

# Payments, Credit and Asset Prices\*

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September 2017

## Abstract

This paper studies a monetary economy with two layers of transactions. In transactions between bank customers, households and institutional investors pay for goods and assets with payment instruments provided by banks. In the bank layer, these payment instructions generate interbank transactions that banks handle with reserves or interbank credit. The model links the payments system and asset markets so that beliefs about asset payoffs matter for the price level, and monetary policy matters for real asset values.

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# 1 Introduction

This paper studies the joint determination of payments, credit, and asset prices. The starting point is that, in modern economies, transactions occur in two layers. In the *bank customer layer*, nonbanks (such as households, firms and institutional investors) trade goods and assets and pay for them using inside money: payment instruments supplied by banks. Inside money includes not only short-term demandable assets such as deposits and money-market fund shares, but also credit lines that can be drawn on demand such as credit cards. Credit lines also play a key role in payment for assets. For example, institutional investors have sweep arrangements with their custodian banks. Participants in the triparty repo market obtain intraday credit from clearing banks.

A common denominator of these different payment instruments is that banks commit to accept payment instructions from their clients. As a result of those payment instructions, transactions in the bank customer layer generate interbank transactions in the *bank layer*. Perhaps the most obvious example are direct payments out of bank deposit accounts by check or wire transfer: payments between customers of different banks generate interbank transfers of funds. In many asset markets, transactions are cleared by specialized financial market utilities such as clearinghouses that provide some netting of transactions. Institutional investors then settle netted positions with those utilities through payment instructions to their banks.

Interbank payments are made with outside money: reserves supplied by the central bank. But they may also be handled through various forms of short-term credit. For example, in the United States, utilities like NSS and CHIPS allow for intraday netting of a share of interbank transactions, so only net positions are periodically settled with reserves. The central bank may also provide intraday overdraft credit to banks. Nevertheless, the bulk of interbank payments goes through gross settlement systems provided by central banks, such as Fedwire in the United States or Target in the Euro Area.

In the aftermath of recent financial crises, central banks have made unprecedented changes to the quantity as well as the price of reserves. Several central banks have dramatically increased the quantity of reserves relative to the value of transactions. These policy shifts have reduced the relative importance of interbank credit. For example, in the United States, the use of intraday overdraft as well as interbank overnight Fed funds borrowing have essentially disappeared. Moreover, a number of central banks have begun charging negative nominal interest rates for the use of reserves.

This paper proposes a stylized model of an economy with two layers of transactions. Households and institutional investors are bank customers who must pay for some goods and assets with inside money: deposits or credit lines supplied by a competitive banking sector. Banks handle their customers' payment instructions and must make some interbank payments with outside money: reserves supplied by the government or interbank credit. Both banks and the government incur costs of leverage that decline with the quantity and quality of available collateral, in particular assets and claims to future taxes.

The model determines asset prices, the nominal price level and agents' portfolios as a function of government policy and investor beliefs about asset payoffs. It also determines the share of resources used up as costs of leverage. An efficient payment system allocates collateral so as to minimize that share of resources and hence maximize consumption. Asset prices reflect not only cash flow expectations and uncertainty premia, but also the collateral and liquidity benefits that assets provide to banks and their customers.

We use the model to think about links between asset markets and the payment system. The key properties that generate such links are illustrated by the quantity equation

$$PT = D + L.$$

Here the total volume of transactions  $T$  includes asset purchases by institutional investors, not simply the value of goods traded. Moreover, the only medium of exchange to buy goods and assets in the bank customer layer are deposits  $D$  and credit lines  $L$ , inside money supplied by banks who rely on assets as collateral. Outside money is only one input to the production of inside money, albeit a special one since it not only serves as collateral, but also provides liquidity for making interbank payments.

Consider an increase in uncertainty about asset payoffs that lowers asset values. As the value of collateral that banks can purchase declines, supplying inside money becomes more costly. A decline in inside money  $D + L$  then puts downward pressure on the price level. At the same time, however, an increase in uncertainty also lowers the demand for inside money by institutional investors, which has the opposite effect. The details of financial structure, including the use of inside money by institutional investors and the scope of netting arrangements, are thus important in order to assess the effects of asset market shocks on inflation.

In our model, some assets are priced exclusively by intermediaries. Segmentation of asset markets arises endogenously because banks receive collateral benefits from assets but households do not. Banks invest in assets to back inside money and thus bid up the prices of those assets. In particular, the real interest rate on short-term credit is so low in equilibrium that households choose not to lend short term. It therefore does not satisfy a consumption-based pricing equation. Instead, banks are the only marginal investors, and bank Euler equations say that interest rates are lower when bank leverage is higher and collateral is scarcer.

We also use the model to think about recent policy shifts, with a focus on two policy tools. First, the government can trade in asset markets to change the mix of collateral available to banks. Second, the government controls the real return on reserves. The central bank sets the nominal interest rate on reserves. Moreover, the inflation rate is given by the growth rate of nominal government liabilities. This result follows from the quantity equation and the fact that prices are flexible. Importantly, what matters is not the growth rate of reserves, but instead the growth rate of nominal inside money  $D + L$ , which in turn depends on nominal collateral available to banks.

The government can select one of two policy regimes. Reserves are *scarce* if banks do not always have sufficient reserves to handle all interbank payments but instead turn to the short-term credit for liquidity. The liquidity benefit of reserves then generates a spread between the short-term interest rate and the reserve rate. Reserves are scarce if the real return on reserves is sufficiently low, which means the opportunity cost of holding reserves is high. Banks then choose higher leverage to maintain a high return on equity in spite of a higher effective tax on reserves.

As long as reserves are scarce, open-market purchases of short-term debt for reserves change the collateral mix towards more liquid bank assets. In our model, open-market purchases lower the real short-term interest rate. Indeed, when more liquid reserves are available, competition drives banks to produce more inside money, pushing the price level up. As a result, the real value of nominal collateral falls, banks become more levered and bid up the prices of all collateral including short bonds, a permanent “liquidity effect” on the real interest rate.

In the second policy regime, reserves are *abundant*: the quantity of reserves is sufficiently large

relative to the volume of transactions that overnight borrowing is never needed. Once reserves lose their liquidity benefit, short-term loans and reserves become perfect substitutes and earn the same interest rate – the economy enters a “liquidity trap” where conventional open-market policy becomes ineffective. If the interest rate on reserves is zero, then reserves become abundant at the zero lower bound. More generally, however, reserves are abundant whenever the real return on reserves is sufficiently high, which can also happen with positive or negative interest on reserves.

The fact that payments occur in two layers has important implications for what it means to be in a “liquidity trap”. The textbook view is that equality of interest rates on outside money and short bonds implies that the medium of exchange and a safe store of value become perfect substitutes for all agents. In our model, this is true only for banks who are the only investors in both reserves and short bonds. In contrast, inside money requires costly bank leverage and never becomes abundant. In particular, they retain their liquidity benefit even when reserves are abundant. At the same time, collateral remains scarce in the liquidity trap, so unconventional policy that exchanges reserves for lower quality collateral can still matter by changing the collateral mix.

Which regime is better depends in our model on the relative leverage costs of banks versus the government. If the government can borrow more cheaply than banks, then it makes sense to move to abundant reserves, as several central banks have done recently. An extreme version would be narrow banking. In contrast, if the government has trouble to credibly commit to a path for nominal debt, then it is beneficial to have banks rely more on collateral other than government bonds or reserves. Since the optimal system depends on the quality of collateral, it may make sense to switch between regimes over time in response to asset market events.

The availability of two separate policy tools implies that the stance of policy cannot be easily summarized by a single variable, such as the short-term nominal interest rate. For example, when reserves are scarce, the government can lower the nominal interest rate either through open-market purchases or by lowering the real return on reserves. However, the effect on real interest rates and inflation is generally different. The reason is that asset values reflect not only liquidity benefits – as in many monetary models – but also collateral benefits. Policy affects interest rates by altering both benefits.

Our model assumes that markets are competitive and all prices are perfectly flexible. Banks and other financial firms maximize shareholder value and operate under constant returns to scale. Moreover, they do not face adjustment costs to equity. The effects we highlight thus do not follow from a scarcity of bank capital. We think of our model as one of large banks that provide payment services in a world where credit markets are highly securitized. This perspective also motivates our leading example for a shock: a change in uncertainty that moves asset premia.

Financial frictions are formally introduced as follows. First, inside money and outside money relax liquidity constraints in the bank customer layer and the bank layer, respectively. In this sense, those assets are more liquid than other assets. By assuming generalized cash-in-advance constraints for households and institutional investors, we abstract from effects of interest rates on the volume of transactions in units of goods and assets, respectively. While adding such effects is conceptually straightforward, our goal here is to provide a tractable setup that zeros in on novel effects for the demand and supply for money.

Second, banks and the government face an upward-sloping marginal cost of making commitments. This cost is smaller the more collateral the institution has available, that is, the larger and safer is a bank’s asset portfolio or the larger the tax base, respectively. It can have either an ex post

or an ex ante interpretation. For example, if more levered banks and governments are more likely to renege on certain promises, more labor may be required to ex post renegotiate those promises so that less labor is available for producing goods. Alternatively, more levered banks and governments may have to exert more effort ex ante to produce costly signals of their credibility.

