in stabilizing the economy. An expansion of the balance sheet through asset purchases enables the central bank to exchange sovereign bonds with limited collateral value in the secured market with newly issued money that can be used by banks to satisfy

expected liquidity outflows in the afternoon. We find that this policy can sizeably reduce the contraction in real activity. An increase in private haircuts from 3 to 40 percent generates an output contraction of 0.2 percent when inflation is kept constant and of 0.03 percent when the price of the bonds is kept constant. However, we also find that no single policy is able to shield the economy from all types of interbank market disruptions.

These numerical results highlight the rich dynamics generated by a novel feature of our model, namely the interaction between several occasionally binding constraints. It is interesting to note how the status of a particular constraint changes with key parameters. In models with one occasionally binding constraint, there are typically two regions - a "constrained" and an "unconstrained" one - and changing parameters moves the economy from one region to another. The implications for policy typically depend on which region the economy is visiting. In our model, when one parameter changes, a constraint can turn from binding to slack, and then to binding again, due to the interaction with other occasionally binding constraints. The particular constraint that binds is crucial for policy interventions, but in our model it is a priori not obvious, which one this will be.

This paper is related to the literature on interbank markets and on the impact of sovereign risk on the macroeconomy. There is an extensive literature in banking on the role of interbank markets in banks' liquidity management, starting with Bhattacharya and Gale (1987). A number of recent papers focus on analyzing frictions that prevent interbank markets from distributing liquidity efficiently within the banking system. Frictions include asymmetric information about banks' assets (Flannery, 1996; Freixas and Jorge, 2008; Heider and Holthausen, 2015), imperfect cross-border information (Freixas and Holthausen, 2005), banks' free-riding on liquidity provision by the central bank (Repullo, 2005), and multiplicity of Pareto-ranked equilibria (Freixas and Skeie, 2011). Papers in this strand of literature tend to be partial equilibrium and static, with links to the real economy modeled in a reduced-form fashion.

Several recent papers build dynamic general equilibrium models which include interbank trade. Afonso and Lagos (2015) and Atkeson and Weill (2015) analyse the trading decisions of banks in an OTC market. Bianchi and Bigio (2013) extend that framework to study the implementation of monetary policy through the banking system. The macroeconomic impact of interbank market trade and money market freezes is analysed by Bruche and Suarez (2010), who focus on the unsecured segment of that market, and more recently by Gertler, Kiyotaki and Prestipino (2016). Our paper contributes to this literature by considering both unsecured

and secured interbank markets, and collateralized lending by the central bank.<sup>3</sup> In our setup, disruptions in the unsecured money market segment may in principle be offset by an increased recourse to private secured markets or to central bank funding. We nevertheless find that disruptions in the unsecured market segment can generate non-negligible output costs. When also the secured market segment is severely malfunctioning, the possibility to tap into central bank can improve macroeconomic outcomes.

In our model, lending from the central bank is backed by government bonds, and increase in haircuts on sovereign bonds capture in a reduced-form the impact of sovereign default risk on collateral value. Our paper is thus also related to the literature on the impact of sovereign default risk on financial intermediation and the macroeconomy. Recent contributions to this strand of literature study the impact of sovereign risk on the funding ability of banks and their lending decisions (Bocola, 2016) as well as the link between government default and financial fragility, including the question of why the banking system may become exposed to government bonds (Gennaioli and Rossi, 2014). We do not model sovereign default risk explicitly, focusing instead on the implications of increased haircuts on government bonds for banks' ability to borrow in secured and unsecured interbank markets, as well as on how central bank policies can help alleviate bank funding problems arising due to increased private market haircuts.

The paper proceeds as follows. In section 2, we describe the model. In section 3, we define the equilibrium. In section 4, we characterize the system of equilibrium conditions. In section 5, we describe the steady state and present some analytical results. Section 6 illustrates the model predictions through a numerical analysis. Section 7 concludes.

## 2 The model

The economy is inhabited by a continuum of households, firms and banks. There is a government and a central bank.

Time is discrete,  $t = 0, 1, 2, \dots$ . We think of a period as composed of two sub-periods (“morning” and “afternoon”). Let us describe each in turn.

At the beginning of each period (in the morning), aggregate shocks occur. Households receive payments from financial assets and allocate their nominal wealth among money, long-term government bonds, and deposits at banks. Households also supply labor to firms in their

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<sup>3</sup>A related paper to ours is Piazzesi and Schneider (2017) which builds a framework where banks use reserves to settle interbank trades and to handle endusers' payment instructions. The authors show that key to the efficiency of a payment system is the provision and allocation of collateral.



country, receiving wages in return. The government taxes the labor income of the households in its country, makes payments on its debt and may change the stock of outstanding debt. Banks accept deposits from households and the central bank and make dividend payments to households. After accepting the deposits, banks learn their afternoon type in the morning. This latter can be either “connected,” in which case banks can borrow in the unsecured interbank market, or “not connected,” in which case they cannot, and the only possibility is to borrow by pledging assets in the secured interbank market. Banks then lend to firms (more precisely, finance their capital) and they hold government bonds and reserves (“cash”). The central bank provides funding to banks that wish to borrow against collateral. As an additional policy tool, the central bank can choose “haircuts” on the collateral pledged to access those funds.

During the afternoon, firms use labor and capital to produce a homogeneous output good which is consumed by households. Banks experience idiosyncratic deposit withdrawal shocks which average out to zero across all banks. Conceptually, these relate to random idiosyncratic consumption needs, additional economic activity and immediate payment for these services, which we shall refrain from modelling. Banks can accommodate those shocks by using their existing reserves, by selling government bonds or by borrowing in the unsecured market from other banks. They can only access the unsecured market, however, if they are “connected”. Banks are assumed to always position themselves so as to meet these liquidity withdrawals, i.e., bank failures are considered too costly and not an option. All banks meet as “one big banker family” at the end of the period. One can think of it as follows. First, the same bank-individual liquidity shock happens “in reverse”, so that banks enter the banker-family meeting in the same state they were in at the beginning of the afternoon. However, there would then still be bank heterogeneity left. Thus, banks all equate their positions at that point and restart the next period with the same portfolio. Alternatively, and equivalently, one can think that there are securities markets which open at the end of the period and allow banks to equate their portfolios. Banks during the period therefore are only concerned with the marginal value of an additional unit of net worth they can produce for the next period.

Firms and banks are owned by households. Similar to Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), banks are operated by bank managers who run a bank on behalf of their owning households. We deviate from those papers in that we assume that banks pay a fixed fraction of their net worth to households as a dividend in the morning of every period.

## 2.1 The households

There is a representative household, indexed by  $i \in (0, 1)$ . At the beginning of time  $t$ , households hold an amount of cash,  $\widetilde{M}_{t-1}^H$ , brought from period  $t - 1$ . They also receive repayment from banks of deposits opened in the previous period gross of the due interest,  $R_{t-1}^D D_{t-1}$ . Holding an amount  $H_t$  of nominal wealth at hand, each household chooses how to allocate it among existing nominal assets, namely money,  $M_t^H$ , and deposits,  $D_t$ .

During the day, beginning-of-period money balances are increased by the value of households' revenues and decreased by the value of their expenses. The amount of nominal balances brought by household  $i$  into period  $t + 1$ ,  $\widetilde{M}_t^H$ , is thus

$$\widetilde{M}_t^H = M_t^H + (1 - \tau_t) W_t l_t + E_t - P_t c_t, \quad (1)$$

where  $P_t$  is the price of the consumption good,  $l_t$  is hours worked,  $\tau_t$  is the labor tax rate,  $W_t$  is the nominal wage level, and  $E_t$  is the profit payout (“earnings”) by banks.

The nominal wealth available at the beginning of period  $t + 1$  for investment in nominal assets is given by

$$H_{t+1} = R_t^D D_t + \widetilde{M}_t^H. \quad (2)$$

The household then solves the problem

$$\max_{\{c_t > 0, l_t > 0, D_t \geq 0, M_t^H \geq 0\}} E_t \sum_{t=0}^{\infty} \beta^t \left[ u(c_t, l_t) + v\left(\frac{M_t^H}{P_t}\right) \right] \quad (3)$$

subject to (2) and

$$D_t + M_t^H \leq H_t.$$

## 2.2 Firms

A final-good firm  $j$  uses capital  $k_{t-1,j}$  and labor  $l_{t,j}$  to produce a homogeneous final output good  $y_{t,j}$  according to the production function

$$y_{t,j} = \gamma_t k_{t-1,j}^\theta l_{t,j}^{1-\theta}$$

where  $\gamma_t$  is a country-specific productivity shock. It receives revenues  $P_t y_{t,j}$  and pays wages  $W_t l_{t,j}$ . Capital is owned by the firms, which are in turn owned by banks: effectively then, the banks own the capital, renting it out to firms and extracting a real “rental rate”  $r_t$  per unit of capital or total nominal rental rate payments  $P_t r_t k_{t-1,j}$  from firm  $j$  on their capital  $k_{t-1,j}$ .

Capital-producing firms buy old capital  $k_{t-1}$  from the banks and combine it with final goods  $I_t$  to produce new capital  $k_t$ , according to

$$k_t = (1 - \delta) k_{t-1} + I_t.$$

New capital is then sold back to banks. Alternatively and equivalently, one may directly assume that the banks undertake the investments.

### 2.3 The government

The government has some outstanding debt with face value  $\bar{B}_{t-1}$ . It needs to purchase goods  $g_t$  and pays for it by taxing labor income as well as issuing discount bonds with a face value  $\Delta\bar{B}_t$  to be added to the outstanding debt next period, obtaining nominal resources  $Q_t\Delta\bar{B}_t$  for it in period  $t$ . We assume that some suitable no-Ponzi condition holds. The government discount bonds are repaid at a rate  $\kappa$ .

The outstanding debt at the beginning of period  $t + 1$  will be

$$\bar{B}_t = (1 - \kappa) \bar{B}_{t-1} + \Delta\bar{B}_t \tag{4}$$

The government budget balance at time  $t$  is

$$P_t g_t + \kappa \bar{B}_{t-1} = \tau_t W_t l_t + Q_t \Delta\bar{B}_t + S_t \tag{5}$$

where  $S_t$  are seigniorage payments from the central bank and  $g_t$  is an exogenously given process for government expenditures.

We assume that the government conducts fiscal policy so as to stabilize the stock of debt at a targeted level  $\bar{B}^*$ . This way, we aim at disentangling the fiscal role of the government from that of issuer of bonds with collateral value. In our analysis, movements in collateral value will be unrelated to fiscal policy decisions.

The government stabilizes the debt by adopting the following rule for the income tax:

$$\tau_t - \tau^* = \alpha \left( B_t - \bar{B}^* \right), \tag{6}$$

where  $\tau_t$  increases above its target level  $\tau^*$ , if the debt level is above  $\bar{B}^*$ . We assume that  $\alpha$  is such that the equilibrium is saddle-path stable and that the fiscal rule ensures a gradual

convergence to the desired stock of debt, following aggregate disturbances.<sup>4</sup> The target value  $\tau^*$  is the level of the income tax necessary to stabilize the debt at  $\bar{B}^*$ .<sup>5</sup>

## 2.4 The central bank

The central bank chooses the total money supply  $\bar{M}_t$  and interacts with banks in the “morning”, providing them with funds. Banks come into the period with total liabilities (F=“funds from the central bank”) at face value  $\bar{F}_{t-1}$ . Banks make payments  $\kappa^F \bar{F}_{t-1}$  on these liabilities and obtain new funds, at face value  $\Delta \bar{F}_t$ . Thus,

$$\bar{F}_t = (1 - \kappa^F) \bar{F}_{t-1} + \Delta \bar{F}_t \quad (7)$$

Banks obtain funds  $Q_t^F \Delta \bar{F}_t$  for these new liabilities, at the common price or discount factor  $Q_t^F$ . This discount factor is a policy parameter set by the central bank. The central bank furthermore buys and sells government bonds outright. Let  $B_{t-1}^C$  be the stock of government bonds held by the central bank (“C”) at the beginning of period  $t$ . The government makes payments on a fraction of these bonds, i.e., the central bank receives cash payments  $\kappa B_{t-1}^C$ . The remaining government bonds in the hands of the central banks are  $(1 - \kappa) B_{t-1}^C$ . The central bank then changes its stock to  $B_t^C$ , at current market prices  $Q_t$ , using cash. Thus,

$$B_t^C = (1 - \kappa) B_{t-1}^C + \Delta B_t^c$$

The central bank balance sheet looks as follows at time  $t$ :

Assets	Liabilities
$Q_t^F \bar{F}_t$ (loans to banks)	$M_t^H$ (currency held by HH)
$Q_t B_t^C$ (bond holdings)	$M_t$ (bank reserves)
	$S_t$ (seigniorage)

Let

$$\bar{M}_t = M_t^H + M_t$$

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<sup>4</sup>In our quantitative section, we provide a comparison of steady state equilibria: in that analysis, the parameter  $\alpha$  plays no role.

<sup>5</sup>Notice that  $\tau^*$  can be obtained by combining equation (4) and (5) in steady state, together with the rule  $B = \bar{B}^*$ , to get

$$\tau^* (1 - \theta) y = g + \kappa (1 - Q) \frac{\bar{b}^*}{\pi} - Q \left(1 - \frac{1}{\pi}\right) \bar{b}^* - s.$$

Here  $\bar{b}^* = \frac{\bar{B}^*}{P}$ ,  $s = \frac{S}{P}$  and  $\pi$  is the steady state inflation rate.

be the total money stock before seigniorage is paid. Note that the seigniorage is paid to the government at the end of the period and therefore becomes part of the currency in circulation next period. The flow budget constraint of the central bank is given by:

$$\begin{aligned}\bar{M}_t - \bar{M}_{t-1} &= S_{t-1} + Q_t^F (\bar{F}_t - (1 - \kappa^F) \bar{F}_{t-1}) - \kappa^F \bar{F}_{t-1} \\ &\quad + Q_t (B_t^C - (1 - \kappa) B_{t-1}^C) - \kappa B_{t-1}^C.\end{aligned}\tag{8}$$

Seigniorage can then be calculated as the residual balance sheet profit,

$$S_t = Q_t^F \bar{F}_t + Q_t B_t^C - \bar{M}_t.\tag{9}$$

## 2.5 Banks

There is a continuum of banks (“Lenders”), indexed by  $l \in (0, 1)$ . Consider a bank  $l$ .

### 2.5.1 Assets and liabilities

At the end of the morning, after earning income on its assets, paying interest on its liabilities and retrading, but just before paying dividends to share holders, the bank holds four type of assets. It additionally and briefly holds an asset in the afternoon, for a total of five. As an overview, the end-of-morning balance sheet of that bank is

Assets	Liabilities
$P_t k_{t,l}$ (capital held)	$D_{t,l}$ (deposits by HH)
$Q_t B_{t,l}$ (bond holdings)	$Q_t^F F_{t,l}$ (secured loans)
$E_{t,l}$ (cash dividends)	$N_{t,l}$ (net worth)
$M_{t,l}$ (cash reserves)	

In detail:

1. Capital  $k_{t,l}$  of firms, or, equivalently, firms, who in turn own the capital. Capital can only be acquired and traded in the morning. Capital evolves according to

$$k_{t,l} = (1 - \delta) k_{t-1,l} + \Delta k_{t,l}$$

where  $\Delta k_{t,l}$  is the gross investment of bank  $l$  in capital.

2. Bonds with a nominal face value  $B_{t,l}$ . A fraction  $\kappa$  of the government debt will be repaid. The bank changes its government bond position per market purchases or sales (“-”)  $\Delta B_{t,l}$

in the morning, so that

$$B_{t,l} = (1 - \kappa) B_{t-1,l} + \Delta B_{t,l}$$

at the end of the morning. If the bank purchases (sells) bonds on the open market, it pays (receives)  $Q_t \Delta B_{t,l}$ . As a baseline, we allow  $\Delta B_{t,l}$  to be negative, indicating a sale. In the afternoon and after the first bank-individual liquidity shock, once again, the bank can change its government bond position per market purchases or sales (“-”)  $\widetilde{\Delta B}_{t,l}$ , so that

$$\widetilde{B}_{t,l} = B_{t,l} + \widetilde{\Delta B}_{t,l}$$

When the second reverse liquidity shock hits, the trade is reversed as well, resulting in

$$B_{t,l} = \widetilde{B}_{t,l} - \widetilde{\Delta B}_{t,l}$$

One can instead think of this as a secured repo market, vis-a-vis other banks. To that end, it is useful to introduce haircut parameters  $0 \leq \widetilde{\eta}_t \leq 1$ , imposed by other lending banks. The bank then receives the cash amount  $\widetilde{\eta}_t Q_t \widetilde{\Delta B}_{t,l}$  in the first of these two transactions, repaying the same amount in the second. Taken literally, there is no risk here that this haircut could reasonably insure against, but this is just due to keeping the model simple. The interest rate is zero.