Our analysis below starts with a baseline model in which the real quantity of both inside money and assets on bank balance sheets are fixed. In two extensions, we then introduce institutional investors whose demand for loans or inside money responds to changes in interest rates. We first consider *carry traders* who hold real assets and borrow against those assets using short-term credit supplied by banks. Carry traders have no inside money demand, but supply collateral to banks in the form of short-term loans. An example are asset-management firms who finance securities holdings with repurchase agreements.

The new feature in an economy with carry traders is that the price level now depends on carry traders' demand for loans. For example, lower uncertainty increases the demand for loans and hence the quantity of collateral for banks, the supply of inside money and the price level. An asset-price boom can thus be accompanied by inflation even if the supply of reserves as well as the amount of goods transacted remains constant and banks hold no uncertain assets themselves. Moreover, monetary policy that lowers the real short rate lowers carry traders' borrowing costs and boosts the aggregate market by allowing more leverage.

The second extension introduces *active traders* who hold not only assets but also inside money, since they must occasionally rebalance their portfolio using cash payments. An example are asset management firms who sometimes want to exploit opportunities quickly before they can sell their current portfolio. Active traders' portfolio choice responds to the deposit interest rate offered by banks and the fee for credit lines they charge: if inside money is cheaper, active traders hold more of it, and the value of their transactions is higher. The strength of their response depends importantly on how much netting takes place among active traders through intraday credit systems.

The new feature in an economy with active traders is that the price level now depends on active traders' money demand. For example, lower uncertainty increases their money demand. As more inside money is used in asset market transactions, fewer instruments are used in goods market transactions and the price level declines. During an asset price boom, we may thus see low inflation even if the supply of reserves increases. Moreover, monetary policy that lowers the real short term interest rate lowers active traders' trading costs and further boosts the aggregate market.

Our model can be interpreted as describing the subset of worldwide transactions in a currency, rather than the closed economy of a country. The former interpretation is appropriate for economies like the United States that have banking systems and financial markets tightly integrated with those of other countries. We thus think of households in our models as agents who pay for goods out of dollar deposit accounts and credit cards, while institutional investors may include foreign firms who obtain credit or payments from banks in terms of dollars. With this perspective in mind, the model can be used to think about how events in worldwide asset markets may affect nominal prices in the US.

The broad questions we are interested in are the subject of a large literature. The main new features of our model are that *(i)* transactions occur in layers, with inside money used exclusively in the bank customer layer and outside money used exclusively in the bank layer, *(ii)* bank customers include institutional investors, and *(iii)* both banks and the government face leverage costs. Relative to earlier work, these properties change answers to policy questions as well as asset pricing results,

as explained in Section 7.

The paper is structured as follows. Section 2 presents a few facts about payments. Section 3 describes the model. Section 4 looks at the baseline model that features only households and banks. It shows how steady state equilibria can be studied graphically and considers different monetary policy tools. Section 6 introduces uncertainty and studies the link between the payment system and asset markets. It also extends the model to accommodate institutional investors. Finally, Section 7 discusses the related literature.

## 2 Facts on payments

This section presents a number of facts that motivate our model. We combine data from the BIS Payments Statistics, the Payments Risk Committee sponsored by the Federal Reserve Bank of New York, the Federal Reserve Board’s Flow of Funds Accounts and Call Reports, as well as publications of individual clearinghouse companies.

### *Transactions in the bank customer layer*

Figure 1 gives an impression of payments in the two layers in US dollars. The left-hand panel shows payments by bank customers with inside money, that is, payment instructions to various types of intermediaries. The blue area labeled “nonfinancial” adds up payments by cheque as well as various electronic means, notably Automated Clearinghouse (ACH) transfers as well as payments by credit card. While the area appears small in the figure, it does amount to several multiple of GDP. For example, in 2011, nonfinancial transactions were \$71 trillion whereas GDP was \$15 trillion. This is what one would expect given that there are multiple stages of production and commerce before goods reach the consumer. Moreover a share of trade in physical capital including real estate also is contained in this category.

Payment for assets in U.S. markets is organized by specialized financial market utilities who clear transactions and see them through to final settlement. A major player is the Depository Trust & Clearing Corporation (DTCC). One of its subsidiaries, the National Securities Clearing Corporation (NSCC) clears transactions on stock exchanges as well as over-the-counter trades in stocks, mutual fund shares and municipal and corporate bonds. NSCC cleared \$221 trillion worth of such trades in 2011. In the left hand panel of Figure 1, transactions cleared by NSCC are shaded in brown.

NSCC has a customer base (“membership”) of large financial institutions, in particular brokers and dealers. When a buyer and a seller member agree on a trade – either in an exchange or in an over-the-counter market – the trade is reported to NSCC which then inserts itself as a counterparty to both buyer and seller. In the short run, members thus effectively pay for assets with credit from NSCC. To alleviate counterparty risk, members post collateral that limits their position relative to NSCC. Over time, NSCC nets opposite trades by the same member. Periodically, members settle net positions via payment instructions to members’ bank which then make (receive) interbank payments to (from) DTCC. Netting implies that settlement payments amount to only a fraction of the dollar value of cleared transactions.

Another DTCC subsidiary, the Fixed Income Clearing Corporation (FICC) offers clearing for Treasury and agency securities. FICC payments are settled on the books of two “clearing banks”, JP Morgan and Bank of New York Mellon. Interbank trades of Treasury and agency bonds can alternatively be made via the Fedwire Securities system offered by the Federal Reserve System to

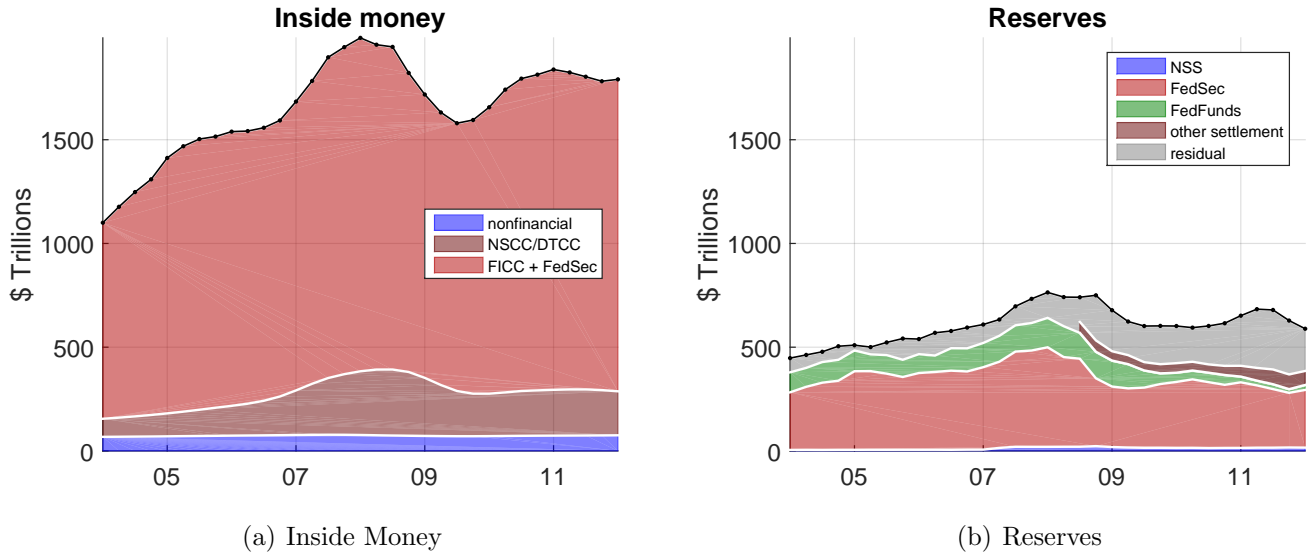


Figure 1: Selected U.S. dollar transactions, quarterly at annual rates.

(a) Inside Money: "nonfinancial" = cheque and electronic payments reported by banks, "NSCC/DTCC" = securities transactions cleared by NSCC, "FICC+FedSec" = securities transactions cleared by FICC and Fedwire Securities Service. (b) Reserves: "NSS" = NSS settlement, "FedSec" = Fedwire Securities settlement, "FedFunds" = estimate of payments for Fed Funds borrowing by U.S. banks, "other settlement" = estimate of payments to financial market utilities.

its member banks. The left hand panel of Figure 1 shows the sum of FICC and Fedwire Securities trades in red. This number is high partially because every repurchase agreement involves two separate security transactions (that is, the lender wires payment for a purchase to the borrower and the borrower wires payments back to the lender at maturity).

Figure 1 does not provide an exhaustive list of US dollar transactions. First, it leaves out financial market utilities handling derivatives and foreign exchange transactions. For example, the Continuous Linked Settlement (CLS) group is a clearinghouse for foreign exchange spot and swap transactions that handled trades worth \$1,440 trillion in 2011. Netting in these markets is very efficient so that CLS payments after netting were only \$3 trillion. Second, even for goods and assets covered, Figure 1 omits purchases made against credit from the seller that involves no payment instruction to a third party. This type of transaction includes trade credit arrangements. In asset markets, a share of bilateral repo trades between broker dealers and their clients is settled on the books of the broker dealers. Finally, the figure also leaves out transaction made with currency.