3. Cash  $E_{t,l}$  earmarked to be distributed to shareholders (E = “earmarked” or “earnings”) at the end of the morning. Note that this does not mean that the households end up being forced to hold money, as everything happens “simultaneously” in the morning. If they want to hold those extra earnings as extra deposits, then  $D_t$  would simply already be higher before they receive the earnings from the banks, in “anticipation” of these earning payments.
4. Reserves (M=“money”)  $M_{t,l} \geq 0$ . They may add to cash (not earmarked for paying shareholders) in the morning,

$$M_{t,l} = M_{t-1,l} + \Delta M_{t,l} \geq 0$$

as well as in the afternoon,

$$\widetilde{M}_{t,l} = M_{t,l} + \widetilde{\Delta M}_{t,l} \geq 0$$

reversing the first-liquidity-shock transaction when the reverse liquidity shock hits,

$$M_{t,l} = \widetilde{M}_{t,l} - \Delta \widetilde{M}_{t,l}$$

5. Unsecured claims on other banks at face value, obtained during the first liquidity shock in the afternoon. They are repaid at zero interest rate during the second reverse-liquidity shock.

Bank  $l$  has four types of liabilities:

1. Deposits  $D_{t,l}$ . This is owed to household and subject to aggregate withdrawals and additions  $\Delta D_{t,l}$  in the morning, so that

$$D_{t,l} = R_{t-1}^D D_{t-1,l} + \Delta D_{t,l}$$

where  $R_{t-1}^D$  is the return on one unit of deposits, agreed at time  $t - 1$ . Additionally, there are idiosyncratic withdrawals and additions in the afternoon, to be described.

2. Secured loans (F=“funding”) from the central bank at face value  $F_{t,l}$ . Secured loans require collateral. A bank  $l$  with liabilities  $F_{t,l}$  to the central banks needs to pledge an amount  $0 \leq B_{t,l}^F \leq B_{t,l}$  of government bonds  $B_{t,l}$  satisfying the collateral constraint

$$F_{t,l} \leq \eta_t Q_t B_{t,l}^F \tag{10}$$

where  $\eta_t$  is a haircut parameter and is set by the central bank. The collateral constraints are set in terms of the market value of securities, as is the case in ECB monetary policy operations. Secured loans from the central bank are obtained in the morning. The change in the secured loans  $\Delta F_{t,l}$  provide the banks with change in liquidity (“cash”) of  $Q_t^F \Delta F_{t,l}$ , in addition to the liquidity carried over from the previous period. Liquidity is needed in the afternoon. Therefore, the discount rate  $Q_t^F$  will not only relate to an intertemporal trade-off, as is common in most models, but importantly also to the intratemporal tradeoff of obtaining potentially costly liquidity in the morning in order to secure sufficient funding in the afternoon.

3. Outstanding unsecured liabilities to other banks issued at the time of the first liquidity shock in the afternoon. Only “connected” banks can issue them. They are repaid at zero interest rate at the time of the reverse liquidity shock.

#### 4. Net worth $N_{t,l}$ .

The sum of assets equals the sum of liabilities, at any point in time.

### 2.5.2 Liquidity needs in the afternoon

At the beginning of the afternoon, households hold total deposits  $D_t$  with banks. We seek to capture the daily churning of deposits at banks, due to cross-household and firm-household payment activities with inside money, as follows. At the start of the afternoon in period  $t$ , deposits get reshuffled across banks, so that bank  $l$  with pre-shuffle end-of-morning deposits  $D_{t,l}$  experiences a withdrawal  $\omega_{t,l}D_{t,l}$ . Here,  $\omega = \omega_{t,l} \in (-\infty, \omega^{\max}]$ , with  $0 \leq \omega^{\max} \leq 1$ , is a random variable, which is iid across banks  $l$  and is distributed according to  $F(\omega)$ . The remaining post-shuffle beginning-of-afternoon deposits  $\tilde{D}_{t,l}$  are thus

$$\tilde{D}_{t,l} = (1 - \omega_{t,l}) D_{t,l}$$

In order to meet withdrawals, banks need to have enough reserves at hand to cover them. We assume that banks will always find defaulting on the withdrawals worse than any precautionary measure they can take against it, and thus rule out withdrawal caps and bank runs by assumption. Reserves can be obtained in the morning by various trades, resulting in bank holdings  $M_{t,l}$ . In the afternoon, additional reserves can be obtained only by new unsecured loans from other banks, maturing at the end of the afternoon, or by selling government bonds. Implicitly, we are assuming that the discount window of the central bank is not open in the afternoon, i.e., that banks need to obtain central bank funding in the morning in precaution to withdrawal demands in the afternoon. This captures the fact that the discount window is rarely used for funding liquidity needs and that these liquidity transactions happen “fast”, compared to central bank liquidity provision.

The withdrawal shock is exactly reversed with a second reverse liquidity shock, so that banks exit the period with the original level of deposits  $D_{t,l}$  and can thus repay their unsecured loans or buy back the government securities originally sold. The same holds if the signs are reversed. Thus, the first liquidity shock creates only a very temporary liquidity need that banks must satisfy.

New unsecured loans can only be obtained by “connected” banks. We assume that banks face an exogenous iid probability  $\xi_t$  of being connected and being able to borrow on the unsecured loan market. We assume this probability to be iid across banks and time. The



draw of the type of the bank (i.e., “connected” or “not connected”, with probability  $\xi_t$ ) happens early in the morning: thus, banks know in the morning, whether they are able to potentially borrow in the afternoon or whether they need to potentially sell government bonds instead. Every bank can lend unsecured, if they so choose.

If banks do not have access to the unsecured loan market, they will need to sell government bonds, in case of liquidity needs. They can only do so with the portion that has not yet been pledged to the central bank. With  $\omega^{\max}$  as the maximal withdrawal shock, non-connected banks therefore have to hold government securities satisfying

$$\omega^{\max} D_{t,l} - M_{t,l} \leq \tilde{\eta}_t Q_t (B_{t,l} - B_{t,l}^F) \quad (11)$$

where  $0 \leq \tilde{\eta}_t \leq 1$  is a haircut parameters imposed by other lending banks, if we interpret this sale of government bonds as a private repo or private secured borrowing, and where the constraint is in terms of the unpledged portion of the government bond holdings  $B_{t,l} - B_{t,l}^F$ .

As all the afternoon transactions are reversed at the end of the afternoon and since all within-afternoon interest rates are zero, banks will be entirely indifferent between using any of the available sources of liquidity: what happens in the afternoon stays in the afternoon. The only impact of these choices and restrictions is that banks need to plan ahead of time in the morning to make sure that they have enough funding in the afternoon, in the worst case scenario. If a bank is unconnected, that worse-case scenario is particularly bad, as it needs to have enough of cash reserves plus unpledged bonds to meet the maximally conceivable afternoon deposit withdrawal.

### 2.5.3 Objective function and leverage constraints

Banks are owned by households in their country. If net worth is nonnegative, they repay a portion  $\phi$  of their net worth to households each period

$$E_{t,l} = \phi N_{t,l}$$

In terms of aggregate bank equity  $N_t$  and resulting dividend payments, the profit payments by banks are  $E_t = \phi N_t$ , if  $N_t \geq 0$ . If net worth is negative, banks declare bankruptcy. In that case, all assets are sold, and the proceeds are returned pro rata to the holders of bank liabilities. We shall consider only shocks and scenarios, so that net worth remains positive.

The net worth of bank  $l$  before payments to shareholders satisfies

$$\begin{aligned} N_{t,l} &= \max\{0, P_t (r_t + 1 - \delta) k_{t-1,l} + M_{t-1,l} + ((1 - \kappa) Q_t + \kappa) B_{t-1,l} - R_{t-1}^D D_{t-1,l} - \kappa^F F_{t-1,l}\} \\ &= \max\{0, P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} - D_{t,l} - Q_t^F F_{t,l} + E_{t,l}\} \end{aligned}$$

where the first equation is the net worth calculated on the balance of assets and their earnings and payments before the bank makes its portfolio decision, while the second equation exploits the equality of assets to liabilities after the portfolio decision.

From these two equations, one can calculate

$$\Delta M_{t,l} = M_{t,l} - M_{t-1,l}.$$

Given the draw of the type according to  $\xi_t = P$  (“connected”), bank  $l$  can be either “connected” or “unconnected” (denoted with the subscripts “ $c$ ” or “ $u$ ”, respectively).

Aggregate net worth at the beginning of the period is

$$\begin{aligned} N_t = \max \{0, & P_t (r_t + 1 - \delta) (\xi_{t-1} k_{t-1,c} + (1 - \xi_{t-1}) k_{t-1,u}) \\ & + (\xi_{t-1} M_{t-1,c} + (1 - \xi_{t-1}) M_{t-1,u}) \\ & + ((1 - \kappa^F) Q_t^F + \kappa^F) (\xi_{t-1} F_{t-1,c} + (1 - \xi_{t-1}) F_{t-1,u}) \\ & + ((1 - \kappa) Q_t + \kappa) (\xi_{t-1} B_{t-1,c} + (1 - \xi_{t-1}) B_{t-1,u}) \\ & - R_{t-1}^D (\xi_{t-1} D_{t-1,c} + (1 - \xi_{t-1}) D_{t-1,u}) \} \end{aligned}$$

which implies that  $N_t = \xi_t N_{t,c} + (1 - \xi_t) N_{t,u}$ . In principle, the second expression could be negative and aggregate net worth could become zero, in which case the banking sector of an entire country becomes insolvent. For those cases, it would be important to specify what happens to the assets and liabilities, and the economy overall. To keep the analysis manageable, we shall entirely focus on shocks and equilibria for now, where this does not happen along the equilibrium paths.

We shall impose that sub-banks get the same net worth, regardless of type (“connected”, “unconnected”), effectively assuming that the net worth is assigned before the type is known<sup>6</sup>,  $N_{t,c} = N_{t,u} = N_t$ , where  $N_{t,c}$  is the net worth per connected bank, i.e., the total net worth in all connected banks is  $\xi_t N_{t,c}$ , the total net worth in all unconnected banks is  $(1 - \xi_t) N_{t,u}$ .

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<sup>6</sup>If the net worth could be assigned after the type is known, obviously only connected banks would get any net worth, and the model would become rather uninteresting.

Correspondingly, all assets and liabilities are likewise distributed equally, regardless of type (again, assuming that this redistribution is done before the new type is drawn for each sub-bank).

Summing this and imposing the two previous equations shows that total net worth is  $N_t$ , as it should be. Therefore, we shall drop the distinction between  $N_{t,c}$ ,  $N_{t,u}$  and  $N_t$ . The sub-bank budget constraint is

$$P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t = D_{t,l} + Q_t^F F_{t,l} + N_t \quad (12)$$

As in Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), we assume that there is a moral hazard constraint in that bank managers may run away with a fraction of their assets in the morning, after their asset trades are completed and after dividends are paid to the household. The constraint is

$$\lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \leq V_{t,l}$$

where  $0 \leq \lambda \leq 1$  is a leverage parameter. Implicitly, we assume that the same leverage parameter holds for all assets, and that bankers can run away with all assets, including government bonds that may have been pledged as collateral vis-a-vis the central bank<sup>7</sup>.

## 2.6 The rest of the world

We assume that a share of the stock of government bonds is held by the rest of the world and that foreigners have an elastic demand for those bonds.<sup>8</sup> Because unconnected banks can buy or sell bonds to foreigners, they can change their bond holdings independently from the government's outstanding stock of debt.

<sup>7</sup>Alternatively, one may wish to impose that banks cannot run away with assets pledged to the central bank as collateral. In that case, the collateral constraint would be

$$\lambda \left[ P_t (k_{t,l} - k_{t,l}^F) + Q_t (B_{t,l} - B_{t,l}^F) + M_{t,l} \right] \leq V_{t,l}$$

or a version in between this and the in-text equation. Since collateral pledged to the ECB remains in the control of banks, we feel that the assumption used in the text is more appropriate.

<sup>8</sup>We introduce the elastic foreign sector demand for two reasons. First, a large fraction of euro area sovereign debt is held by non-euro area residents, and these bondholders actively rebalance their bond positions. For example, Koijen et al. (2016) document that during the Public Sector Purchase Programme implemented by the ECB since March 2015, for each unit of sovereign bonds purchased by the ECB, the foreign sector sold 0.64 of it. Second, when solving the model we will focus on the parameter space in which connected banks choose not to hold bonds. In a closed economy, therefore, unconnected banks would have to absorb whatever amount of bonds is issued by the government (after deducting the fixed amount held by the central bank). The price of the bond would have to adjust to clear the market. Such direct link between the bond market and the unconnected banks' decisions would be unrealistic.

We do not wish to model the foreign sector explicitly. We simply assume that international investors have a demand for domestic bonds that reacts to movements in the real return on these bonds,

$$B_t^W = P_t \left( \varkappa - \frac{1}{\varrho} \log Q_t \pi_t \right), \quad (13)$$

where  $\varrho > 0$  and  $\varkappa \geq 0$ . Notice that this functional form allows foreign bond holdings to become negative, e.g., in case domestic bond demand exceeds government bond supply, while  $Q_t$  is always positive. If  $\rho = 0$ , bond demand becomes infinitely elastic. In that case, the real return  $1/(Q_t \pi_t)$  is fixed and foreign holdings take whatever value is needed to clear the bond market. The flow budget constraint of the foreign sector is

$$Q_t B_t^W + P_t c_t^W = [\kappa + (1 - \kappa) Q] B_{t-1}^W. \quad (14)$$

### 3 Equilibrium

An equilibrium is a vector of sequences such that:

1. Given  $P_t, \tau_t, W_t, R_{t-1}^D, E_t$ , the representative household chooses  $c_t > 0, l_t > 0, D_t \geq 0, M_t^H \geq 0$  to maximize their objective function

$$\max_{\{c_t > 0, l_t > 0, D_t \geq 0, M_t^H \geq 0\}} E_t \left[ \sum_{t=0}^{\infty} \beta^t \left[ u(c_t, l_t) + v \left( \frac{M_t^H}{P_t} \right) \right] \right]$$

subject to

$$D_t + M_t^H \leq H_t$$

where

$$H_{t+1} = R_t^D D_t + M_t^H + (1 - \tau_t) W_t l_t + E_t - P_t c_t.$$

2. Final good firms choose capital and labor to maximize their expected profits from production, which makes use of the technology

$$y_{t,j} = \gamma_t k_{t-1,j}^\theta l_{t,j}^{1-\theta}.$$

3. Capital-producing firms choose how much old capital  $k_{t-1}$  to buy from banks and to combine with final goods  $I_t$  to produce new capital  $k_t$ , according to the technology

$$k_t = (1 - \delta) k_{t-1} + I_t.$$

4. Bank families aggregate the assets and liabilities of the individual family members:

$$V_t = \xi_t V_{t,c} + (1 - \xi_t) V_{t,u} \quad (15)$$

$$k_t = \xi_t k_{t,c} + (1 - \xi_t) k_{t,u} \quad (16)$$

$$D_t = \xi_t D_{t,c} + (1 - \xi_t) D_{t,u} \quad (17)$$

$$B_t = \xi_t B_{t,c} + (1 - \xi_t) B_{t,u} \quad (18)$$

$$F_t = \xi_t F_{t,c} + (1 - \xi_t) F_{t,u} \quad (19)$$

$$M_t = \xi_t M_{t,c} + (1 - \xi_t) M_{t,u} \quad (20)$$

5. Given the stochastic paths for the endogenous variables  $c_t$ ,  $l_t$ ,  $r_t$ ,  $P_t$ ,  $Q_t$ ,  $Q_t^F$ ,  $\eta_t$ , and stochastic exogenous sequence for  $\tilde{\eta}_t$  and the draw of the type according to  $\xi_t$ , the representative date- $t$  connected bank chooses  $k_{t,c}$ ,  $B_{t,c}$ ,  $B_{t,c}^F$ ,  $F_{t,c}$ ,  $D_{t,c}$ ,  $M_{t,c}$  and the representative date- $t$  unconnected bank chooses  $k_{t,u}$ ,  $B_{t,u}$ ,  $B_{t,u}^F$ ,  $F_{t,u}$ ,  $D_{t,u}$ ,  $M_{t,u}$  to maximize the banks' objective function, i.e. to maximize

$$V_{t,l} = P_t E \left[ \phi \sum_{s=0}^{\infty} (\beta (1 - \phi))^s \frac{u_c(c_{t+s}, l_{t+s})}{u_c(c_t, l_t)} \frac{N_{t+s}}{P_{t+s}} \right] \quad (21)$$

where

$$\begin{aligned} N_t = \max\{0, & \quad P_t (r + 1 - \delta) (\xi_{t-1} k_{t-1,c} + (1 - \xi_{t-1}) k_{t-1,u}) \\ & + (\xi_{t-1} M_{t-1,c} + (1 - \xi_{t-1}) M_{t-1,u}) \\ & + ((1 - \kappa^F) Q_t^F + \kappa^F) (\xi_{t-1} F_{t-1,c} + (1 - \xi_{t-1}) F_{t-1,u}) \\ & + ((1 - \kappa) Q_t + \kappa) (\xi_{t-1} B_{t-1,c} + (1 - \xi_{t-1}) B_{t-1,u}) \\ & - R_{t-1}^D (\xi_{t-1} D_{t-1,c} + (1 - \xi_{t-1}) D_{t-1,u}) \} \end{aligned} \quad (22)$$

s.t. for  $l = c, u$ ,

$$\begin{aligned} V_{t,l} & \geq \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \\ 0 & \leq B_{t,l} - B_{t,l}^F \\ P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t & = D_{t,l} + Q_t^F F_{t,l} + N_t \\ F_{t,l} & \leq \eta_t Q_t B_{t,l}^F \end{aligned}$$

as well as

$$\omega^{\max} D_{t,u} - M_{t,u} \leq \tilde{\eta}_t Q_t (B_{t,u} - B_{t,u}^F)$$

for the unconnected banks.

6. The central banks chooses the total amount of money supply  $\bar{M}_t$ , the haircut parameter  $\eta_t$ , the discount factor on central bank funds  $Q_t^F$ , the bond purchases  $B_t^C$  as well as the seigniorage payment  $S_t$ . It satisfies the balance sheet constraint

$$S_t = Q_t^F \bar{F}_t + Q_t B_t^C - \bar{M}_t \quad (23)$$

and the budget constraint

$$\begin{aligned} \bar{M}_t = & Q_{t-1}^F \bar{F}_{t-1} + Q_{t-1} B_{t-1}^C + Q_t^F (\bar{F}_t - (1 - \kappa^F) \bar{F}_{t-1}) \\ & - \kappa^F \bar{F}_{t-1} + Q_t (B_t^C - (1 - \kappa) B_{t-1}^C) - \kappa B_{t-1}^C \end{aligned} \quad (24)$$

7. The government satisfies the debt evolution constraint, the budget constraint and the tax rule

$$\bar{B}_t = (1 - \kappa) \bar{B}_{t-1} + \Delta \bar{B}_t \quad (25)$$

$$P_t g_t + \kappa \bar{B}_{t-1} = \tau_t W_t l_t + Q_t \Delta \bar{B}_t + S_t \quad (26)$$

$$\tau_t W_t l_t = \alpha \bar{B}_{t-1}. \quad (27)$$

8. The foreign sector chooses the amount of domestic bonds to hold

$$B_t^W = \varkappa - \frac{1}{\rho} \log Q_t \pi_t, \quad (28)$$

and satisfies the budget constraint

$$Q_t B_t^W + P_t c_t^W = [\kappa + (1 - \kappa) Q] B_{t-1}^W. \quad (29)$$

9. Markets clear:

$$c_t + g_t + I_t + c_t^W = y_t \quad (30)$$

$$\bar{B}_t = B_t + B_t^C + B_t^W \quad (31)$$

$$\bar{F}_t = F_t \quad (32)$$

$$\bar{M}_t = M_t + M_t^H \quad (33)$$

## 4 Analysis

We characterize the decision of households, firms and banks in turn.

### 4.1 Households

The household budget constraint at time  $t$  writes as

$$D_t + M_t^H \leq R_{t-1}^D D_{t-1} + M_{t-1}^H + (1 - \tau_{t-1}) W_{t-1} l_{t-1} + E_{t-1} - P_{t-1} c_{t-1} \quad (34)$$

The household's problem is subject to a non-negativity condition,

$$M_t^H \geq 0. \quad (35)$$

Note also that  $c_t > 0$ ,  $l_t > 0$ , and  $D_t \geq 0$ . We do not list these constraints separately for the following reasons. For  $c_t > 0$  and  $l_t > 0$ , we can assure nonnegativity with appropriate choice for preferences and per the imposition of Inada conditions. We constrain the analysis a priori to  $D_t > 0$ , despite the possibility in principle that it could be zero or negative when allowing for more generality<sup>9</sup>.

Let  $\mu_t^{HH}$  denote a Lagrange multiplier on the period- $t$  household budget constraint (34), and  $\hat{\mu}_{t,h}^M$  the multiplier on the constraint (35). The optimality conditions are given by:

$$\begin{aligned} -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} &= (1 - \tau_t) \frac{W_t}{P_t} \\ v_M(m_t^h) &= u_c(c_t, l_t) (R_t^D - 1) - \mu_{t,h}^M \\ \frac{u_c(c_{t-1}, l_{t-1})}{P_{t-1}} &= \beta R_t^D \left[ \frac{u_c(c_t, l_t)}{P_t} \right] \end{aligned}$$

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<sup>9</sup>We have not yet fully analyzed this matter for the dynamic evolution of the economy. It may well be that net worth of banks temporarily exceeds the funding needed for financing the capital stock, and that therefore deposits ought to be negative, rather than positive. For now, the attention is on the steady state analysis, however, and on returns to capital exceeding the returns on deposits.

where  $\mu_{t,h}^M = P_t \widehat{\mu}_{t,h}^M$ , together with the complementary slackness condition  $\widehat{\mu}_{t,h}^M M_t^H = 0$ .

## 4.2 Firms

First-order conditions arising from the problem of the firms are

$$\begin{aligned} y_t &= \gamma_t k_{t-1}^\theta l_t^{1-\theta}, \\ W_t l_t &= (1 - \theta) P_t y_t, \\ r_{t,A} k_{t-1,A} &= \theta y_t, \\ k_t &= (1 - \delta) k_{t-1} + I_t. \end{aligned}$$

## 4.3 Banks

The run-away constraint (assuming it always binds) is

$$V_{t,l} = \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \quad (36)$$

The value of the mother bank is  $V_t$ , which is given by

$$V_t = \xi_t V_{t,c} + (1 - \xi_t) V_{t,u} \quad (37)$$

**Proposition 1 (linearity)** *The problem of bank  $l$  is linear in net worth and*

$$V_{t,l} = \psi_t N_{t,l} \quad (38)$$

*for any bank  $l$  and some factor  $\psi_t$ . In particular,  $V_{t,l} = 0$  if  $N_{t,l} = 0$ .*

Proof: Since there are no fixed costs, a bank with twice as much net worth can invest twice as much in the assets. Furthermore, if a portfolio is optimal at some scale for net worth, then doubling every portion of that portfolio is optimal at twice that net worth. Thus the value of the bank is twice as large, giving the linearity above.

We need to calculate  $V_{t,l}$ . The proposition above implies

$$V_t = \psi_t N_t \quad (39)$$

giving us a valuation of a marginal unit of net worth at the beginning of period  $t$ , for a representative bank.



Suppose, at the end of the period, the representative “mother bank” has various assets,  $k_t$ ,  $B_t$ , and  $M_t$ , brought to it by the various sub-banks as they get together again at the end of the period. The end-of-period value  $\tilde{V}_t$  of the “mother bank” then satisfies

$$\begin{aligned}\tilde{V}_t &= \beta(1-\phi) E_t \left[ \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} N_{t+1} \right] \\ &= \tilde{\psi}_{t,k} P_t k_t + \tilde{\psi}_{t,B} B_t + \tilde{\psi}_{t,M} M_t - \tilde{\psi}_{t,D} D_t - \tilde{\psi}_{t,F} F_t\end{aligned}\quad (40)$$

Per inspecting (22) as well (38), we obtain

$$\tilde{\psi}_{t,k} = \beta(1-\phi) E_t \left[ \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \psi_{t+1} (r_{t+1} + 1 - \delta) \right] \quad (41)$$

$$\tilde{\psi}_{t,B} = \beta(1-\phi) E_t \left[ \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} ((1-\kappa) Q_{t+1} + \kappa) \right] \quad (42)$$

$$\tilde{\psi}_{t,D} = \beta(1-\phi) E_t \left[ \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} R_t^D \right] \quad (43)$$

$$\tilde{\psi}_{t,F} = \beta(1-\phi) E_t \left[ \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} ((1-\kappa^F) Q_{t+1}^F + \kappa^F) \right] \quad (44)$$

$$\tilde{\psi}_{t,M} = \beta(1-\phi) E_t \left[ \frac{u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} \frac{P_t}{P_{t+1}} \psi_{t+1} \right] \quad (45)$$

For the sub-banker of type  $l$ , write

$$V_{t,l} = \phi N_{t,A} + \tilde{V}_{t,l} \quad (46)$$

The sub-bankers contribute to  $\tilde{V}_t$  per

$$\tilde{V}_{t,l} = \tilde{\psi}_{t,k} k_{t,l} + \tilde{\psi}_{t,B} B_{t,l} + \tilde{\psi}_{t,M} M_{t,l} - \tilde{\psi}_{t,D} D_{t,l} - \tilde{\psi}_{t,F} F_{t,l} \quad (47)$$

The run-away constraint for bank  $l$  can then be rewritten as

$$\phi N_{t,A} + \tilde{V}_{t,l} \geq \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \quad (48)$$

Banks will pledge just enough collateral to the central bank to make the collateral constraint binding, nothing more (even if indifferent between that and pledging more: then, “binding” is an assumption). For both types of banks,

$$F_{t,l} = \eta_t Q_t B_{t,l}^F \quad (49)$$

with

$$0 \leq B_{t,l} - B_{t,l}^F \quad (50)$$

There are also nonnegativity constraints for investing in cash, in bonds, and for financing from the central bank, for both types of banks:

$$0 \leq M_{t,l} \quad (51)$$

$$0 \leq B_{t,l} \quad (52)$$

$$0 \leq F_{t,l}. \quad (53)$$

Note that we are interested in cases where banks choose to raise deposits and to extend loans. The former requirement ensures that banks have liquidity shocks in the afternoon and thus provides a meaningful role for interbank markets. The latter requirement generates an active link between financial intermediation and real activity in our economy.

We can have cases, however, when banks decide not to raise central bank finance, as in the case of connected banks that can always get afternoon zero-interest rate unsecured loans from other banks, if the need arises (this is assuming that  $Q_t^F \leq 1$ , otherwise there would be arbitrage possibilities for banks!). Similarly, banks can decide not to hold bonds, if their liquidity value is too low and the cost of satisfying the afternoon constraint with cash is sufficiently low. Alternatively, they can decide not to hold cash, if they have access to afternoon unsecured or secured finance, and if the expected return on capital is higher than the expected return on money.

To simplify the analysis, we assume (and verify in appendix A) that the economy is in an interior equilibrium for  $D_{t,l}$  and  $k_{t,l}$  in all the interesting cases we consider. In light of the considerations above, we explicitly allow for corner solutions for  $F_{t,l}$ ,  $B_{t,l}$  and  $M_{t,l}$ .

As for the afternoon, there is no need to keep track of trades, except to make sure that the afternoon funding constraints for the unconnected banks hold,

$$\omega^{\max} D_{t,u} - M_{t,u} \leq \tilde{\eta}_t Q_t (B_{t,u} - B_{t,u}^F). \quad (54)$$

Banks  $l = u$  and  $l = c$  who are given  $N_t$  maximize (47) subject to the sub-bank budget constraint (12) and the run-away constraint (48), the collateral constraints (49), (50), as well as (54) only for the unconnected banks. Let  $\mu_{t,l}^{BC}$  denote the Lagrange multiplier on the budget constraint (12),  $\mu_{t,l}^{RA}$  the Lagrange multiplier on the run-away constraint (48),  $\mu_{t,l}^{CC}$

the Lagrange multiplier on the collateral constraint (49),  $\mu_{t,u}$  the Lagrange multiplier on the afternoon funding constraint of the unconnected banks,  $\mu_{t,l}^M \geq 0$ ,  $\mu_{t,l}^F \geq 0$ ,  $\mu_{t,l}^C \geq 0$  and  $\mu_{t,l}^B \geq 0$  the Lagrange multipliers on the constraints  $M_{t,l} \geq 0$ ,  $F_{t,l} \geq 0$ , the collateral constraint (50), and the non-negativity constraint for bonds  $B_{t,l}$ , respectively.

The first-order conditions characterizing banks' choices for capital, bonds, and money, are

$$\begin{aligned} (1 + \mu_{t,l}^{RA}) \frac{\tilde{\psi}_{t,k}}{P_t} &= \mu_{t,l}^{BC} + \lambda \mu_{t,l}^{RA} \\ \left(1 + \mu_{t,l}^{RA}\right) \frac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^C \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \frac{\tilde{\psi}_{t,B}}{Q_t} &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^C - \mu_{t,u} \tilde{\eta}_t \quad \text{for } l = u \\ \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,M} + \mu_{t,c}^M &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda \quad \text{for } l = c \\ \left(1 + \mu_{t,l}^{RA}\right) \tilde{\psi}_{t,M} + \mu_{t,c}^M &= \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,u} \quad \text{for } l = u \end{aligned}$$

Those characterizing banks' choices for deposits, central bank funding, and bonds to be pledged at the central bank, are

$$\begin{cases} (1 + \mu_{t,l}^{RA}) \tilde{\psi}_{t,D} = \mu_{t,l}^{BC} & \text{for } l = c \\ (1 + \mu_{t,l}^{RA}) \tilde{\psi}_{t,D} = \mu_{t,l}^{BC} - \omega^{\max} \mu_{t,u} & \text{for } l = u \end{cases} \quad (55)$$

$$(1 + \mu_{t,l}^{RA}) \tilde{\psi}_{t,F} = \mu_{t,l}^{BC} Q_t^F - \mu_{t,l}^{CC} + \mu_{t,c}^F \quad (56)$$

$$\begin{aligned} \mu_{t,l}^{CC} \eta_t &= \mu_{t,l}^C \quad \text{for } l = c \\ \mu_{t,l}^{CC} \eta_t &= \mu_{t,u} \tilde{\eta}_t + \mu_{t,l}^C \quad \text{for } l = u \end{aligned}$$

The complementary slackness conditions are

$$\mu_{t,l}^F F_{t,l} = 0 \quad (57)$$

$$\mu_{t,l}^M M_{t,l} = 0 \quad (58)$$

$$\mu_{t,l}^C (B_{t,l} - B_{t,l}^F) = 0 \quad (59)$$

$$\mu_{t,l}^{RA} \left[ \phi N_{t,A} + \tilde{V}_{t,l} - \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \right] = 0 \quad (60)$$

$$\mu_{t,l}^B B_{t,l} = 0 \quad (61)$$

for  $l = u, c$ , and

$$\mu_{t,u} \left[ \omega^{\max} D_{t,u} - M_{t,u} - \tilde{\eta}_t Q_t (B_{t,u} - B_{t,u}^F) \right] = 0$$

for unconnected banks only.