Even given these omissions, the message from the left panel of Figure is clear: transaction volume is large, and especially so in asset markets. The volume in asset markets also exhibits pronounced fluctuations in the recent boom-bust episode. We also emphasize that not all of these payment instructions are directly submitted to traditional banks. Financial market utilities that provide netting are also important. Moreover, customers of money market mutual funds may also pay by cheque or arrange ACH transfers. The payment instruction is then further relayed by the money market fund to its custodian bank.

### *Transactions in the bank layer*

The right-hand panel of Figure 1 shows transactions over two settlement systems provided by the Federal Reserve Banks. The blue area represents interbank payments via the National Settlement Service, which allows for multilateral netting of payments by cheque and ACH. To a first approximation, one can think of it as the counterpart of the blue area in the left hand panel, that is, non-securities payments after netting. All other areas in the right panel represent interbank payments over Fedwire, the real time gross settlement system of the Federal Reserve. Fedwire is accessed by participating banks who send reserves to each other.

The coloring of areas is designed to indicate roughly how the interbank payments were generated. The red area represents payments for Treasury and agency securities over Fedwire Securities. Since there is no netting involved, large securities transfers correspond to large transfers of reserves. For the years after 2008, the brown area is an estimate of payments made over Fedwire to settle positions with financial market utilities. The estimate includes not only NSCC and FICC, but also CHIPS, a private large value transfer system used by about 50 large banks. CHIPS uses a netting algorithm to simplify payments among its member banks; in 2011, it handled \$440 trillion worth of transactions.

The green area in the figure represents payments for interbank credit in the Fed Funds market, also sent over Fedwire. As for repo transactions, a relatively small amount of outstanding overnight credit can generate a large number for annual Fedwire transfers. The transition from a regime of scarce reserves to one with abundant reserves after the financial crisis is apparent by the drop in Fed Funds transactions. The presence of government sponsored enterprises and Federal Home Loan banks implies that the Fed Funds market has not dried up completely.

The red and brown areas suggest that payment instructions generated by asset trading are responsible for a large share of interbank payments. This is true even though netting by financial market utilities reduced the cleared transactions from the left panel to much smaller numbers. At the same time, during times of scarce reserves, bank liquidity management via the Fed Funds market also generates a large chunk of payments. The figure also contains a gray area which we cannot assign to one of the payment types.

### *Payments by custodian banks*

Figure 2 provides a closer look at the activities of banks who make a lot of payments. We consider 27 bank holding companies that are "systematically important" and hence report individual data on payments to regulators. Along the horizontal axis, we measure 2014 payments via large value transfer systems like Fedwire and CHIPS, normalized by bank assets. The 27 banks' joint payments are large: they account for over 75% of total payments over CHIPS and Fedwire.

Along the vertical axis, we measure assets held in custody, again normalized by assets. There is a strong positive relationship: banks who have more assets in custody also tend to make more payments. The fact that the relationship holds after normalization by assets suggests that it is not merely a scale effect. Moreover, the color of a dot indicates the size of the bank in terms of total assets. It is not obviously related to the two ratios measured along the axes.

In fact, the largest banks – JP Morgan, Citi, Wells Fargo and Bank of America all appear as bright pink dots – do not have the largest payments relative to assets. Instead, it is somewhat smaller banks specializing in the custodian business – State Street, Northern Trust and BoNY Mellon appear in the top right corner of the figure – who have the largest payments/assets ratios.

There are two plausible reasons why banks who are large custodians might be expected to make



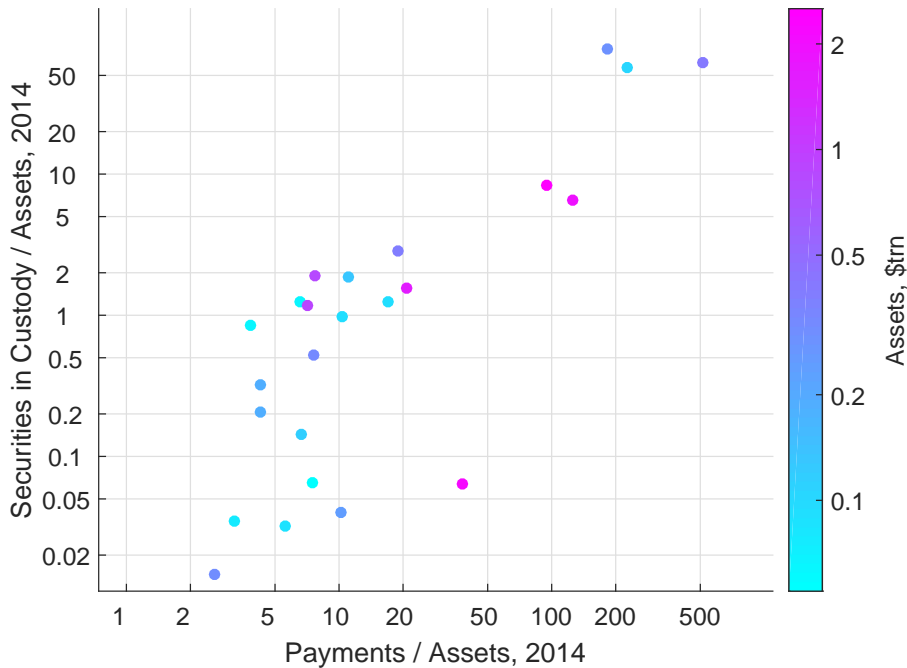


Figure 2: Payments / Assets vs Securities in Custody / Assets for systemically important bank holding companies. Flows for the year 2014, stocks for 2014 Q4. Color indicates assets in \$ trillion.

lots of payments. One is already apparent in Figure 1: there is simply a lot of "churn" in asset markets, in part due to frequent short term changes in positions. This possibility motivates the inclusion of liquidity constraints for asset traders together with a netting system, in our model below.

A second reason is that custodian banks hold the portfolios of money market funds, who in turn offer payment instruments. While the front office of the money market fund receives payment instructions, those instructions are still executed by banks who access large value transfer systems, in particular Fedwire. For the purposes of our model, our perspective on money market funds is thus to consolidate them with the banking system.

### 3 Model

Time is discrete, there is one good and there are no aggregate shocks. Households consume an endowment of goods  $\Omega$  as well as fruit from trees  $x$ . Total output  $Y = \Omega + x$  is constant. Households also own competitive financial firms. For now, the only financial firms are banks who issue payment instruments. Below we introduce different types of asset management companies. All financial firms issue equity and participate in tree and credit markets along with households.

#### *Layers and frictions*

The model describes transactions and asset positions in two layers. In the layer of bank customers, households and nonbank financial firms trade goods and assets. In the bank layer, banks trade assets and also borrow from and lend to each other. A key connection between layers is that when customers trade, they make payment instructions to banks.

The model incorporates two frictions. First, it is costly for all agents to commit to make

future payments, and less so if they own more assets that can serve as collateral. These "leverage costs" apply when banks provide inside money (such as deposits), and when financial firms and the government issue debt. It implies that assets are valued in part for their collateral benefits.

Second, there are liquidity constraints in both layers. In particular, in the customer layer, goods and assets must be paid for with *inside* money that banks provide. In this section, the only payment instruments are deposits.<sup>1</sup> In the bank layer, payments generated by customer payment instructions must be paid with *outside* money that is provided by the government. This outside money consists of reserves. Liquidity constraints in the customer and bank layers imply that inside and outside money, respectively, are valued for their liquidity benefits.

We assume that financial firms can be costlessly recapitalized every period and that their objective function exhibits constant returns to scale. As a result, a firm's history does not constrain its future portfolio and capital structure decisions. As in Lagos and Wright (2005), the distribution of heterogeneous agents' (here firms') histories thus plays no role in the model.

### 3.1 Households

Households have linear utility, discount the future at the rate  $\delta = -\log \beta$  and receive an endowment  $\Omega_t$  every period. Households enter the period with deposits  $D_t^h$  and buy consumption  $C_t$  at the nominal price  $P_t$  measured in units of reserves. Their liquidity constraint is

$$P_t C_t \leq D_t^h. \quad (1)$$

A cash-in-advance approach helps zero in on the role of endogenous inside money. It is not difficult to extend the model so that the money demand by households is elastic, but it would make the new mechanisms in our model less transparent.

In addition to deposits, households can invest in safe short bonds ("overnight credit") that earn an interest rate  $i_t$ . Households can also buy trees, which are infinitely lived assets that provide fruit  $x_t$  and trade at a nominal price  $Q_t$ . The household budget constraint is

$$\begin{aligned} P_t C_t = & P_t \Omega_t + D_t^h (1 + i_{t-1}^D) - D_{t+1}^h \\ & + (1 + i_t) B_t^h - B_{t+1}^h + (Q_t + P_t x_t) \theta_{t-1}^h - Q_t \theta_t^h dj \\ & + \text{dividends} + \text{government transfers}. \end{aligned} \quad (2)$$

Expenditure on goods must be financed through either (i) the sale of endowment, (ii) changes in household asset positions in deposits, overnight credit or trees, or (iii) exogenous income from dividends, fees or government transfers, described in more detail below.<sup>2</sup>

Households cannot borrow overnight or sell trees short, that is, we impose  $\theta^h, B^h \geq 0$ . We interpret the endowment as payoff from all assets that are not traded by banks or the government, including labor income (payoffs from human capital) and claims to housing services. In contrast, trees are assets that are also traded by banks and the government, including mortgage bonds. Their

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<sup>1</sup>Section 3.5 introduces credit lines and shows that the model continues to work similarly. The key property of either payment instrument is that it provides liquidity to endusers and requires costly commitment on the part of banks. Our model is about a modern economy where currency plays a negligible role in all (legal) transactions.

<sup>2</sup>We assume that an interest rate  $i^D$  is earned on deposits regardless of whether they are used for payment. This assumption helps simplify the algebra. More detailed modeling of the fee structure of deposit accounts is possibly interesting but not likely to be first order for the questions we address in this paper.

ownership will affect the production of inside money. It is an equilibrium outcome which sector in the economy ends up holding trees. The liquidity constraint (1) makes bonds and trees less liquid because they cannot be used to pay for consumption.

### *Notation for rates of return*

We think of our model period as a short period such as a day, and we do not study hyperinflation periods, so that nominal and real rates of return are always small decimal numbers. We thus simplify formulas throughout by using the approximation  $\exp(r) = 1 + r$  for any small rate of return  $r$ , and setting any products of rates of return to zero. For example, with an inflation rate  $\pi_t = \log P_t/P_{t-1}$ , the real rate of return on deposits between dates  $t - 1$  and  $t$  is  $i_{t-1}^D - \pi_t$ .

We write households' marginal utility of wealth as  $\exp(-\gamma_t)$ . The log marginal rate of substitution  $\hat{\delta}_t := \delta - (\gamma_{t+1} - \gamma_t)$  between wealth at dates  $t$  and  $t + 1$  is the effective discount rate for payoffs. Since utility is linear in consumption, deviations of the log MRS from the discount rate  $\delta$  are due only to variation in the Lagrange multiplier on the liquidity constraint. For example, if liquidity is more expensive next period, agents effectively discount the future at a lower rate.

### *Household choices*

We focus on equilibria in which the liquidity constraint (1) is binding. The first-order-conditions for consumption and deposits then imply

$$\hat{\delta}_t - (i_t^D - \pi_{t+1}) = \gamma_{t+1}. \quad (3)$$

Since utility is linear in consumption, the benefit to households of an additional unit of liquidity arranged for next period is measured by  $\gamma_{t+1}$ . They equate this marginal benefit of liquidity to the opportunity cost of holding deposits, that is, the difference between the effective discount rate and the real return on deposits.

The household first-order condition for short bonds is

$$\hat{\delta}_t - (i_t - \pi_{t+1}) \geq 0. \quad (4)$$

Household only hold bonds if the real overnight rate is at least as high as their discount rate. The condition holds with equality if the household lends overnight. We will see below that households will not hold bonds or trees in the presence of banks in those markets.

## **3.2 Banks**

Households own many competitive banks. We describe the problem of a typical bank which maximizes shareholder value

$$\sum_{t=0}^{\infty} \exp\left(-\sum_{\tau=0}^{t-1} \hat{\delta}_{\tau}\right) y_t^b. \quad (5)$$

Here bank dividends  $y_t^b$  are discounted at the rate  $\hat{\delta}_t$ . The dividends are positive when banks distribute profits or negative when banks recapitalize.

Table 1 illustrates a bank's balance sheet at the beginning of day  $t$ . The asset side consists of reserves, overnight lending, and trees. The liability side has equity, overnight borrowing, and deposits. Section 3.5 will add loan commitments  $L_t$  (credit lines) that are formally off-balance sheet. Another off-balance sheet item that we include in our definition of inside money are money-market mutual funds sponsored by the bank. Shares in these funds are typically held in trust for the client. These items make our concept of leverage quite different from an accounting leverage ratio.

Table 1: Bank balance sheet at beginning of day  $t$

Assets	Liabilities
Reserves $M_t$	Equity
Overnight lending $B_t$	Overnight borrowing $F_t$
Trees $Q_t\theta_{t-1}$	Deposits $D_t$

off balance sheet: loan commitments  $L_t$

### *Liquidity management*

The typical bank enters period  $t$  with deposits  $D_t$  and reserves  $M_t$ . We want to capture the fact that customer payment instructions may lead to payments between banks. For example, a payment made by debiting a deposit account may be credited to an account holder at a different bank. We thus assume that a bank receives an idiosyncratic withdrawal shock: an amount  $\tilde{\lambda}_t D_t$  must be sent to other banks, where  $\tilde{\lambda}$  is iid across banks with mean zero and cdf  $G$ . We also assume that  $\tilde{\lambda}$  is bounded above: the cdf  $G$  is increasing only up to a bound  $\bar{\lambda}$  with  $\bar{\lambda} < 1$ .

In the cross section, some banks draw shocks  $\tilde{\lambda}_t > 0$  and must make payments, while other banks draw shocks  $\tilde{\lambda}_t < 0$  and thus receive payments. Since  $E[\tilde{\lambda}_t] = 0$ , any funds that leave one bank arrive at another bank; there is no aggregate flow into or out of the banking system. The distribution of  $\tilde{\lambda}_t$  depends on the structure of the banking system as well as the pattern of payment flows among customers.<sup>3</sup>

Banks that need to make a transfer  $\tilde{\lambda}_t D_t > 0$  can send reserves they have brought into the period, or they can borrow reserves from other banks. The bank liquidity constraint is

$$\tilde{\lambda}_t D_t \leq M_t + F_{t+1}, \quad (6)$$

where  $F_{t+1} \geq 0$  is overnight borrowing of reserves from other banks. If the marginal cost of overnight borrowing is larger than other sources of funding available to the bank, it is optimal to borrow as little as necessary. Banks then choose a threshold rule: they do not borrow unless  $\tilde{\lambda}_t$  is so large that the withdrawal  $\tilde{\lambda}_t D_t$  exhausts their reserves.

For a bank that enters the period with reserves  $M_t$ , deposits  $D_t$ , the liquidity constraint (6) implies a threshold shock

$$\lambda_t := \frac{M_t}{D_t}. \quad (7)$$

We refer to  $\lambda_t$  as the *liquidity ratio* of a bank. It is the inverse of a money multiplier that relates the total value of deposits to the quantity of reserves.

For a given liquidity ratio, the liquidity constraint (6) binds if the bank's liquidity shock is sufficiently large, that is,  $\tilde{\lambda}_t > \lambda_t$ . Reserves then provide a liquidity benefit, measured by the multiplier on the constraint. Moreover, the bank borrows reserves overnight

$$F_{t+1} = \tilde{\lambda}_t D_t - M_t = \left( \frac{\tilde{\lambda}_t}{\lambda_t} - 1 \right) M_t > 0. \quad (8)$$

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<sup>3</sup>The likelihood of payment shocks is the same across banks, regardless of bank size. Since the size distribution of banks is not determinate in equilibrium below, little is lost in thinking about equally sized banks. Alternatively, one may think about large banks consisting of small branches that cannot manage liquidity jointly but instead each must deal with their own shocks.

Since liquidity shocks have an upper bound  $\bar{\lambda}$ , banks can in principle choose a high enough liquidity ratio,  $\lambda_t > \bar{\lambda}$ , so that they cannot run out of reserves. Banks who do this have a zero multiplier on their liquidity constraint, which means they do not attach special value to reserves for their payment services.

#### *Portfolio and capital structure choice*

Banks adjust their portfolio and capital structure subject to leverage costs. They invest in reserves, overnight credit and trees while trading off returns, collateral values and liquidity benefits. They issue deposits and adjust equity capital, either through positive dividend payouts or negative recapitalizations  $y_t^b$ . Capital structure choices trade off returns, leverage costs and liquidity costs.

The bank budget constraint says that net payout to shareholders must be financed through changes in the bank's positions in reserves, deposits, overnight credit, or trees:

$$\begin{aligned} P_t y_t^b = & M_t (1 + i_{t-1}^R) - M_{t+1} - D_t (1 + i_{t-1}^D) + D_{t+1} \\ & + (B_t - F_t) (1 + i_{t-1}) - (B_{t+1} - F_{t+1}) + ((Q_t + P_t x_t) \theta_{t-1} - Q_t \theta_t) dj \\ & - e^{\pi_t} c(\kappa_{t-1}) (D_t + F_t) + D_t^b (1 + i_{t-1}^D) - D_{t+1}^b. \end{aligned} \quad (9)$$

In the second line,  $B \geq 0$  represents lending in overnight credit. The last line collects bank leverage costs and credit lines that banks use to pay those costs, both discussed in detail below.