These are linear programming problem, maximizing a linear objective subject to linear constraints. So, the solution is either a corner solution or there will be indifference between certain asset classes, resulting in no-arbitrage conditions.

## 5 Steady state analysis

We characterize a stochastic steady state where prices grow at the rate  $\pi$  and all shocks are zero except for the idiosyncratic liquidity shock  $\omega$  faced by banks. We denote with small letters all real variables, i.e. the corresponding variables in capital letter divided by the price of the consumption good,  $P_t$ . The steady state is characterized by the set of conditions reported in Appendix A.

### 5.1 Analytical characterization of the bank problem

In this section, we provide some analytical results for the bank problem. We focus on the set of parameters such that:

1. Both bank types choose to extend loans and to raise deposits,  $k_l > 0$  and  $d_l > 0$ . The requirement  $k_l > 0$  ensures an active link between activity of all banks and the real activity. This requires capital to be sufficiently productive compared to the cost of deposits,  $\tilde{\psi}_k > \tilde{\psi}_D$ , which after substituting for  $\tilde{\psi}_k$  and  $\tilde{\psi}_D$  yields:

$$\theta \frac{y}{(\xi k_c + (1 - \xi) k_u)} + 1 - \delta > \frac{1}{\beta}. \quad (62)$$

The requirement that  $d_l > 0$  means that both bank types will be subject to liquidity shocks in the afternoon and thus liquidity management will play an important role for both bank types. Different bank types may still choose to manage their liquidity differently (through interbank markets and/or by borrowing from the central bank and saving cash for the afternoon). For households to deposit with banks we need that  $R^D > 1$  or, equivalently,

$$\frac{\pi}{\beta} > 1. \quad (63)$$

2. The central bank conducts monetary policy by conducting open market operations, i.e. by changing the amount of bonds  $b_C$  held on its balance sheet. It also sets the price of central bank funding,  $Q^F$ , and the haircut  $\eta$ .

3. Connected banks do not borrow from the central bank,  $\mu_c^F > 0$  and  $f_c = 0$ . Note: In reality, when banks can easily borrow unsecured, they use central bank funding only to manage their expected liquidity needs, like reserve requirements. Those are set to zero in the model. In our model, banks will only access central bank funding when their access to interbank markets is impaired. Indeed, historically, banks have made precautionary use of central bank funding to satisfy (unexpected) liquidity needs only in crisis periods.

A sufficient condition for  $\mu_c^F > 0$  and  $f_c = 0$  is

$$\frac{(1 - \kappa^F) Q^F + \kappa^F}{Q^F} > \frac{\pi}{\beta} = R^D. \quad (64)$$

Note that for  $\kappa^F = 1$ , this condition is equivalent to

$$\frac{1}{Q^F} > \frac{\pi}{\beta}.$$

The condition is intuitive: if the interest rate on central bank funding is higher than the rate on deposits, central bank funding will not be used. It is both more expensive in terms of the interest rate and it requires collateral.

When conditions (62)-(64) hold, we can characterize decisions of connected banks as follows (the proof is in the Appendix).

**Proposition 2 (connected banks)** *Suppose conditions (62)-(64) hold. Then, a connected bank does not borrow from the central bank. A connected bank does not hold any cash. Moreover, if the afternoon constraint of unconnected banks binds,  $\mu_u > 0$ , then a connected bank does not hold any bonds, i.e.,  $b_c = 0$ .*

Connected banks have access to the unsecured market in which they can smooth out liquidity shocks without a need for collateral. Given condition (64), central bank funding is more expensive than the cost of deposits so connected banks will not use it for funding purposes. Similarly, connected banks will not hold any precautionary cash reserves since holding cash carries an opportunity cost. Whenever the afternoon constraint of unconnected banks binds, physical return on bonds is lower than the return on capital as bonds command a collateral premium. However, since connected banks do not need any collateral, they prefer to invest solely in capital.

Decisions of unconnected banks are as follows (the proof is in the Appendix).

**Proposition 3 (unconnected banks)** *Suppose conditions (62)-(64) hold. If the afternoon constraint is slack,  $\mu_u = 0$ , then an unconnected bank does not borrow from the central bank,  $\mu_u^F > 0$  and  $f_u = 0$ . Also, if condition*

$$\tilde{\eta} \geq \eta Q^F \omega^{\max} \quad (65)$$

*holds, then an unconnected bank does not borrow from the central bank. If the afternoon constraint binds and condition*

$$\tilde{\eta} < \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M} \quad (66)$$

*holds, then an unconnected bank does not borrow in the secured market and instead it borrows only from the central bank and holds money,  $m_u > 0$ .*

If the afternoon constraint is slack, unconnected banks are unconstrained in their afternoon borrowing in the secured market. Therefore, they do not borrow from the central bank. Similarly, they do not borrow from the central bank whenever (65) holds, as private haircut  $\tilde{\eta}$  on bonds is favorable compared to the cost of central bank funding  $\eta Q^F$  weighted by the maximum afternoon withdrawals  $\omega^{\max}$ . Since  $\eta Q^F \leq 1$  holds, an even simpler sufficient condition for (65) to hold is  $\tilde{\eta} \geq \omega^{\max}$ . By contrast, whenever (66) holds, private haircut  $\tilde{\eta}$  is so unfavorable that unconnected banks do not use secured market and borrow from the central bank instead. Since unconnected banks borrow from the central bank, their afternoon constraint binds,  $\mu_u > 0$ . It follows that money holdings are positive

$$m_u = \omega^{\max} d_u > 0.$$

## 6 Numerical analysis

In the numerical analysis, we calibrate the model to euro area data. We then use it to evaluate the macroeconomic impact of money market disruptions and the role played by the central bank in mitigating their adverse consequences.

First, we carry out comparative statics exercises under three alternative scenarios: 1) increased haircuts in the secured market; 2) reduced access to the unsecured money market; and 3) increase in possible deposit withdrawals. For each type of disruption, we characterize the effects on financial conditions and real activity. Second, we evaluate the role of the central bank and analyse the effectiveness of alternative monetary policies. More specifically, we compare

policies that stabilize the amount of bonds purchased by the central bank to policies that use asset purchases to stabilize inflation or sovereign bond prices.

Our results highlight the rich dynamics generated by a novel feature of the model, namely the interaction between various occasionally binding constraints. Notice that the model accounts for the combination of twelve such constraints (equation (35) for households, and equations (48) and (50)-(54) for banks). In the numerical analysis, however, we restrict the attention to regions of the parameter space where conditions (62)-(63) are satisfied, and connected banks hold neither bonds nor money, nor do they borrow at the central bank, i.e.  $b_c = m_c = f_c = 0$ . This effectively limits the number of interacting occasionally binding constraints to eight (equations (35), (48) for  $l = u, c$ , and (50)-(54) for  $l = u$ ). It should be noticed that in our model, when a single parameter changes, constraints can turn from binding to slack, and then to binding again, due to the interaction with other similar constraints. The particular constraint that binds is typically crucial for policy interventions, but it is a priori not obvious, which one this will be.

## 6.1 Calibration

In the model, each period is a quarter. We set the depreciation rate at  $\delta = 0.02$ , the capital income share  $\theta$  at 0.33 and the discount factor at  $\beta = 0.995$ .<sup>10</sup>

The fraction of government bonds repaid each period,  $\kappa$ , is 0.11, corresponding to an average maturity of the outstanding stock of sovereign bonds of 9 quarters. The parameters determining the value of collateral in the private market and at the central bank are set equal to each other, at  $\tilde{\eta} = \eta = 0.97$  (where  $1 - \tilde{\eta} = 1 - \eta = 0.03$  corresponds to a 3% haircut). The value for the central bank corresponds to the haircut imposed by the ECB on sovereign bonds with credit quality 1 and 2 (corresponding to a rating AAA to A-) in 2010. The private haircut value is taken from LCH Clearnet, a large European-based multi-asset clearing house, and refers to an average haircut on French, German and Dutch bonds across all maturities in 2010.

Two novel parameters of our model, which capture frictions in the funding markets and are key to determining banks' choices, are the share of "connected" banks,  $\xi$ , and the maximum fraction of deposits that households can withdraw in the afternoon,  $\omega^{\max}$ . We compute the

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<sup>10</sup>The inverse of the discount factor  $1/\beta$  determines the real rate on household deposits. This rate has been very low in the euro area (in fact, it was negative for overnight deposits both before and after the onset of the financial crisis). To match this stylized fact, we choose a relatively high discount rate  $\beta$ .

average pre-crisis value of  $\xi$  using data from Euro Area Money Market Survey (2013), which covers a panel of 98 euro area credit institutions.<sup>11</sup> We set  $\xi$  at 0.42, the average ratio of the annual cumulative quarterly turnover in the unsecured market segment over the sum of the annual cumulative turnover in the secured and unsecured segments, over the period 2004-2007 (where 2004 is the first year with an observation in the survey, and 2007 is the last year before the crisis). When we assess the impact of money market freezes, we also compute the same average for the crisis period, i.e. over the period 2008-2013 (where 2013 is the last available observation in the survey). The average value for that period is 0.24.

We compute  $\omega^{\max}$  using the information embedded in the liquidity coverage ratio (LCR) - a prudential instrument that requires banks to hold high-quality liquid assets (HQLA) in an amount that allows banks to meet 30-days liquidity outflows under stress. As we are interested in maximum outflows, the “stressed” scenario as considered in the LCR appears to be an appropriate empirical counterpart for  $\omega^{\max}$ . The data is taken from the report of the European Banking Authority (December 2013), which provides data for 2012Q4 and covers 357 EU banks from 21 EU countries, 50 Group 1 banks (large banks, CT1 capital of EUR 3 billion or above), and 307 Group 2 banks (CT1 capital below EUR 3 billion). Their total assets sum to EUR 33000 billion, the aggregate HQLA to EUR 3739 billion and their net monthly cash outflows to EUR 3251 billion. We calculate  $\omega^{\max}$  as the ratio of the monthly net cash outflows over total assets so that  $\omega^{\max} = 0.1$ .<sup>12</sup>

We choose the parameter of the foreign demand for bonds,  $\varkappa$ , to ensure that, if foreign bond holdings take a value consistent with their observed share in total debt (as discussed below), then  $Q$  and  $\pi$  also take their average value at that steady state (.961 and 1.005, respectively). The steady state calibration cannot inform us about  $\rho$ , so we pick a reasonably low value to ensure an elastic demand function that stabilizes the return on the bond, i.e. we set  $\rho = 0.09$ .<sup>13</sup> We check robustness to alternative values (not reported) and find little impact on our quantitative assessment.

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<sup>11</sup>The survey provides information on annual cumulative quarterly turnover in the secured and unsecured market segments, as reported at the end of each year’s second quarter. The unsecured segment comprises all unsecured transactions (total maturities and total turnover). The secured market segment is the repo market (and also includes total maturities and turnover).

<sup>12</sup>In our model, banks hold liquid assets in the amount of  $\tilde{\eta}Q (B_u - B_u^F) + M_u$  to cover afternoon withdrawals  $\omega^{\max}D$ . Since  $F = 0$  in our calibrated steady state, and net worth is a small fraction of total liabilities,  $D$  can be approximated with total assets. Assuming that the afternoon constraint binds, we can calculate  $\omega^{\max}$  as the share of the net monthly cash outflow over total assets.

<sup>13</sup>A high elasticity of the foreign demand for bonds is supported by the evidence provided in Koijen et al. (2016). The authors document the change in foreign holdings of euro area sovereign bonds during the Public Sector Purchase Programme implemented by the ECB in March 2015. They find that the foreign sector decreased its holdings by 5.4% during the period 2015Q2-2015Q4, relative to the average holdings during the



We are left with six parameters that we calibrate to match the model-based predictions on some key variables from their empirical counterparts: the share of net worth distributed by banks as dividends,  $\phi$ , the share of assets bankers can run away with,  $\lambda$ , the coefficient determining the utility from money holdings for households,  $\chi$ , the expenditure on public goods,  $g$ , the amount of government bonds purchased by the central bank,  $B^C$ , and the targeted stock of debt in the economy,  $\bar{B}^*$ . The targeted variables are: i) average debt to GDP; ii) bank leverage ; iii) lending spread; iv) share of banks' bond holdings in total debt; v) share of foreign sector's bond holdings in total debt; and vi) average inflation. Table 2 summarizes all parameter values. Table 3 reports the value taken by the six variables in the data (computed over the pre-crisis period, 1999-2006, unless otherwise indicated) and the model prediction under the chosen parameterization.<sup>14</sup>

## 6.2 Macroeconomic impact

We assess the macroeconomic impact of money market disruptions of the size observed in the euro area by means of a comparative statics analysis.

We assume here that the central bank sets the price of its loans to banks,  $Q_F$ , the haircut on collateralized operations,  $\eta$ , and the stock of sovereign bonds on its balance sheet,  $b_C$ , at constant values. We will evaluate alternative policies in section 6.2.3.

### 6.2.1 Disruptions of the secured interbank market

We analyse the impact of disruptions on the secured market by changing the haircut set on collateral,  $1 - \tilde{\eta}$ . Figures 4 and 5 show the results of a comparative statics exercise in which the haircut moves from the benchmark pre-crisis value of 3 percent (denoted with a red solid line) to 70 percent. Figure 4 reports the response of quantities in the first four rows and prices in the last row. Figure 5 reports in the first column the bank return to investing a unit of net worth in each type of assets (capital,  $\tilde{\psi}_k$ , bonds,  $\tilde{\psi}_B/Q$ , and money,  $\tilde{\psi}_M$ ) and the cost related to each type of liabilities (deposits,  $\tilde{\psi}_k$ , and central bank funding,  $\tilde{\psi}_F/Q_F$ ). In the second column, it shows the lagrangean multipliers associated to the occasionally binding constraints of unconnected banks (the run away constraint,  $\mu_u^{RA}$ , the collateral constraint for central bank

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period 2013Q4-2014Q4. Over the same period, the average yield on euro area sovereign bonds declined by 63 bps, indicating a high elasticity of the foreign demand to the nominal bond return.

<sup>14</sup>Both the value of leverage and the average lending spread for the euro area are taken from Andrade et al. (2016). The share of banks' and of foreign sector's bond holdings in total debt is computed using data for 2015 reported in Koijen et al. (2016).

funding,  $\mu_u^C$ , the afternoon constraint,  $\mu_u$ , the non-negativity constraint on cash holdings,  $\mu_u^M$ , and on central bank funding,  $\mu_u^F$ ).<sup>15</sup> The figures report three dashed coloured lines. The cyan and magenta dotted lines denote respectively the level of the private haircut at which the multipliers  $\mu_u^M$  and  $\mu_u^F$  become zero. At those points, the non-negativity conditions on cash holdings and central bank funding, equations (51) and (53), become slack for unconnected banks. In correspondence of the black line,  $\mu_u^C$  turns positive, implying that for higher values of the private haircut, the collateral constraint of unconnected banks (equation (50)) becomes binding, so that  $B_{t,u} = B_{t,u}^F$ . Unconnected banks pledge the entire amount of bonds they hold at the central bank.

To understand the complex interaction among the different occasionally binding constraints, it is useful to consider figure 4 and start from the calibrated steady state which is marked with a red line in correspondence to a 3 percent haircut. At that level, the collateral premium (the wedge between the return on capital and the return on bonds,  $\tilde{\psi}_k - \tilde{\psi}_B/Q$ ) is positive and the afternoon constraint binds for unconnected banks ( $\mu_u$  is positive). As the haircut increases between the red and cyan lines, the afternoon constraint becomes tighter for unconnected banks ( $\mu_u$  increases). These banks maintain their stock of deposits largely unchanged, and satisfy the afternoon constraint by pledging higher amounts of bonds. Notice that the stock of bonds accepted as collateral is in given supply, and held by three types of agents: the central bank (in fixed amounts), the foreign sector, and unconnected banks. As the bond price  $Q$  increases to reflect the higher demand, the real return to holding bonds,  $\tilde{\psi}_B/Q$  falls. This induces foreign investors to sell their bonds to domestic unconnected banks, satisfying their higher demand.