The first and second lines in (9) collect payoffs from payment instruments and other assets, respectively. The bank receives interest  $i_{t-1}^R$  on reserves that are held overnight, regardless of whether those reserves were used to make a payment. Similarly, the bank pays deposit interest  $i_{t-1}^D$  on deposits issued in the previous period, regardless of whether its customers used the deposits to make a payment. Both conventions could be changed without changing the main points of the analysis, but at the cost of more cluttered notation.

#### *Leverage costs*

If the last line in the bank budget constraint (9) were omitted, the cost of debt would be independent of leverage. We assume instead that the commitment to make future payments is costly. It takes resources to convince overnight lenders that debt will be repaid, as well as to convince customers that the bank will indeed accept and execute payment instructions. Moreover, we assume that convincing lenders and customers is cheaper if the bank owns more assets to back up the commitments, especially if those assets are safe.

The cost of commitment depends on the *collateral ratio*

$$\kappa_t := \frac{M_{t+1} + \rho Q_t \theta_t + B_{t+1}}{D_{t+1} + F_{t+1}}, \quad (10)$$

where  $\rho$  is a fixed parameter. The collateral ratio divides weighted assets by debt – its inverse is a measure of leverage. Banks choose the collateral ratio  $\kappa_{t-1}$  at date  $t - 1$  through their choice of nominal positions at that date. Banks then have to purchase real resources  $c(\kappa_{t-1}) (D_t + F_t) / P_t$  in the goods market at date  $t$ . The cost function  $c$  is smooth, strictly decreasing and convex. We further assume below that it slopes down sufficiently so that banks choose  $\kappa > 1$ .

Bank assets in the numerator of (10) have a collateral value: the resources needed to convince customers about future commitments are smaller if the bank owns more assets. The weight  $\rho$  allows a distinction between safe assets (reserves and overnight lending) and trees, which we will

later assume to be uncertain. The presence of a weight implies that leverage computed as the inverse of the collateral ratio does not generally correspond to accounting measures of leverage.

Since leverage costs take up real resources, we need to address how banks pay for them. The details of this process are not essential and we choose an approach that simplifies formulas. Resources that support leverage chosen at date  $t - 1$  are purchased by banks in the date  $t$  goods market at the price  $P_t$ . In order to pay for those goods, banks must hold deposits  $D_t^b$  at other banks. They face an additional liquidity constraint that is analogous to that for households:

$$e^{\pi_t} c(\kappa_{t-1})(D_t + F_t) \leq D_t^b. \quad (11)$$

Here the factor  $\exp(\pi_t)$  converts nominal debt at date  $t - 1$  dollars into nominal expenditure on goods at date  $t$ . As long as the opportunity cost on deposits is positive, the constraint binds in equilibrium: banks arrange for a line that is just large enough to cover the leverage costs that will accrue next period. When we combine the household and bank liquidity constraints, we get a quantity equation that relates total deposits to output, regardless of how output is split into consumption and leverage costs.

### *Bank optimal choices*

Bank first-order conditions describe the key trade-offs of portfolio and capital structure choice. Risk neutral shareholders compare effective rates of return on assets and liabilities to the required return on equity  $\hat{\delta}$ . Effective rates of returns take into account not only future payoffs, but also how the asset or liability position changes the leverage cost and the liquidity constraint. The presence of frictions thus creates connections between bank balance sheets and rates of return.

Consider first bank portfolio choice. If the asset portfolio of the bank is chosen optimally, the marginal benefit from any asset cannot be larger than  $\hat{\delta}$ : if not, then the bank would choose to invest more. Moreover, the marginal benefit is equal to  $\hat{\delta}$  if the bank optimally holds a positive position in the asset. If the marginal benefit is strictly below  $\hat{\delta}$  then the bank does not invest: it is better to pay dividends instead. All bank first-order conditions are derived in the appendix. They take the form of weak inequalities that hold with equality if and only if the bank holds a positive position.

We illustrate here the optimal choice of overnight credit and reserves, which imply

$$\begin{aligned} \hat{\delta}_t &\geq i_t - \pi_{t+1} + \text{mb}(\kappa_t), \\ \hat{\delta}_t &\geq i_t^R - \pi_{t+1} + \text{mb}(\kappa_t) + e^{-\hat{\delta}_t - \pi_{t+1}} E[\mu_{t+1}], \end{aligned} \quad (12)$$

where  $\mu_t$  is the Lagrange multiplier on the liquidity constraint (6) and  $\text{mb}(\kappa_t) := -c'(\kappa_t)$  represents the marginal benefit of an extra unit of collateral. Since  $c$  is convex, this collateral benefit is a decreasing function of the existing collateral ratio. The first equation shows how banks value overnight credit not only for its pecuniary return, which is the real interest rate  $i_t - \pi_{t+1}$ , but also for its collateral benefit.

The second equation shows that reserves provide the same collateral benefit, but in addition offer a liquidity benefit captured by the expected discounted multiplier on the liquidity constraint. The liquidity benefit depends on the liquidity ratio  $\lambda_{t+1}$  chosen at date  $t$  since that ratio changes the distribution of the future Lagrange multiplier  $\mu_{t+1}$ . For example, higher  $\lambda_{t+1}$  lowers the probability that the liquidity constraint becomes binding. Bank first-order conditions thus relate rates of return to the bank ratios  $\kappa_t$  and  $\lambda_{t+1}$  chosen at date  $t$ .

Consider now the choice of capital structure. If the liability composition of the bank is chosen optimally, the marginal cost of issuing any liability cannot be larger than  $\hat{\delta}$ , and it must be exactly  $\hat{\delta}_t$  if the bank issues that liability. In particular, the first-order condition for deposits is

$$\hat{\delta}_t \leq i_t^D - \pi_{t+1} + \text{mc}(\kappa_t) + e^{-\hat{\delta}_t - \pi_{t+1}} E \left[ \mu_{t+1} \tilde{\lambda}_{t+1} \right], \quad (13)$$

where  $\text{mc}(\kappa_t) = -c'(\kappa_t) \kappa_t + c(\kappa_t)$  is the marginal leverage cost of an additional unit of debt, again decreasing in  $\kappa$ . Deposits are costly not only because the bank must pay interest on them, but also because they increase the leverage cost the bank must pay, and because they tighten future liquidity constraints.

The household and bank first order conditions for deposits illustrates the connection of our model to the standard "tradeoff theory" of capital structure. The household condition (3) implies that  $\hat{\delta}_t > i_t^D - \pi_{t+1}$ : the liquidity benefit of deposits implies that households are happy with a low deposit rate. From the perspective of the bank, this benefit works much like a tax advantage on debt and makes issuing deposits cheaper than issuing equity. At the same time, however, issuing deposits entails leverage and liquidity costs. The tradeoff between the two effects gives rise to a determinate optimal leverage ratio.

Since the bank problem exhibits constant returns to scale, the first-order conditions only pin down the collateral and liquidity ratios. As long as those ratios are chosen optimally, banks are indifferent between positions in all assets and liabilities that have effective rates of returns equal to the rate of return on equity  $\hat{\delta}$ . The appendix further shows that the distribution of  $\mu_{t+1}$  is the same across banks, which implies that  $\kappa$  and  $\lambda$  are equated across banks. Intuitively, the bank problem has no history dependence since the banks can be costlessly recapitalized every period. Since two first-order conditions are enough to pin down the two ratios, it follows that bank optimization implies restrictions across different interest rates; we derive those restrictions below.

### 3.3 Government

We treat the government as a single entity that comprises the central bank and the fiscal authority. The government issues reserves  $M_t$ , borrows  $B_t^g$  in the overnight market and chooses the reserve rate  $i_t^R$ . The government also makes lump-sum transfers to households so that its budget constraint is satisfied every period. Below we further consider particular policies that target endogenous variables such as the overnight interest rate. Such policies are still implemented using the basic tools  $M_t$ ,  $B_t^g$  and  $i_t^R$ .

Just like financial firms, the government incurs a cost of issuing debt, above and beyond the pecuniary cost. The government differs from firms in that it has the power to tax and hence the (implicit) collateral that is available to it. We define the government's collateral ratio as  $\kappa_t^g = P_{t+1} \Omega_{t+1} / (M_{t+1} + B_{t+1}^g)$  and denote the date  $t$  government leverage cost as  $e^{\pi_t} c_g(\kappa_{t-1}^g) (M_t + B_t^g)$  where  $c_g$  is strictly decreasing and convex, as is the bank leverage cost function  $c$ . The more real debt  $(M_t + B_t^g) / P_t$  the government issues relative to the labor income tax base  $\Omega_t$ , the more resources it must spend to convince lenders that it will repay. In order to pay leverage cost, the government is required to hold deposits  $D_t^g$  at banks.

### 3.4 Equilibrium

Equilibrium requires that markets clear at the optimal choices of banks and households, taking into account government policy. Tree market clearing requires that banks or households hold all trees.

The overnight credit market clears if borrowing by banks  $F_t$  plus government borrowing  $B_t^g$  equals aggregate bank lending  $B_t$ . Banks must hold all reserves. Since the cross sectional distribution of bank portfolios is indeterminate, we now use the symbols  $M_t$ ,  $L_t$ ,  $D_t$  etc to denote *aggregate* bank positions.

The goods market clears if households consume the endowment and all fruit from trees, net of any resources spent by banks and the government as leverage costs. We denote the total quantity of goods sold at date  $t$  by  $T_t$ . In nominal terms, goods market clearing means

$$P_t Y_t = P_t C_t + e^{\pi_t} c_g (\kappa_{t-1}^g) M_t + e^{\pi_t} c (\kappa_{t-1}) (D_t + F_t).$$

Since output is exogenous, only the use of goods for consumption or leverage cost is determined in equilibrium. For example, if banks and the government are more levered, then consumption must be lower.

Deposit market clearing means that deposits supplied by banks equal those demanded by households, banks and the government. If the three liquidity constraints (for households, banks and the government) all bind, we obtain the quantity equation

$$P_t T_t = D_t. \tag{14}$$

So far, real transactions only consist of goods trades  $T_t = Y_t$ . (Section 6 will add asset trades.) In order for society to handle these transactions, banks must supply a positive amount of inside money in real terms. Given a finite real value of amount of collateral, banks thus incur leverage costs. As a result, inside money is costly for customers and their liquidity constraints bind.

While customer liquidity constraints always bind, banks' liquidity constraints may or may not bind, depending on how many real reserves are available relative to transactions  $Y_t$  as well as other collateral. We say that reserves are *scarce* at date  $t$  if the threshold shock is smaller than the upper bound of the shock distribution,  $\lambda_{t+1} < \bar{\lambda}$ , so that the bank liquidity constraint binds with positive probability at date  $t + 1$ . In contrast, reserves are *abundant* if  $\lambda_{t+1} \geq \bar{\lambda}$  so banks are sure that the constraint will not bind.

### *Characterizing equilibrium*

The appendix derives a system of equations that characterize equilibrium. It describes the dynamics of the endogenous prices – the interest rates  $i_t$  and  $i_t^L$ , the price of trees  $Q_t$ , the nominal price level  $P_t$  – and the two ratios that describe bank balance sheets, the liquidity ratio  $\lambda_t$  and the collateral ratio  $\kappa_t$ , which are equal across banks in equilibrium. The appendix further states assumptions such that all rates of return are small numbers, so that the approximations used throughout the paper are good.

The model provides only limited scope for transition dynamics. Indeed, the only state variables are the liquidity ratio  $\lambda_t$  and the exogenous level of reserves  $M_t$ . Appendix A.4 provides mild restrictions on policy such that the transition from one steady state to the next takes only one period. The key assumption underlying this result is that banks face no adjustment cost, neither to portfolio holdings nor to equity. As a result, they can respond quickly to any change in the environment.<sup>4</sup>

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<sup>4</sup>The reason that there is any scope for transition dynamics comes from the presence of interbank credit in bank balance sheets when reserves are scarce. We show that open market policy can offset this effect and guide the economy to the new steady state after one period. The size of the extra policy depends on outstanding interbank credit; it is zero when reserves are abundant.



Our analysis below focuses on comparative statics of steady states. The time period in the model should be thought of as very short, such as a day. Given the lack of transition dynamics, we are comfortable using the model for thinking about the behavior of asset prices, payments and credit over a sequence of episodes, such as the recent boom bust cycle.

### 3.5 Credit lines as inside money

So far, our liquidity constraint requires households to pay with deposits. We now show that the basic logic of the model extends to another payment instrument that is important in practice, namely credit lines. The common denominator of both types of payment instruments is that banks make costly commitments to households that must be backed up by bank assets. In our analysis below, we thus apply the general term "inside money" to deposits and credit lines.

Suppose that households can pay either with deposits, or by drawing down a credit line  $L_t^h$ :

$$P_t C_t \leq D_t^h + L_t^h.$$

Credit lines must be arranged one period in advance with a bank, but require no investment – they represent intraday credit extended by banks on demand. In exchange for the commitment to accept payment instructions, banks charge a fee  $i_{t-1}^L L_t$  proportional to the credit amount.<sup>5</sup>

We consider equilibria such that the liquidity constraint binds and households are indifferent between alternative payment instruments. In any such equilibrium we have

$$\hat{\delta}_t - i_t^D - \pi_{t+1} = i_t^L > 0. \quad (15)$$

Households who invest in deposits must provide funds a period in advance on which they receive the real return  $i_t^D - \pi_{t+1}$ . Households who arrange a credit line can instead invest the funds in trees that yield  $\hat{\delta}$ , but must then pay the fee  $i^L$ . The inequality says that inside money is costly and households' optimal choice is then to hold as little money as necessary.

On the bank side, we assume for simplicity that the same collateral is required to back a dollar of credit line and a dollar of deposits. Moreover, the distribution of liquidity shocks is same per dollar of both payment instruments. Since the cost of liquidity (15) is also the same, banks are similarly indifferent between issuing the two instruments. In particular, both issuing deposits and issuing equity together with extending credit lines is cheaper than just issuing equity. The appendix formally states the problem with credit lines and works through the algebra.

## 4 Steady state equilibrium

In a steady state equilibrium, output  $Y = \Omega + x$  and rates of return are constant. We restrict attention to policies that imply the same constant growth rates for the nominal quantities  $M_t$  and  $B_t^g$  so that the bond-reserve ratio  $b := B_t^g/M_t$  is constant as well. With fixed rates of return, the marginal rate of substitution of wealth across dates is given by  $\hat{\delta}_t = \delta$ . The key bank ratios  $\lambda$  and  $\kappa$  are also constant but endogenously determined; they summarize the role of the payment system for asset prices and the price level.

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<sup>5</sup>We assume that an interest rate  $i^D$  is earned on deposits regardless of whether they are spent, and that the fee  $i^L$  is paid on credit lines regardless of whether they are drawn. These assumptions help simplify the algebra. More detailed modeling of the fee structure of different payment instruments is possibly interesting but not likely to be first order for the questions we address in this paper.

The determination of the bank ratios can be illustrated graphically by reducing the system of equations characterizing equilibrium to two equations describing two curves in a  $(\lambda, \kappa)$  plane. The *liquidity-management curve* follows from bank first-order conditions: it says what collateral ratio banks should choose when their liquidity ratio is  $\lambda$ , given current market prices. The *capital-structure curve* exploits the fact that the ratios are connected via bank balance sheets: it says how large of a liquidity ratio is needed to achieve a collateral ratio  $\kappa$  given the quantities of different types of collateral. We now derive both curves. To derive the liquidity-management curve, we start with a discussion of asset valuation in general equilibrium.

## 4.1 Collateral benefits and asset market participation

Except for the restriction that reserves must be held by banks, we have not made assumptions on asset market segmentation. Instead, participation patterns of intermediaries and households are determined endogenously in equilibrium. Throughout, the basic principle at work is familiar from other models with short sale constraints: assets are held – and hence priced – by the investor who likes them the most, either because that investor derives nonpecuniary (liquidity or collateral) benefits from them or later also because that investor has more optimistic beliefs.

Who participates in the overnight credit market? The first-order conditions for households (4) and banks (12) indicate that banks obtain a collateral benefit from overnight credit, whereas households do not. As long as banks issue some deposits, their collateral benefit is positive. It follows that banks like the asset more: their first-order condition holds with equality while that of household holds with strict inequality. In other words, households do not participate in the overnight credit market in equilibrium, because they perceive the equilibrium interest rate as too low. Therefore, all bonds are held by banks.<sup>6</sup>

The steady state real overnight interest rate satisfies the bank Euler equation

$$\delta - (i - \pi) = \text{mb}(\kappa). \quad (16)$$

Since  $\text{mb}'(\kappa) = -c''(\kappa) < 0$ , the interest rate is positively related to the collateral ratio. If collateral is scarce on bank balance sheets – or equivalently, if banks are highly levered – then interest rates are low. Intuitively, banks look for safe collateral and bid up the price of short bonds. As in other models of intermediary asset pricing, we obtain a connection between returns and leverage. An interesting feature here is that banks endogenously become the only marginal investor pricing short bonds.

Pricing of bank trees works analogously. The collateral benefit implies that trees are held by banks, not households. Cash flow on trees is discounted at a low rate that accounts for the collateral benefit: the steady state value of trees is

$$v(\kappa) = \frac{x}{\delta - \rho \text{mb}(\kappa)}. \quad (17)$$

It is important here that the representative household owns the bank, so the same discount rate  $\delta$  enters both first order conditions and the collateral benefit is the only difference between the first-order conditions.

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<sup>6</sup>It is important here that the representative household owns the bank, so the same discount rate  $\delta$  enters both first-order conditions and the collateral benefit. Another way to view the result is that shareholders find short bonds more useful inside the bank, and hence hold them only indirectly via banks.

## 4.2 Liquidity benefits and liquidity-management curve

How much collateral do banks optimally hold when they want a liquidity ratio  $\lambda$ ? The bank first-order condition (12) shows how optimal liquidity management equates the opportunity cost of reserves to their collateral and liquidity benefits. In general equilibrium, the liquidity benefit is in turn related to the aggregate collateral ratio:

$$\delta - (i^R - \pi) = \text{mb}(\kappa) + (1 - G(\lambda))(\text{mc}(\kappa) - \text{mb}(\kappa)). \quad (18)$$

The appendix derives this equation. For a given real return on reserves, it represents pairs  $(\lambda, \kappa)$  such that banks optimally choose  $\lambda$  and  $\kappa$  given market prices. We refer to it as the *liquidity-management curve*.