At the cyan line, the return on holding bonds has lowered to the point that unconnected banks become indifferent between holding bonds or cash. The non-negativity condition on cash holdings of unconnected banks (equation (51)) becomes slack. In the region between the cyan and magenta dotted lines, unconnected banks reduce their stock of bonds and satisfy the afternoon constraint by holding positive amount of cash instead. The bond price falls, increasing the nominal return to holding bonds. Inflation rises in order to induce households to hold less money and to ensure clearing of the money market, given the constant stock of money injected by the central bank. Overall, the real return to holding bonds for foreign

<sup>15</sup>The multiplier on the run-away constraint of connected banks,  $\mu_c^{RA}$ , is not shown as it remains positive in our exercises, indicating that connected banks always lever up to the maximum in order to invest in capital. The multiplier of the non-negativity constraints for bond holdings of unconnected banks,  $\mu_{t,u}^B$ , is not shown either, as it is always zero in the analysis presented in this section.

investors rises, inducing them to increase their demand and domestic banks to shift even further into cash holdings. Unconnected banks reduce somewhat their investment in capital as a consequence of a smaller deposit base. Connected banks also mildly reduce it because the overall supply of households' deposits decreases.

When the haircut increases to the right of the magenta line, it becomes attractive for unconnected banks to use central banking funding. Although the interest rate on central bank funding is higher than the one on private secured market funding, the central bank haircuts become increasingly favourable relative to those imposed in the private market. Unconnected banks gradually reduce the share of bonds pledged in the private market and increase the share pledged at the central bank. In the area between the red and black dotted lines, the collateral constraint of unconnected banks is slack. Those banks pledge at the central bank an amount of bonds lower than the amount held on the balance sheet, so that private secured market funding and central bank funding coexist.

Only when the private haircut exceeds the level at the black dotted line, the difference between private and central bank haircuts is sufficiently high to induce unconnected banks to pledge their entire stock of bonds at the central bank. The collateral constraint (equation (50)) becomes binding. The availability of central bank funding helps to stabilize the economy. As there is no use of bonds as collateral in the secured market, the level of the private haircut becomes irrelevant. Unconnected banks are able to stabilize the amount of deposits they raise and thus their investment in capital. Inflation remains constant because there is no need to reallocate money from households to banks. Connected banks receive a stable share of total deposits and are therefore able to keep their investment in capital constant.

Overall, an increase of  $1 - \tilde{\eta}$  from 3 to 45 percent, i.e. the point where unconnected banks use only central bank funding, generates an output contraction of 0.65 percent. The reason for the contained macroeconomic impact is that the availability of central bank funding sets a floor to the decline in deposits and capital. In section 6.3 below, we show that the contractionary impact on output would be much more severe (2 percent when private haircuts are around 40%, 5.5% when they are 70 percent), were the central bank not to provide direct funding to banks.

## 6.2.2 Disruptions of the unsecured interbank market

The second exercise we conduct aims at analyzing the macroeconomic effects of a drying up of the unsecured money market segment. Figure 6 shows the results of a comparative statics

exercise in which the share of unconnected banks,  $1 - \xi$ , is increased from 0.5 to 0.9. The figure reports a selection of the variables shown in figure 4. The solid red line denotes the share of unconnected banks under our benchmark calibration ( $1 - \xi = 0.58$ ). The dashed red line marks the share of unconnected banks observed on average over the period 2008-2013 ( $1 - \xi = 0.76$ ).

At the red line, the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. The amount of deposits raised by connected and unconnected banks is of a broadly comparable magnitude. Unconnected banks, however, have a lower capital stock than connected banks, as they need to invest part of the funds in bonds to be pledged in the secured market in the afternoon. At that point, the return on bonds is higher than the return on money (not shown), and unconnected banks choose not to hold cash for the afternoon. As the share of unconnected banks increases moving rightward in the figure, a larger number of banks needs to satisfy possible afternoon liquidity withdrawals, raising the aggregate demand for bonds and the bond price. In the region where  $1 - \xi < 0.7$ , the real return on bonds falls for foreign investors, inducing them to sell part of their bond holdings to domestic banks. The amount of bonds held by each unconnected bank,  $b_u$ , nonetheless falls, as more banks need to hold bonds as collateral, and the supply of bonds is fixed. When the share of unconnected banks increases further, i.e. when  $1 - \xi$  exceeds .7, the increase in the price of bonds reduces further the return on holding bonds relative to cash. Unconnected banks start reducing their stock of bonds and holding cash instead to satisfy the afternoon constraint. As the central bank keeps the amount of sovereign bonds on its balance sheet constant, as well as the supply of money, households need to decrease money holdings to clear the market, which is facilitated by an increase in inflation. The limited availability of bonds and the increasing cost of holding money implies that unconnected banks also contract deposits, as an additional way to satisfy afternoon constraint. Lower deposits reduce unconnected banks' liquidity needs in case of unexpected withdrawals but also their investment in capital. Despite the high return to capital, connected banks need to reduce investment because the overall amount that households deposit at banks fall. As a consequence, aggregate capital and output fall in the economy.

In the benchmark calibration with  $\omega^{\max} = 0.1$ , the contraction in real activity induced by disruptions in the unsecured market is moderate. A fall in the share of connected banks from  $1 - \xi = 0.42$  (pre-crisis average share of unsecured transactions in total) to  $1 - \xi = 0.24$  (post-crisis average) generates a decline in real activity of around 0.5 percent. Planning for moderate liquidity outflows in the afternoon does not constrain unconnected banks too much.

The amount of resources diverted from investment in capital to investment in unproductive assets (bonds) is limited.

It should be noticed that disruptions in the unsecured market are typically accompanied by fears of higher customers' deposit withdrawals, as in the first phase of the financial crisis. In a scenario where the share of unconnected banks,  $1 - \xi$ , increases, and  $\omega^{\max}$  is also raised from 0.1 to 0.2, results (not reported) are qualitatively similar. However, the reduction in output is much larger, amounting to about 4 percent. This higher contraction is driven by a larger distortion in the allocation of savings, whereby funds are diverted away from productive capital into unproductive assets.

### 6.2.3 Increased risk of a depositor run

We explore the macroeconomic impact of higher risk of runs on deposits by increasing the maximum level of expected withdrawals,  $\omega^{\max}$ , from 0.1 (the value under our benchmark calibration) to 0.6. Figure 7 shows the results. As the maximum share of possible deposit withdrawals,  $\omega^{\max}$ , rises above its calibrated level, unconnected banks become increasingly constrained in the afternoon and this generates a sharp initial increase in the collateral premium. For  $\omega^{\max} < 0.12$ , the return on bonds falls but remains above the return on money, so that unconnected banks choose to satisfy the tighter afternoon constraint by increasing their stock of bonds. When  $\omega^{\max} = 0.12$ , the two returns equalize and the non-negativity condition on cash held by unconnected banks becomes slack. Unconnected banks start holding money to satisfy their afternoon constraint, and this allows them to mildly reduce their stock of bonds. Clearing of the money market requires households to hold less money. This is induced by a rise in inflation which increases the cost of holding money and the return on households' deposits. The progressive increase in the collateral premium and in inflation reduces the value to the bank of investing in assets (money and bonds), up to the point  $\omega^{\max} = .18$ , where  $\mu_u^{RA}$  becomes zero and the run-away constraint (equation (48)) becomes slack for unconnected banks. Indeed, for higher levels of  $\omega^{\max}$ , these banks do not to lever up to the maximum. They rather reduce deposits, since their main use is to purchase assets with low return (bonds and money). Both the stock of deposits and the amount of cash held for the afternoon thus decline. When  $\omega^{\max} = .48$ , cash of unconnected banks is down to zero. Any further reduction in deposits thus enables banks to reduce their holdings of bonds. The bond price falls, inducing foreigners to buy the share of bonds not held by domestic banks or the central bank. Connected banks

hold the amount of deposits supplied by the households and not held by unconnected banks, and use these funds to increase their investment in capital.

Overall, the economy experiences a reduction in the aggregate amount of deposits that severely limits productive investment and reduces the capital stock. A doubling of the expected deposit withdrawals compared to the calibrated steady state value, i.e. an increase in  $\omega^{\max}$  from 0.1 to 0.2, generates output losses of 3 percent.

### 6.3 Central bank policies

What role does the central bank play in mitigating the macroeconomic impact of money market disruptions? To answer this question, we first evaluate the impact on real activity of a secured market freeze of the type analysed in section 6.2.1, in the absence of central bank funding.

We then assess the scope for using two alternative monetary policies. The first is one where the central bank changes the amount of asset purchases and the size of its balance sheet to stabilize inflation. This policy captures the main features of quantitative easing, a non-standard measure widely used by central banks during the crisis. The second is a policy where the central bank uses the same instruments to maintain a constant price of bonds. This policy resembles the Securities Markets Programme, which was implemented by the Eurosystem at the peak of the sovereign crisis and consisted in interventions in public debt securities markets.

#### 6.3.1 The role of central bank lending

We run a counterfactual exercise where banks do not fund themselves at the central bank because this latter charges prohibitively high haircuts on the collateral pledged ( $1 - \eta = .99$ ). Figure 8 shows the results of the same comparative statics exercise shown in figure 4, where the private haircut,  $1 - \tilde{\eta}$ , moves from zero to 70 percent.

In this exercise, the cyan and black dotted lines indicate the point where the non-negativity constraint on cash holdings and run-away constraint (equations (51) and (48), for  $l = u$ ), respectively, become slack for unconnected banks. At the right of the cyan dotted line, when haircuts imposed by the private market become increasingly unfavourable, unconnected banks are unable to access central bank funding, and have to increase either cash or bonds to be able to satisfy possible afternoon withdrawals. Inflation rises to induce households to reallocate their wealth from money to deposits, so that unconnected banks are able to satisfy their increasing demand for cash. Since the real return on bonds rises, foreign investors buy domestic bonds, forcing banks to decrease their bond holdings and to increase demand for cash further.

At  $1 - \tilde{\eta} = 0.42$ , inflation has induced households to run down to zero money holdings so that unconnected banks cannot keep increasing their stock of cash. Their run-away constraint becomes slack and these banks stop leveraging up to the maximum. They instead reduce deposits and sharply increase their bond holdings for insurance purposes, which induces them to scale down their investment in capital. The contraction in deposits by unconnected banks is larger than the reduction in the supply of deposits by households. Connected banks are thus able to increase their deposits and investment in capital. The overall effect on the economy is nonetheless a sharp contraction in investment. Output falls by 2 percent when the haircut increases to 40 percent, and by 5.4 percent when the haircut is at 70 percent.

These results point to the importance of central bank funding to offset disruptions in secured money markets. In particular, they suggest that collateral policy can be an effective instrument to counteract fluctuations in private market asset valuation. It should be noticed, however, that central bank operations are unable to mitigate the adverse effects arising from unsecured money market freezes. Figure 6 shows that, even when the share of unconnected banks increases and higher demand for bonds raises their liquidity value, unconnected banks never choose to fund themselves at the central bank. This latter option is too expensive relative to the alternative of funding themselves in the private secured markets. The reason is that, in our benchmark calibration, the haircut applied by the private market and the central bank are identical. At the same time, the private market is active within the period and therefore funding in that market imposes a gross interest rate of one. The central bank, instead, provides funding whose repayment becomes due the following period and charges a higher interest rate.<sup>16</sup> Similar considerations hold for the case of fears of increased afternoon withdrawals. Central bank lending operations provide funds that can be used to satisfy bank's liquidity needs. However, the same funds can be obtained at more favourable terms in the private markets.

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<sup>16</sup>This feature of our model captures an important difference between funds available in the private market or at the central bank. For instance, the Eurosystem provides regular main refinancing operations with a weekly frequency and longer-term operations with a duration of three months. Transactions taking place in the interbank market are typically of a shorter maturity (a large share being overnight).

### 6.3.2 Alternative monetary policies

The analysis conducted in section 6.2 assumes that the central bank maintains its three policy instruments constant, namely the price of bank loans,  $Q_F$ , the haircut on collateralized operations,  $\eta$ , and the stock of sovereign bonds on its balance sheet,  $b_C$ . We analyse here the role of two alternative monetary policies.

**Stabilizing inflation** In all the comparative exercises presented so far, money market disruptions generate sizeable movements in inflation. These movements are needed to ensure clearing of the money market when unconnected banks demand more cash. During the crisis, inflation remained subdued, possibly due to a variety of non-standard policies conducted by central banks with the aim of stabilizing inflation. We conduct the comparative statics exercise reported in section 6.2.1, under an alternative policy that stabilizes the inflation rate at the level taken in the calibrated steady state. The central bank thus chooses to keep constant the price of the loans,  $Q_F$ , and the haircut,  $\eta$ , while changing the amount of bonds purchased to maintain inflation at its long-run average.

Figure 9 reports the results of the exercise when the private haircut is moved from 3 to 70 percent, as in figure 4. For  $1 - \tilde{\eta} < 0.1$ , the behaviour of the economy is unchanged under this alternative monetary policy. Despite the increase in the private haircut and in the price of bonds, unconnected banks can still satisfy their afternoon constraint by holding only bonds. Inflation remains constant as it does not need to reallocate money holdings from households to banks. When the private haircut takes the value of 0.1, the collateral value of bonds become so low that unconnected banks choose to hold also money. At that point, the central bank starts expanding the balance sheet via purchases of sovereign bonds,  $b_C$ . As foreign investors do not change substantially their bond position, the government essentially buys bonds from unconnected banks, giving them newly created money in exchange. The increase in the stock of money in circulation helps to satisfy the higher demand for cash by unconnected banks without the need for households to decrease their money holdings and for inflation to rise. Because asset purchases and the money injected by the central bank increases with  $1 - \tilde{\eta}$ , unconnected banks can satisfy their afternoon constraint without reducing their deposits and thus investment in capital. At the point where  $1 - \tilde{\eta} = .4$ , the central bank has fully replaced private banks in holding sovereign bonds.



Under this policy, the negative impact of the secured money market freeze is only one third of the one arising under a policy that keeps constant the amount of bonds on the central bank's balance sheet. An increase in the private haircut from 3 to 40 percent generates an output contraction of -0.2 percent.

**Stabilizing the price of government bonds** We conduct the same comparative statics exercise reported above under a policy that stabilizes the bond price at the level taken in the calibrated steady state. The central bank keeps constant the price of the loans,  $Q_F$ , and the haircut,  $\eta$ , while changing the amount of bonds purchased,  $b_C$ .

As the private haircut increases from the level taken at the steady state, banks initially manage higher liquidity needs by increasing their stock of bonds, while leaving deposits broadly unchanged and holding no cash. The higher demand for bonds is satisfied through bond sales from foreigners, whose return has lowered due to higher inflation. The price of bonds thus remains constant. The increased funds needed by unconnected banks to purchase bonds generates a mild reduction in investment.

At the point where  $1 - \tilde{\eta} = 0.1$ , it is not possible to satisfy the increased bond demand with foreign investor's supply, while keeping the price of bonds constant, so the central bank increases its own bond purchases. Domestic banks sell their bonds to the central bank, receiving in return cash that can be used to satisfy their afternoon constraint. The lower diversion of resources towards unproductive assets enables unconnected banks to invest more in capital and to better stabilize output, relative to a policy with a constant size of the balance sheet.

A policy that stabilizes the price of bonds requires a less gradual expansion of the balance sheet, relative to the case when inflation is kept constant. As banks are able to substitute bonds with low liquidity value with newly issued money at lower levels of the private haircut, this policy almost completely offset the negative impact of a secured market freeze. The change in output private haircut from 3 to 40 percent is -0.03 percent.

## 7 Conclusions

We presented a general equilibrium model where banks can finance their liquidity needs in the unsecured or secured interbank markets, and where they have also access to collateralized central bank funding. The model accounts for the reduced ability of banks to access the unsecured market during the financial and sovereign crisis, and their shift to secured market funding. It

also accounts for the impaired functioning of the secured market during the sovereign crisis, increased fears of customers' deposit withdrawals, and higher reliance on central bank liquidity provision, particularly for banks located in countries with a vulnerable sovereign.

Our model shows that disruptions of different segments of the money markets transmit differently to the real economy. The specific channel of transmission depends on the complex interaction among multiple occasionally binding constraints faced by banks. In all cases, the macroeconomic impact can be sizeable.