The left-hand side of the curve (18) are banks' opportunity costs of holding reserves, relative to shrinking the balance sheet and paying back shareholders who demand the return  $\delta$  on equity. The first term on the right-hand side is the collateral benefit  $\text{mb}(\kappa)$ , which is the same as for overnight credit. The second term is the liquidity benefit: with probability  $1 - G(\lambda)$ , the bank runs out of reserves, the liquidity constraint binds and the bank has a cost of borrowing  $\text{mc}(\kappa) - \text{mb}(\kappa)$  which is decreasing in  $\kappa$ . Comparing the bank Euler equation (16) with the liquidity-management curve, we see that the liquidity benefit is equal to the spread  $i - i^R$  between the overnight rate and the reserve rate.

The liquidity benefit of reserves is positive if and only if reserves are scarce, which happens when the liquidity ratio  $\lambda$  is below the upper bound  $\bar{\lambda}$  of the liquidity shock distribution. If reserves are abundant (formally,  $\lambda \geq \bar{\lambda}$  and hence  $G(\lambda) = 1$ ), banks never run out of reserves when handling payments, and they do not value reserves for their liquidity. The spread  $i - i^R$  shrinks to zero as reserves and short bonds become perfect substitutes and must earn the same return. As long as reserves are scarce, the liquidity benefit is decreasing in  $\lambda$ : if the liquidity ratio  $\lambda$  is higher, banks run out of reserves less often and achieve a lower expected benefit.

A higher collateral ratio  $\kappa$  reduces the liquidity benefit since it lowers the cost of overnight borrowing. A bank that runs out of reserves must turn to the overnight market to make payments and incurs a marginal cost of leverage  $\text{mc}(\kappa)$ . At the same time, one bank's overnight credit serves as collateral for another bank. By the bank's Euler equation (16), the collateral benefit of overnight credit is  $\text{mb}(\kappa)$  and lowers the equilibrium overnight rate. The cost of borrowing is the difference between the two terms.

Figure 3 shows the shape of the liquidity-management curve. The  $(\lambda, \kappa)$  plane splits into two halves: for  $\lambda > \bar{\lambda}$ , reserves are abundant and the curve is flat, whereas for  $\lambda < \bar{\lambda}$ , reserves are scarce and the curve slopes down. Intuitively, the liquidity-management curve tells us how much collateral banks should hold if they want to maintain a liquidity ratio  $\lambda$ . With scarce reserves, the optimal collateral ratio responds to  $\lambda$  since they may have to borrow overnight: in particular, for higher  $\lambda$ , less collateral is needed since the bank runs out of reserves less often. Once reserves are abundant, banks do not borrow overnight and changes in  $\lambda$  no longer affect the optimal  $\kappa$ .

### *Banks' demand for money and overnight borrowing*

The liquidity-management curve (18) can be viewed as a money demand function. From the bank Euler equation (16), the collateral ratio  $\kappa$  moves one-for-one with the overnight interest rate. Banks' desired liquidity ratio  $\lambda$  is thus decreasing in the spread  $i - i^R$  as long as reserves are scarce. Since the real quantity of deposits is pinned down by output  $Y$ , banks' real demand for reserves  $\lambda Y$

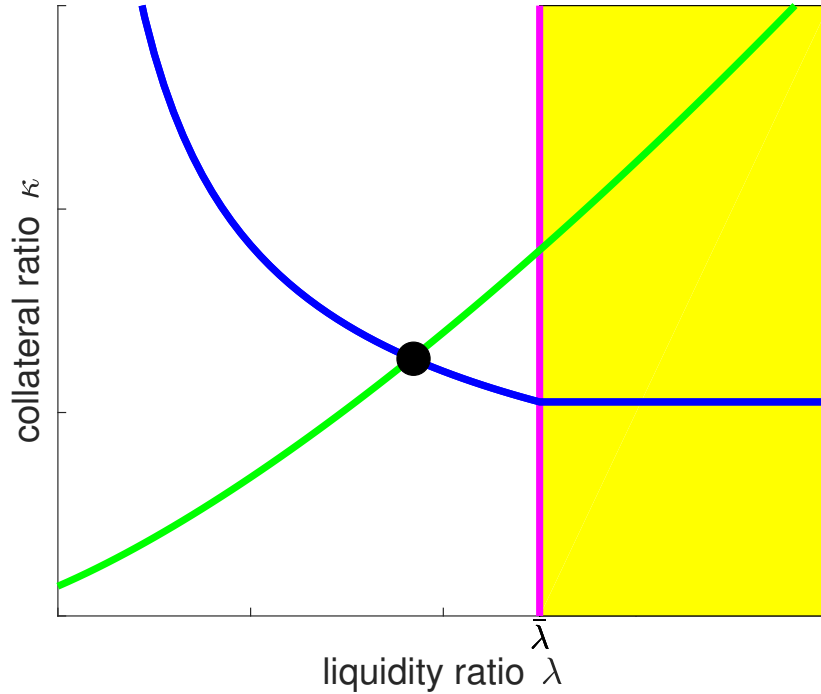


Figure 3: The blue line is the liquidity management curve. The green line is the capital structure curve. In the yellow shaded region, reserves are abundant,  $\lambda \geq \bar{\lambda}$ .

moves one for one with the liquidity ratio: it slopes down when reserves are scarce and becomes perfectly elastic at  $i = i^R$  when reserves are abundant.

We emphasize that bank demand for outside money as described by the liquidity-management curve is quite different from the demand for inside money. Indeed, in the current version of the model, the demand for inside money is inelastic at  $Y$ . As a result, the demand for outside money does not depend on the liquidity preference of households. Instead, it is shaped by banks' technology for handling payments. For example, when bank's money demand becomes flat depends on the maximal liquidity shock  $\bar{\lambda}$  banks can face, as opposed to say, the level of real balances at which households are satiated.

Since banks handle payments with either reserves or overnight borrowing, the equilibrium liquidity ratio affects the outstanding amount of interbank overnight credit. When reserves are abundant, banks do not borrow overnight. With scarce reserves, aggregate overnight credit follows from integrating over all banks that receive a liquidity shock beyond  $\lambda_t$ :

$$\frac{F_t}{D_t} = \int_{\lambda}^{\bar{\lambda}} (\bar{\lambda} - \lambda) dG(\tilde{\lambda}) := f(\lambda). \quad (19)$$

The function  $f$  is decreasing: if the banking system has more real reserves to handle a given amount of transactions, then less overnight credit is needed. In Figure 3, overnight borrowing by banks can therefore be read on the horizontal axis going left.

### 4.3 Capital-structure curve

How much liquidity  $\lambda$  is required in order for banks to manage transactions  $T$  with a collateral ratio  $\kappa$ ? Regardless of whether reserves are abundant or not, the ratios are mechanically related via

bank balance sheets. Substituting (17) and (19) into the collateral ratio (10), we obtain the general equilibrium relationship

$$\kappa = \frac{\lambda(1+b) + \rho v(\kappa)/T + f(\lambda)}{1 + f(\lambda)}. \quad (20)$$

We refer to pairs  $(\lambda, \kappa)$  that satisfy this relationship as the *capital-structure curve*. Since the bond-reserve ratio  $b = B_t^g/M_t$  is a fixed parameter, the curve also tells us how much government leverage is required to handle transactions with given bank leverage.

The capital-structure curve slopes up in the  $(\lambda, \kappa)$ -plane; it is shown as a light green line in Figure 3. The basic intuition is that reserves contribute to collateral and the real quantity of deposits is pinned down by the volume of transactions  $T$ . Other things equal, a bank that wants more collateral thus needs to hold more reserves. As a stark example, consider a “narrow bank” which holds no trees or overnight credit – its only collateral is reserves or short term government debt. The collateral ratio is then simply the ratio of reserves to inside money:  $\kappa = (1+b)\lambda$ .

Two more subtle effects further contribute to an upward slope. First, banks with higher liquidity ratios run out of reserves less often, which results in lower outstanding interbank credit and hence a higher collateral ratio – the  $f(\lambda)$  terms in (20). Indeed, since every dollar of interbank credit is both an asset and a liability to the banking sector, a reduction in interbank credit also increases overall collateral. Second, more collateralized banks compete less for trees, so tree prices fall – the  $v(\kappa)$  term in (20). Achieving a higher collateral ratio thus requires even more reserves.

#### 4.4 Equilibrium and comparative statics

We can now summarize the determination of bank positions, asset prices and the price level in steady state equilibrium. The liquidity and collateral ratios are determined by the intersection of the capital structure and liquidity-management curves. Reserves can be scarce – as displayed in Figure 3 – but they can also be abundant. Which case obtains depends both on policy parameters such as  $i^R$  and  $b$ , and on features of financial structure, in particular, the availability of trees and distribution of liquidity shocks. We discuss these forces in detail below.