We find that the central bank plays a key role in shielding the economy from the adverse impact of disruptions in money markets. Changes in the haircut set by the central bank on collateral used in financing operations, or expansions in the central bank's balance sheet, are desirable policy instruments when tensions arise in the secured market. However, no single policy is beneficial under all types of money market disruptions.

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**Table 1: ECB vs private haircuts on sovereign bonds**

	ECB		Private	
	CQS1-2	CQS3	Germany	Portugal
2010	2.8	7.8	2.7	8.1
2011	2.8	7.8	3.0	10.1
2012	2.8	7.8	3.0	80.0
2013	2.8	7.8	3.0	80.0
2014	2.2	9.4	3.0	80.0

Source: ECB and LCH Clearnet. CQS1-2 refers to sovereign bonds with credit quality 1 and 2, corresponding to a rating AAA to A-. CQS3 refers to bonds with credit quality 3, corresponding to a rating BBB+ to BBB-. Data for private haircuts for 2010 (under the column 'Germany') refers to an average haircut on bonds from France, Germany and the Netherland.

**Table 2: Calibration targets**

Variable	Data	Model
Debt/GDP	0.57	0.61
Bank leverage	6.00	5.81
Loan spread (annual)	0.021	0.021
Share bonds unconnected banks	0.23	0.24
Share bonds foreign sector	0.64	0.60
Inflation (annual)	0.02	0.02

**Table 3: Parameter values**

Parameter	Description	Value
$\theta$	Capital share in income	0.33
$\delta$	Capital depreciation rate	0.02
$\beta$	Discount rate households	0.995
$\chi$	Coefficient in households' utility	0.006
$g$	Government spending	0.442
$\kappa^{-1}$	Average maturity bonds (quarters)	9
$\phi$	Fraction of net worth paid as dividends	0.031
$\xi$	Fraction of banks with access to unsecured market	0.42
$\tilde{\eta}$	Haircut on bonds set by banks	0.97
$\eta$	Haircut on bonds set by central bank	0.97
$\lambda$	Share of assets bankers can run away with	0.153
$\omega^{\max}$	Max possible withdrawal as share of deposits	0.1
$B_C$	Bonds held by central bank	1.045
$B^*$	Stock of debt	6.58
$\varrho$	Parameter foreign bond demand	0.1

Figure 4: Comparative statics: changing the value of the private haircut,  $1 - \tilde{\eta}$

The figure shows the response of quantities and prices, as the private haircut is increased from 3 to 70 percent. The red solid line denotes the calibrated steady state. The cyan and magenta dotted lines denote respectively the level of the haircut at which the non-negativity conditions on cash holdings and central bank funding become slack for unconnected banks. The black dotted line denotes the level of the private haircut at which their collateral constraint becomes binding.

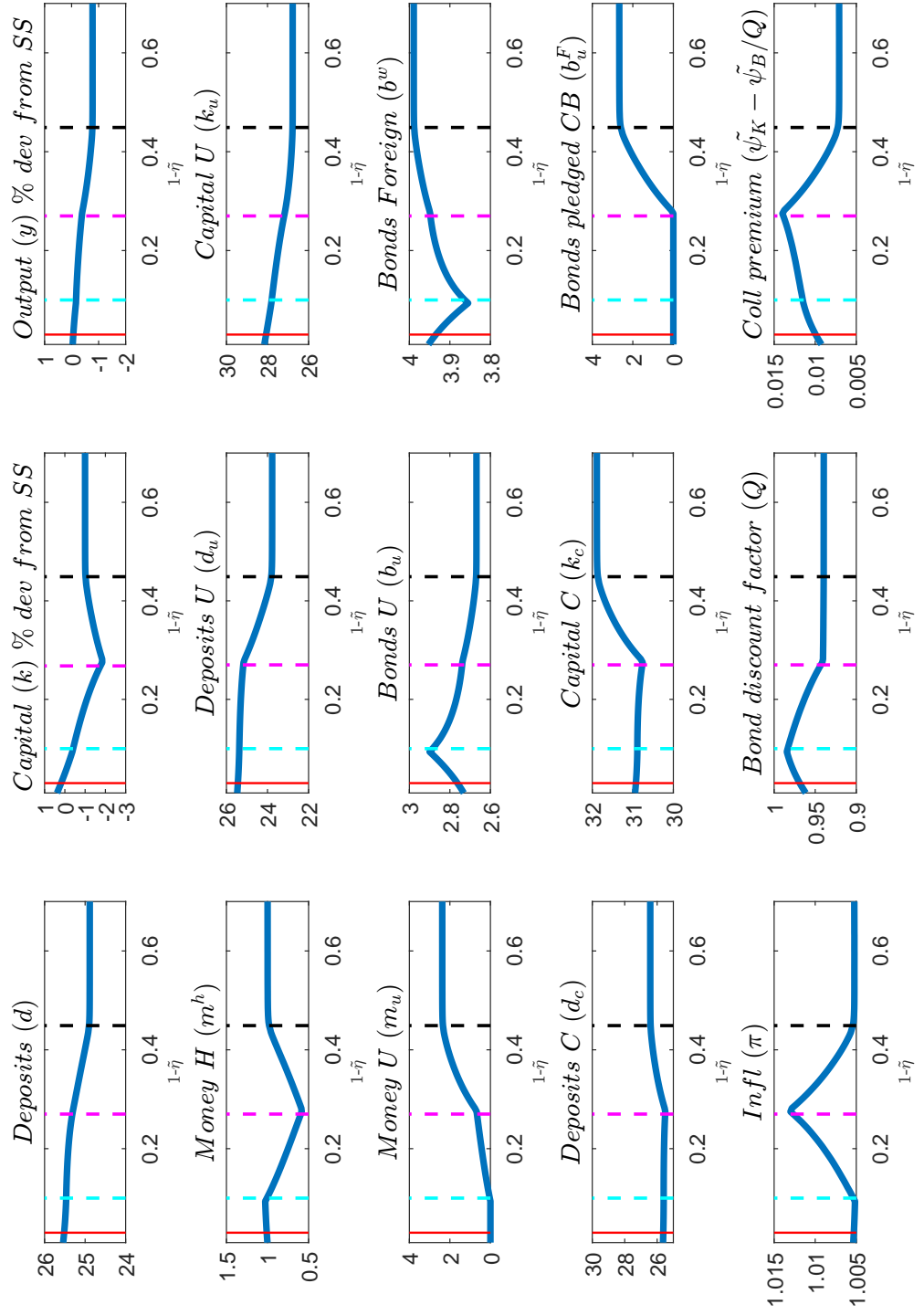


Figure 5: Comparative statics: changing the value of the private haircut,  $1 - \tilde{\eta}$  [cont'd]

The figure shows the returns on assets and liabilities, and the Lagrange multipliers, as the private haircut is increased from 3 to 70 percent. The red solid line denotes the calibrated steady state. The cyan and magenta dotted lines denote respectively the level of the haircut at which the non-negativity conditions on cash holdings and central bank funding become slack for unconnected banks. The black dotted line denotes the level of private haircut at which their collateral constraint becomes binding.

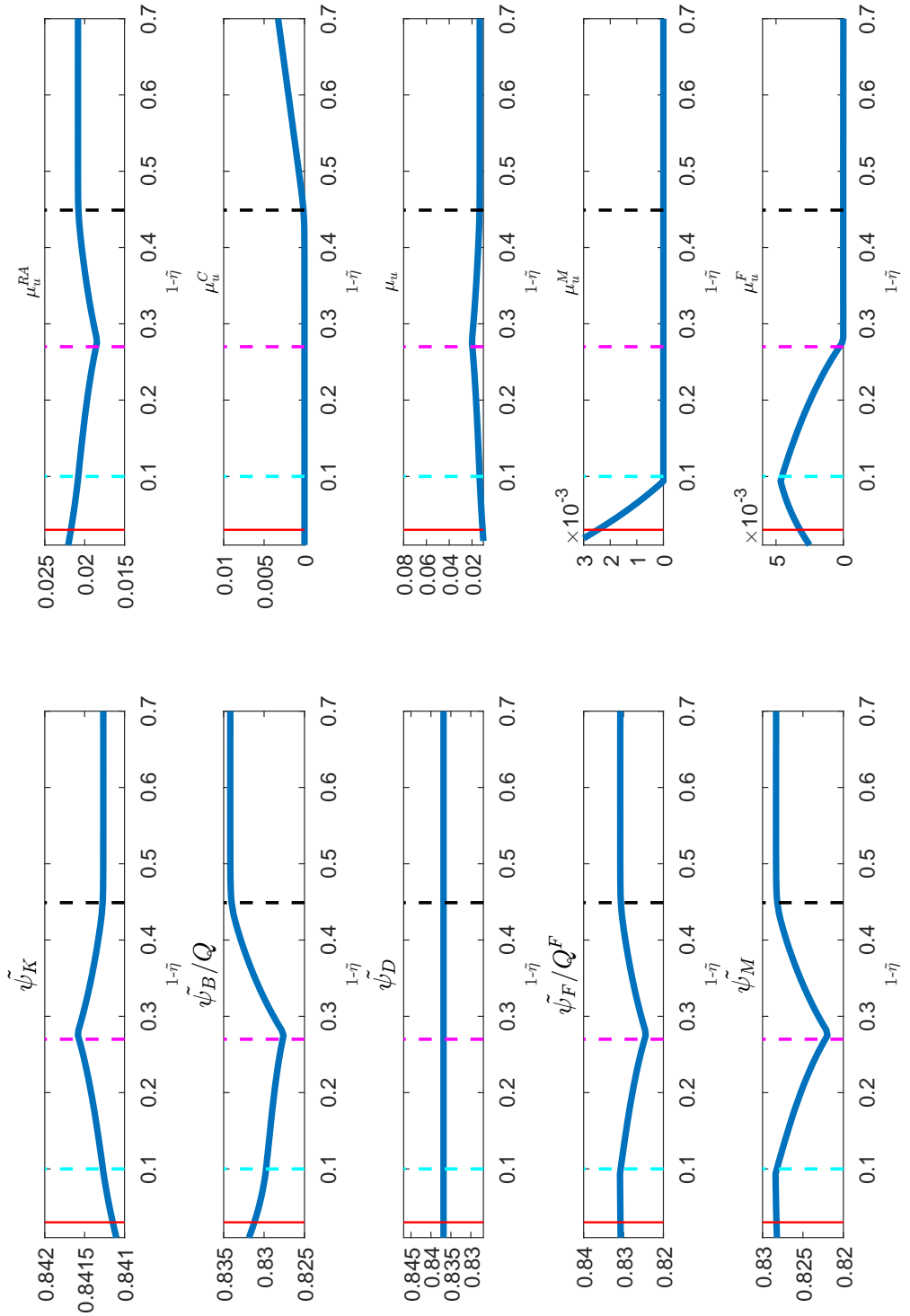


Figure 6: Comparative statics: changing the share of banks with no access to unsecured funding,  $1 - \xi$

The figure shows the behaviour of the economy as the share of unconnected banks is increased from 50 to 90 percent. The red solid line denotes the calibrated steady state, where  $1 - \xi = 0.58$ . The red dotted line denotes the lower post-crisis average of connected banks,  $1 - \xi = 0.76$ .

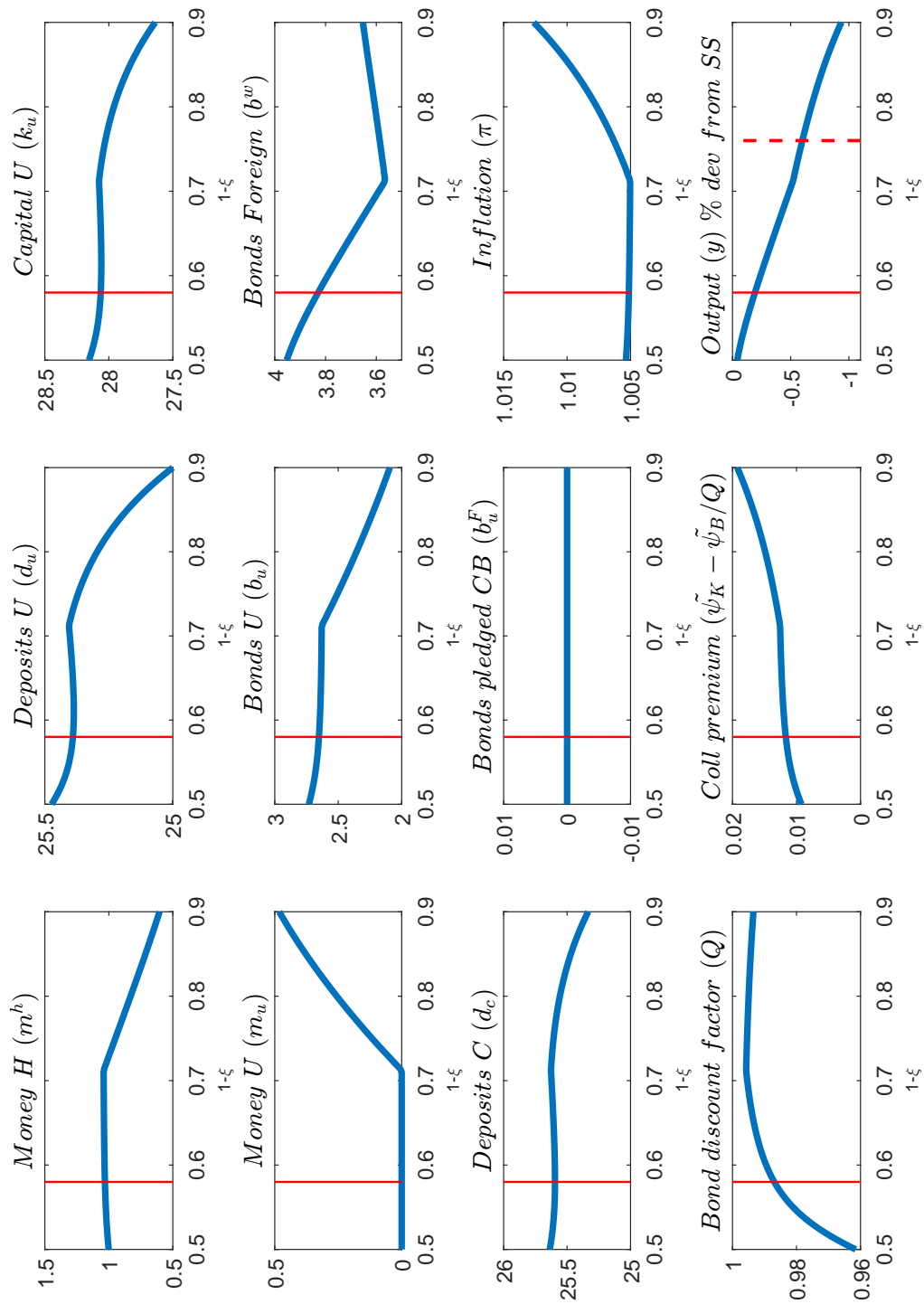




Figure 7: Comparative statics: changing the share of possible afternoon withdrawals,  $\omega^{\max}$

The figure shows the behaviour of the economy as the share of possible afternoon withdrawals is increased from 10 to 50 percent. The red solid line denotes the calibrated steady state, where  $\omega^{\max} = 0.1$ .

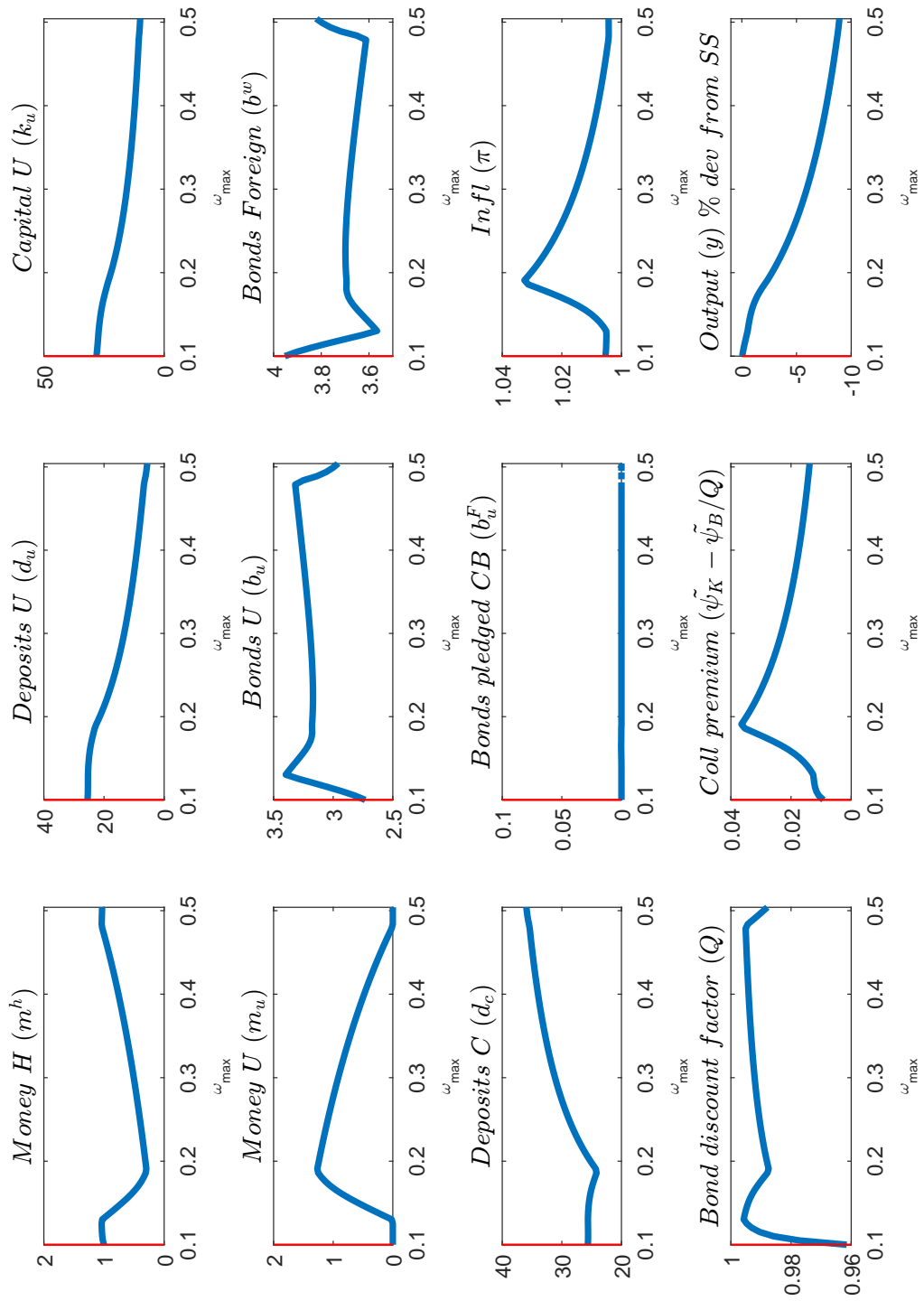


Figure 8: Changes in private haircut without central bank refinancing operations

The figure shows the behaviour of the economy as the private haircut is increased from 3 to 70 percent. The cyan and black dotted lines denote respectively the levels of the haircut at which the non-negativity conditions on cash holdings and the run away constraint of the unconnected banks respectively become slack. The red solid line denotes the calibrated steady state.

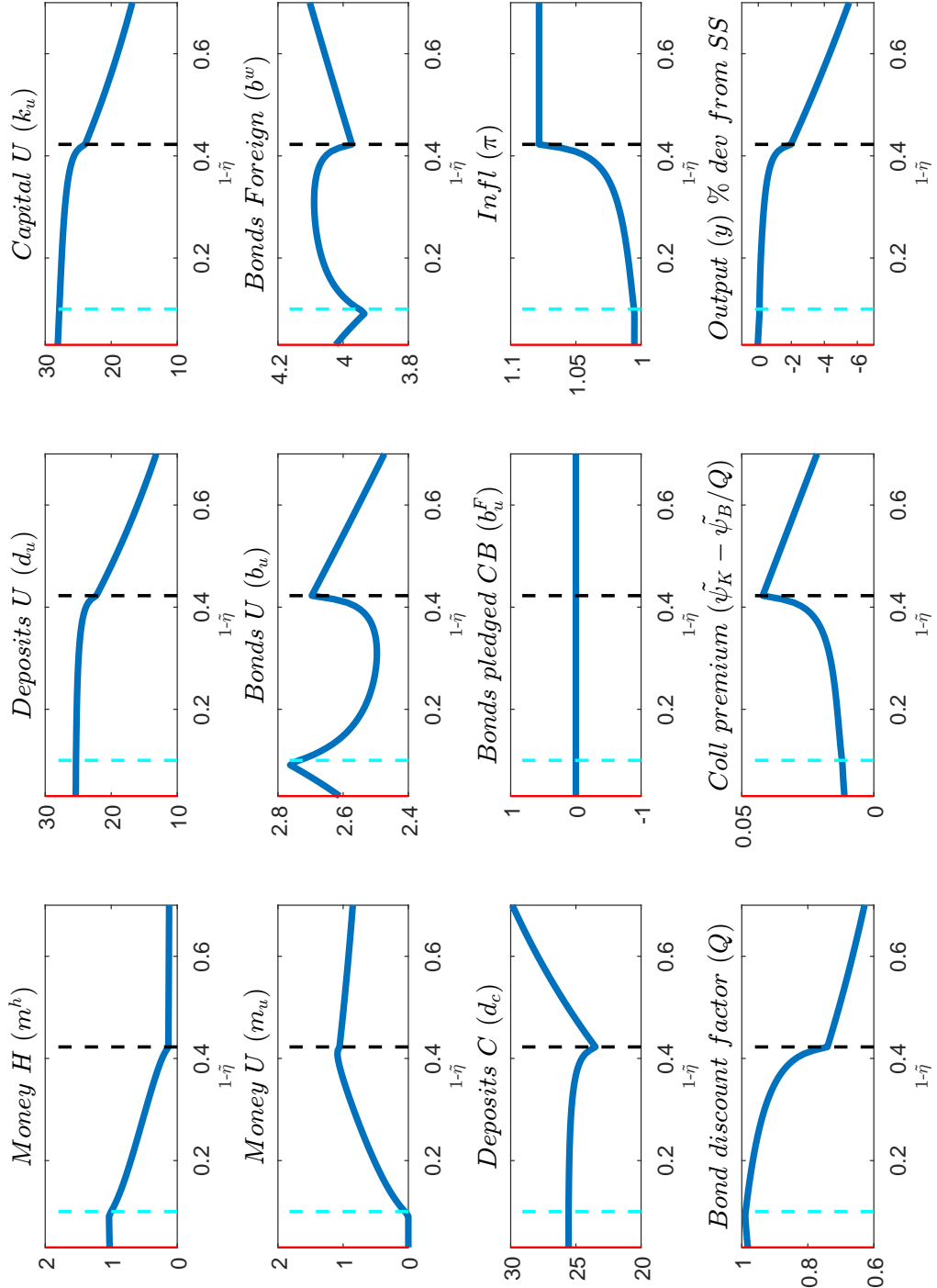


Figure 9: Changes in private haircut under constant inflation

The figure shows the behaviour of the economy as the private haircut is increased from zero to 70 percent and the central bank keeps inflation constant. The red solid line denotes the calibrated steady state.

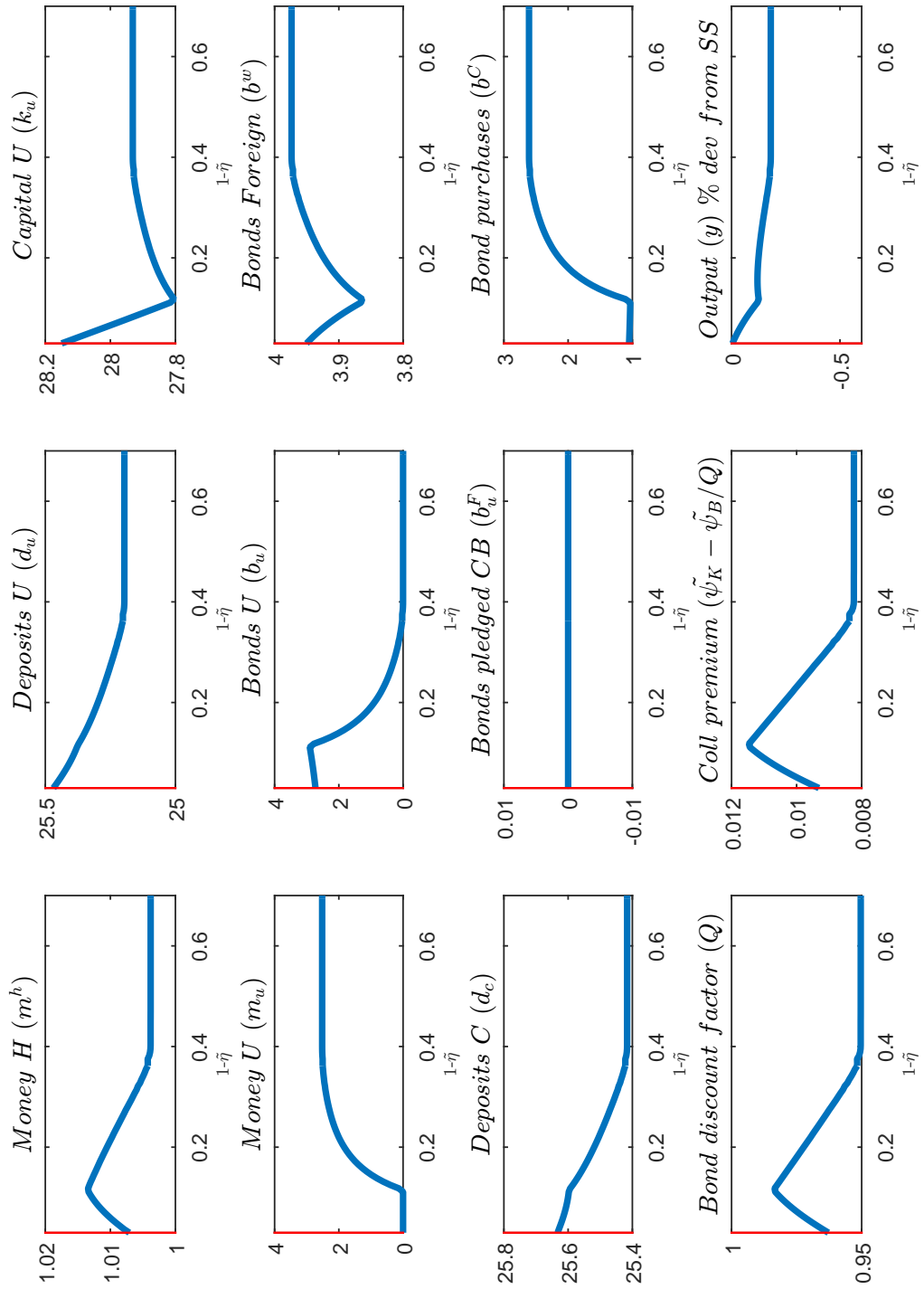
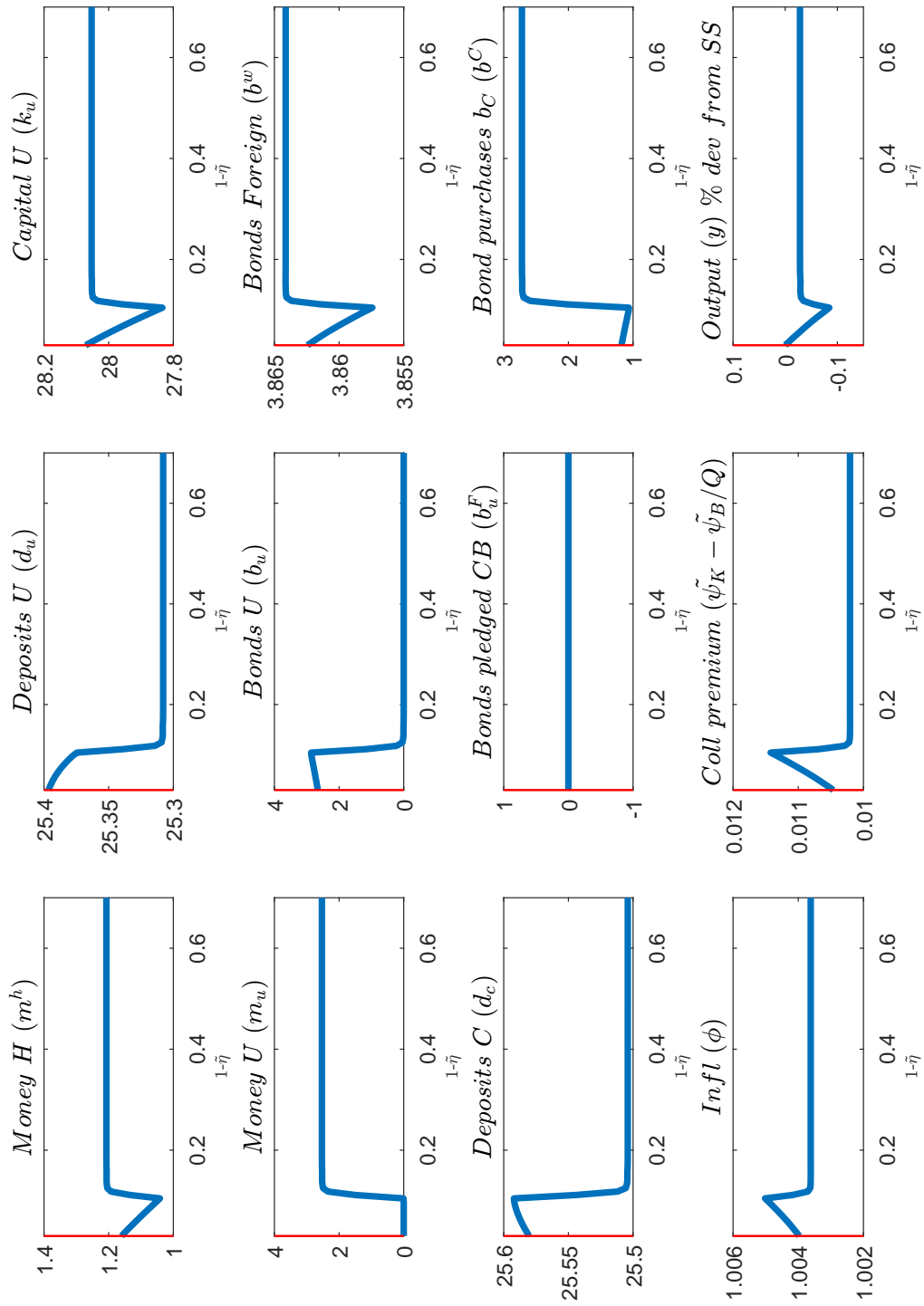


Figure 10: Changes in private haircut under constant bond prices

The figure shows the behaviour of the economy as the private haircut is increased from zero to 70 percent and the central bank keeps bond prices constant. The red solid line denotes the calibrated steady state.



## A The equations characterizing the steady state

We characterize the steady state of the model. For simplicity, we focus on the case when capital is not accepted as collateral at the central bank,  $\eta^K = 0$ . Recall that we already assume that capital is not accepted as collateral in the private secured market,  $\tilde{\eta}^k = 0$ .

Define a generic variable as the corresponding capital letter variable, divided by the contemporaneous price level, i.e.  $x_t = \frac{X_t}{P_t}$ . The steady state is characterized by the following conditions:

1. 4 household equations:

$$\begin{aligned}
 R^D &= \frac{\pi}{\beta} \\
 -\frac{u_l(c, l)}{u_c(c, l)} &= (1 - \tau) w \\
 v_M(m^h) &= u_c(c, n) (R^D - 1) - \mu_h^M \\
 c &= (1 - \tau) w l + \left(\frac{1}{\beta} - 1\right) \pi d + (1 - \pi) m^h + \phi n
 \end{aligned}$$

2. 3 firms' equations:

$$\begin{aligned}
 y &= \gamma k^\theta l^{1-\theta} \\
 w l &= (1 - \theta) y \\
 r k &= \theta y
 \end{aligned}$$

and

$$I = \delta k.$$

3. 5 central bank equations: 2 equations

$$\begin{aligned}
 s &= Q^F \bar{f} + Q b_C - \bar{m} \\
 \bar{m} &= \left[ Q^F - \kappa^F \frac{1}{\pi} (1 - Q^F) \right] \bar{f} + \left[ Q - \kappa \frac{1}{\pi} (1 - Q) \right] b_C
 \end{aligned}$$

plus the value of 3 variables (policy instruments):  $\eta, Q^F, b_C$ .

Note that the seigniorage revenue of the central bank is given by the interest rate payments on its assets:

$$s = \kappa^F \frac{1}{\pi} (1 - Q^F) \bar{f} + \kappa \frac{1}{\pi} (1 - Q) b_C.$$

4. 2 government equations:

$$\bar{b} = \bar{b}^*$$

$$\tau^* (1 - \theta) y = g + \kappa (1 - Q) \frac{\bar{b}^*}{\pi} - Q \left(1 - \frac{1}{\pi}\right) \bar{b}^* - s.$$

where  $g$  is exogenous.

5. 4 market clearing equations:

$$\bar{f} = f$$

$$\bar{m} = m + m^h$$

$$\bar{b} = b + b_C + b_H + b^W$$

$$y = c + c^W + g + I$$

where the market clearing condition for the goods market (last equation above) is redundant due to the Walras law.

6. 45 bank equations:

8 equations common to  $c$  and  $u$  banks,

$$\nu = \psi n$$

$$n = \max \left\{ 0, (r + 1 - \delta) (\xi k_c + (1 - \xi) k_u) \right. \\ \left. + (\xi m_c + (1 - \xi) m_u) \frac{1}{\pi} \right. \\ \left. + ((1 - \kappa) Q + \kappa) (\xi b_c + (1 - \xi) b_u) \frac{1}{\pi} \right. \\ \left. - ((1 - \kappa^F) Q^F + \kappa^F) (\xi f_c + (1 - \xi) f_u) \frac{1}{\pi} \right. \\ \left. - \frac{1}{\beta} (\xi d_c + (1 - \xi) d_u) \right\},$$

$$\tilde{v} = \tilde{\psi}_k k + \tilde{\psi}_B b + \tilde{\psi}_M m - \tilde{\psi}_D d - \tilde{\psi}_F f$$

$$\begin{aligned}
\tilde{\psi}_k &= \beta(1-\phi)\psi(r+1-\delta) \\
\tilde{\psi}_B &= \beta(1-\phi)\frac{1}{\pi}\psi[(1-\kappa)Q+\kappa] \\
\tilde{\psi}_D &= \beta(1-\phi)\frac{1}{\beta}\psi \\
\tilde{\psi}_F &= \beta(1-\phi)\frac{1}{\pi}\psi((1-\kappa^F)Q^F+\kappa^F) \\
\tilde{\psi}_M &= \beta(1-\phi)\frac{1}{\pi}\psi
\end{aligned}$$

18 equations for  $l = c, u$ :

$$k_l + Qb_l + m_l + \phi n = d_l + Q^F f_l + n$$

$$\phi n + \tilde{v}_l = \lambda(k_l + Qb_l + m_l)$$

$$v_l = \phi n + \tilde{v}_l$$

$$\tilde{v}_l = \tilde{\psi}_k k_l + \tilde{\psi}_B b_l + \tilde{\psi}_M m_l - \tilde{\psi}_D d_l - \tilde{\psi}_F f_l$$

$$f_l = \eta Q b_l^F$$

$$\mu_l^F f_l = 0$$

$$\mu_l^M m_l = 0$$

$$\mu_l^C (b_l - b_l^F) = 0$$

$$\mu_l^B b_l = 0$$

7 equations for unconnected banks:

$$(1 + \mu_u^{RA}) \tilde{\psi}_k = \mu_u^{BC} + \lambda \mu_u^{RA} \quad (67)$$

$$(1 + \mu_u^{RA}) \frac{\tilde{\psi}_B}{Q} + \mu_u^B = \mu_u^{BC} + \lambda \mu_u^{RA} - \mu_u^C - \mu_u \tilde{\eta} \quad (68)$$

$$(1 + \mu_u^{RA}) \tilde{\psi}_M + \mu_u^M = \mu_u^{BC} + \lambda \mu_u^{RA} - \mu_u \quad (69)$$

$$(1 + \mu_u^{RA}) \tilde{\psi}_D = \mu_u^{BC} - \omega^{\max} \mu_u \quad (70)$$

$$(1 + \mu_u^{RA}) \frac{\tilde{\psi}_F}{Q^F} = \mu_u^{BC} - \mu_u^C \frac{1}{Q^F} + \mu_u^F \frac{1}{Q^F} \quad (71)$$

$$\mu_u^C = \mu_u^{CC} \eta - \mu_u \tilde{\eta} \quad (72)$$

$$\mu_u [\omega^{\max} d_u - m_u - \tilde{\eta} Q (b_u - b_u^F)] = 0 \quad (73)$$

6 equations for connected banks:

$$(1 + \mu_c^{RA}) \tilde{\psi}_k = \mu_c^{BC} + \lambda \mu_c^{RA} \quad (74)$$

$$(1 + \mu_c^{RA}) \frac{\tilde{\psi}_B}{Q} + \mu_c^B = \mu_c^{BC} + \lambda \mu_c^{RA} - \mu_c^C \quad (75)$$

$$(1 + \mu_c^{RA}) \tilde{\psi}_M + \mu_c^M = \mu_c^{BC} + \lambda \mu_c^{RA} \quad (76)$$

$$(1 + \mu_c^{RA}) \tilde{\psi}_D = \mu_c^{BC} \quad (77)$$

$$(1 + \mu_c^{RA}) \frac{\tilde{\psi}_F}{Q^F} = \mu_c^{BC} - \mu_c^C \frac{1}{Q^F} + \mu_c^F \frac{1}{Q^F} \quad (78)$$

$$\mu_c^C = \mu_c^{CC} \eta \quad (79)$$

6 bank aggregation equations:

$$v = \xi v_c + (1 - \xi) v_u$$

$$k = \xi k_c + (1 - \xi) k_u$$

$$d = \xi d_c + (1 - \xi) d_u$$

$$b = \xi b_c + (1 - \xi) b_u$$

$$f = \xi f_c + (1 - \xi) f_u$$

$$m = \xi m_c + (1 - \xi) m_u.$$

7. 2 rest of the world equations

$$b^W = \varkappa - \frac{1}{\rho} \log Q_t \pi_t$$

$$Qb^W + c^W = [\kappa + (1 - \kappa) Q] \frac{b^W}{\pi}.$$

These are 66 equations (one redundant by the Walras law) in 65 endogenous variables:

$$\left\{ \begin{array}{l} y, k, c, c^W, l, d, n, m^h, b, b^W, f, m, v, \tilde{v}, \bar{b}, \tau^*, \\ \psi, \tilde{\psi}_k, \tilde{\psi}_B, \tilde{\psi}_M, \tilde{\psi}_D, \tilde{\psi}_F, \mu_u, w, r, Q, R^D, \pi, I, s, \bar{f}, \bar{m} \end{array} \right\}$$

plus

$$\{k_l, m_l, f_l, b_l, b_l^F, d_l, v_l, \tilde{v}_l, \mu_l^F, \mu_l^M, \mu_l^{RA}, \mu_l^{BC}, \mu_l^{CC}, \mu_l^C, \mu_l^B\},$$

plus the value of the three policy instruments

$$\eta^A, Q^F, b_C,$$



and of the following exogenous variables:  $g, \xi$ .

The bank first-order conditions can be further simplified as follows. For the unconnected banks, conditions (74)-(79) can be simplified to:

$$\mu_u [\omega^{\max} d_u - m_u - \tilde{\eta} Q (b_u - b_u^F)] = 0 \quad (80)$$

$$\omega^{\max} \mu_u = (1 + \mu_u^{RA}) (\tilde{\psi}_k - \tilde{\psi}_D) - \lambda \mu_u^{RA} \quad (81)$$

$$\mu_u^{CC} \eta = (1 + \mu_u^{RA}) \left( \tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q} \right) - \mu_u^B \quad (82)$$

$$\mu_u^M = (1 + \mu_u^{RA}) (\tilde{\psi}_k - \tilde{\psi}_M) - \mu_u \quad (83)$$

$$\mu_u^F \frac{1}{Q^F} = (1 + \mu_u^{RA}) \left( \frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D \right) - \omega^{\max} \mu_u + \mu_u^{CC} \frac{1}{Q^F} \quad (84)$$

For the connected banks, conditions (74)-(79) can be simplified to,

$$\mu_c^{RA} = \frac{\tilde{\psi}_k - \tilde{\psi}_D}{\lambda - (\tilde{\psi}_k - \tilde{\psi}_D)} \quad (85)$$

$$\mu_c^B = (1 + \mu_c^{RA}) \left( \tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q} \right) - \mu_c^{CC} \eta \quad (86)$$

$$\mu_c^M = (1 + \mu_c^{RA}) (\tilde{\psi}_k - \tilde{\psi}_M) \quad (87)$$

$$\mu_c^F \frac{1}{Q^F} = (1 + \mu_c^{RA}) \left( \frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D \right) + \mu_c^{CC} \frac{1}{Q^F} \quad (88)$$

## B Proofs

### Proof of Proposition 2

By (64), we have  $\mu_c^F > 0$  and  $f_c = 0$ . We first claim that  $\mu_c^C = 0$ . Intuitively, if a bank  $l$  does not borrow from the central bank, it cannot be collateral-constrained at its borrowing from the central bank. Consider the following complementary slackness conditions in the bank problem:

$$\mu_l^C (b_l - b_l^F) = 0$$

Since  $f_l = 0$ , we have that  $b_l^F = 0$  since  $f_l = \eta Q b_l^F$  and  $b_l^F \geq 0$ . Therefore, the above complementary slackness condition simplifies to

$$\mu_l^C b_l = 0.$$

There are two possibilities: either the bond holdings are positive,  $b_l > 0$  or they are zero,  $b_l = 0$ . In the former case, it follows that  $\mu_l^C = 0$ , which proves the claim. In the latter case, we have  $b_l = 0$ , and a bank does not hold any bonds, does not pledge any bonds, and is not constrained by the bond collateral constraint,  $\mu_l^C = 0$ . Therefore,  $\mu_l^C = 0$  holds for a bank that does not borrow from the central bank.

We next show that connected banks do not hold any cash. First-order condition (87) for connected banks implies that whenever

$$\tilde{\psi}_{A,k} > \tilde{\psi}_{A,M}$$

holds, we have  $\mu_{A,c}^M > 0$  and thus  $m_{A,c} = 0$ . The above condition is equivalent to

$$\theta \frac{y_A}{(\xi_A k_{A,c} + (1 - \xi_A) k_{A,u})} + 1 - \delta > \frac{1}{\pi}.$$

Given (62) and (63), the condition above is always satisfied.

Finally, we show that if the afternoon constraint of unconnected banks binds,  $\mu_u > 0$ , then a connected bank does not hold any bonds, i.e.,  $b_c = 0$ . Combining (67) with (68), we get

$$\mu_u^B = (1 + \mu_u^{RA}) \left( \tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q} \right) - \mu_u^C - \mu_u \tilde{\eta}.$$

Since  $\mu_u > 0$ ,  $\mu_u^C \geq 0$  and  $\mu_u^B \geq 0$ , it follows that

$$\tilde{\psi}_k > \frac{\tilde{\psi}_B}{Q} \tag{89}$$

must hold.

Now turning to the connected banks, combine (74) and (75), and use  $\mu_c^C = 0$  (since connected banks do not borrow from the central bank), to get

$$\mu_c^B = (1 + \mu_c^{RA}) \left( \tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q} \right)$$

Since  $\tilde{\psi}_k > \frac{\tilde{\psi}_B}{Q}$ , we have that  $\mu_c^B > 0$  implying that  $b_c = 0$ .

### **Proof of Proposition 3**

We first show that if the afternoon constraint is slack,  $\mu_u = 0$ , then unconnected banks do not borrow from the central bank,  $\mu_u^F > 0$  and  $f_u = 0$ . The claim follows from the first-order condition (84) and condition (64): since  $\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D$  and  $\mu_u^{CC} \geq 0$ , we have  $\mu_u^F > 0$  and  $f_u = 0$ .

We next show that if  $\tilde{\eta} \geq \eta Q^F \omega^{\max}$  holds, then unconnected banks do not borrow from the central bank. We prove the claim by contradiction: suppose  $\tilde{\eta} \geq \eta Q^F \omega^{\max}$  holds and yet unconnected banks borrow from the central bank so that  $f_u > 0$ ,  $\mu_u^F = 0$  and  $b_u^F > 0$ . Using (84), we get

$$\omega^{\max} \mu_u = (1 + \mu_u^{RA}) \left( \frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D \right) + \mu_u^{CC} \frac{1}{Q^F}.$$

Since  $\mu_u^C \geq 0$  we have, by (72), that  $\mu_u^{CC} \geq \mu_u \frac{\tilde{\eta}}{\eta}$ . Therefore,

$$\omega^{\max} \mu_u = (1 + \mu_u^{RA}) \left( \frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D \right) + \mu_u^{CC} \frac{1}{Q^F} \geq (1 + \mu_u^{RA}) \left( \frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D \right) + \mu_u \frac{\tilde{\eta}}{\eta} \frac{1}{Q^F}$$

so that

$$\mu_{A,u} \left( \omega^{\max} - \frac{\tilde{\eta}}{\eta} \frac{1}{Q^F} \right) \geq (1 + \mu_u^{RA}) \left( \frac{\tilde{\psi}_F}{Q^F} - \tilde{\psi}_D \right).$$

The right-hand side of the expression above is positive since  $\frac{\tilde{\psi}_F}{Q^F} > \tilde{\psi}_D$  by (64). The left-hand side is non-positive since  $\omega^{\max} \eta Q^F \leq \tilde{\eta}$ . A contradiction.

Finally, we show that if  $\tilde{\eta} < \frac{\tilde{\psi}_k - \tilde{\psi}_B}{\tilde{\psi}_k - \tilde{\psi}_M}$  holds, then unconnected banks do not borrow in the secured market and instead they borrow only from the central bank and hold money,  $m_u > 0$ . We prove the claim by contradiction: Suppose that  $\tilde{\eta} < \frac{\tilde{\psi}_k - \tilde{\psi}_B}{\tilde{\psi}_k - \tilde{\psi}_M}$  and yet unconnected banks use bonds to borrow from the secured market so that  $b_u^F < b_u$  and  $\mu_u^C = 0$ . Since  $b_u > 0$ , we have  $\mu_u^B = 0$ . Since  $\mu_u^C = 0$ , we have by (72) that  $\mu_u^{CC} = \mu_u \frac{\tilde{\eta}}{\eta}$ . Using this to substitute out  $\mu_u^{CC}$  in (82), we have:

$$\mu_u = (1 + \mu_u^{RA}) \left( \tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q} \right) \frac{1}{\tilde{\eta}}$$

By (83), we have

$$\mu_u^M = (1 + \mu_u^{RA}) \left( \tilde{\psi}_k - \tilde{\psi}_M \right) - \mu_u \geq 0$$

so that

$$(1 + \mu_u^{RA}) \left( \tilde{\psi}_k - \tilde{\psi}_M \right) \geq \mu_u.$$

Then, we have that

$$(1 + \mu_u^{RA}) \left( \tilde{\psi}_k - \tilde{\psi}_M \right) \geq (1 + \mu_u^{RA}) \left( \tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q} \right) \frac{1}{\tilde{\eta}}$$

or, equivalently,

$$\tilde{\eta} \geq \frac{\tilde{\psi}_k - \frac{\tilde{\psi}_B}{Q}}{\tilde{\psi}_k - \tilde{\psi}_M}.$$

A contradiction.

Since unconnected banks borrow from the central bank, their afternoon constraint binds,  $\mu_u > 0$ . Since they do not borrow from the secured market, we have  $b_u^F = b_u$ . The binding afternoon constraint then implies that

$$m_u = \omega^{\max} d_u > 0.$$

This completes the proof.