Steady state values of  $\lambda$  and  $\kappa$  then imply equilibrium rates of return or prices for all assets. In particular, the overnight rate and the price of trees follow from the bank Euler equation (16) and the tree valuation equation (17), respectively. In addition, we obtain the equilibrium deposit rate  $i^D$  from

$$\delta - (i^D - \pi) = mc(\kappa) + (1 - G(\lambda))(mc(\kappa) - mb(\kappa)). \quad (21)$$

When banks issue deposits, they incur a marginal leverage cost  $mc(\kappa)$ . If reserves are scarce, they incur in addition a liquidity cost since they have to tap the overnight market. The cost of liquidity for households is the opportunity cost of holding deposits  $\delta - (i^D - \pi)$ , which is decreasing in both the collateral and liquidity ratios.

Finally, the quantity equation (14), implies that the equilibrium price level grows over time at the same constant rate as nominal liabilities:

$$P_t = \frac{M_t}{\lambda Y}. \quad (22)$$

Bank liquidity management determines the money multiplier  $1/\lambda$  and hence the real quantity of reserves  $\lambda Y$  demanded by banks. The rate of inflation follows from government policy.

Comparative statics of steady states captures the response to an unanticipated "shock", a change in the environment such as the growth rate of nominal liabilities. Suppose the economy is initially in steady state and the shock hits at date  $t$ . As of date  $t$ , reserves  $M_t$  and the liquidity ratio are predetermined, so that the price level  $P_t$  still lies on the old steady state path. From date  $t + 1$  on, however, the economy is in the new steady state, where all ratios and rates of returns take new values, and the price level (22) is evaluated at the new steady state liquidity ratio.

For inflation, comparative statics of steady states thus captures both a short run and a long run response to a shock. Between dates  $t$  and  $t + 1$  the price level jumps to achieve the new steady state real quantity of reserves  $\lambda Y$  (the short run response), followed by the evolution of the price level at the new growth rate of government liabilities (the long run response). A similar pair of responses can be derived for returns on trees: prices of trees experience a short run adjustment and then settle at new values that reflect the rates of return in the new steady state.

## 4.5 Financial structure and the payment system

This section highlights three properties of a layered payment system as studied in this paper. First, the cost of liquidity differs across layers and policy only indirectly affects the cost of liquidity in the bank customer layer. Second, how many reserves it takes to achieve "abundance" depends on the organization of the banking system, in particular on interbank netting agreements. Finally, details of financial structure – in particular, netting and the denomination of collateral – matter for the cost of liquidity and the connection between asset prices and inflation.

### *Cost of liquidity*

In many monetary models, the cost of liquidity to all agents is measured by the spread between the short interest rate and the interest rate on money. In a "liquidity trap" with  $i = i^R$ , bonds and money are then perfect substitutes for all agents. In a layered payment system, the spread  $i - i^R$  only represents the cost of liquidity of banks. When reserves are abundant, bonds and outside money are perfect substitutes *for banks*. However, the spread  $i - i^R$  does not matter directly to households, who do not hold reserves and do not participate in the overnight credit market.

The cost of liquidity for bank customers is given by the opportunity cost of deposits (21). In general, it depends on banks' liquidity and collateral ratios. It reflects the spread between bonds and reserves only to the extent banks pass on the cost of producing liquidity to their customers. For example, when reserves become abundant, the cost of liquidity falls. However, even in that case it remains positive: it is still true that liquidity relies on costly bank leverage, so deposits and other assets are not perfect substitutes for households.

### *Interbank netting and abundance vs scarcity of reserves*

How many reserves does it take to make reserves abundant? Relatedly, if a central bank wants to reduce its balance sheet to return to scarcity, how far does it have to go? Our model says that the answer depends crucially on how the banking system handles liquidity shocks. One concrete way in which banks influence the size of liquidity shocks is by participating in netting systems that economize on the use of reserves. We now illustrate the role of how the banking system is organized by introducing netting into the model.<sup>7</sup>

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<sup>7</sup>More generally, the changes in the industrial organization of banking can have similar effects. For example, if banking becomes more concentrated through, say, a merger wave, we would also expect fewer interbank payments to handle the same flow of bank customers' transactions and hence a lower demand for reserves.

The liquidity constraint (6) assumes that all bank payments must be made with reserves. Suppose that banks can also use intraday credit  $I_t$  to pay, but that their intraday credit position is limited by a downpayment into the netting system that has to be made in reserves, either held overnight or borrowed from other banks. We thus replace the liquidity constraint by

$$\begin{aligned}\tilde{\lambda}_t D_t &= M_t + F_{t+1} + I_t, \\ I_t &\leq \gamma (M_t + F_{t+1})\end{aligned}$$

and subtract intraday credit  $I_t$  from the date  $t$  cash flow (9).

A bank that runs out of reserves will exhaust its intraday credit limit before tapping the overnight market, and will continue to borrow as little as possible. We can thus combine the two constraint into

$$\tilde{\lambda}_t D_t \leq (1 + \gamma) (M_t + F_{t+1}). \quad (23)$$

The bank's problem is then to maximize shareholder value subject to (23). The appendix shows that the structure of the solution is the same as above, but with the new liquidity ratio (7) defined as  $\lambda_t := (1 + \gamma) M_t / D_t$ . The price level is then determined as  $P_t = M_t (1 + \gamma) / \lambda Y$ .

The new formula for  $\lambda_t$  links the amount of reserves required to achieve abundance (that is,  $\lambda_t \geq \bar{\lambda}$ ) to the extent of interbank netting: more efficient netting (higher  $\gamma$ ) implies that fewer reserves are sufficient to deliver abundance. In general, the fewer interbank payments are required to handle a given volume of bank customer transactions, the fewer reserves are needed a medium of exchange in the bank layer. At the same time, the extent of netting also affects the capital-structure curve, an effect discussed below.

#### *Nominal collateral, netting, interest rates and inflation*

In a layered payment system, bank balance sheets provide a link between asset prices – the value of collateral held by banks – and the nominal price level, which is determined by banks' issuance of inside money. In our model, this link is represented by the slope of the capital-structure curve: it connects the collateral ratio  $\kappa$  – and hence, from the bank Euler equation (16) and the valuation of trees (17), the rates of return on assets held by banks – to the liquidity ratio  $\lambda$  that drives the money multiplier and hence the price level. We now show how financial structure – in particular the denomination of collateral and the presence of netting – shapes this link.

To clarify the role of nominal collateral, we introduce a second type of trees: it promises a payment stream of  $X_t = nM_t$  dollars. We think of this tree as, say, nominal long term mortgages; payments grow with the supply of reserves to ensure a constant share of nominal trees in steady state. Other than the denomination of payments, nominal trees work exactly like the "real trees" studied so far – both enter the collateral ratio with weight  $\rho$ . In equilibrium, nominal trees are thus held exclusively by banks, and their nominal value is  $nM_t v(\kappa)$ .

Allowing for netting and nominal collateral does not affect the liquidity-management curve: banks' choice of ratios given prices does not change. However, the appendix derives the new capital-structure curve as

$$\kappa = \frac{\frac{\lambda}{1+\gamma} (1 + b + n\rho v(\kappa)) + \rho v(\kappa) + \frac{f(\lambda)}{1+\gamma}}{1 + \frac{f(\lambda)}{1+\gamma}}. \quad (24)$$

Comparing with our previous expression for the capital-structure curve (20), there are two differences: more efficient netting (higher  $\gamma$ ) makes the curve flatter, whereas more nominal collateral (higher  $n$ ) makes it steeper.

Intuitively, financial structure determines the link between collateral and liquidity ratios because the price level feeds back to the real value of collateral on the balance sheet. Indeed, a lower money multiplier  $1/\lambda$  depresses the price level  $P_t = M_t(1 + \gamma)/Y\lambda$  and thereby increases the real value of all nominal assets. If there are nominal assets other than reserves, then this revaluation is stronger and it takes a smaller decline in  $1/\lambda$  to achieve any given increase in  $\kappa$ : the curve becomes steeper. In contrast, if there is more netting then a lower money multiplier  $1/\lambda$  depresses the price level by less, the revaluation is weaker and the curve is flatter.

Importantly, what matters for revaluation is only the denomination of collateral, not the identity of the borrower. The slope of the capital-structure curve increases both when there are more nominal trees, which we interpret as private liabilities denominated in dollars, and when there are more nominal government bonds (higher  $b$ ). The feature that the quantity of nominal bonds matters for the price level connects our model to the fiscal theory of the price level. However, the role of government debt here is different: it is only one of the various assets that banks use to back inside money, and this is what gives it a special role.

## 5 Monetary/fiscal policy and interest on reserves

In this section we discuss the effect of government policy on equilibrium asset prices and inflation. The government has two distinct policy tools. First, it can set the real return on reserves  $i^R - \pi$ : it controls the nominal rate on reserves, and it can change the inflation rate via the growth rate of nominal government liabilities. Second, the government can alter the mix of collateral available to banks. For example, the central bank can perform open-market trades in government bonds, thus changing the bond-reserve ratio  $b$ . It could also purchase other assets such as trees.

A key feature of our model is that the two policy tools affect asset prices and inflation in different ways. A quick way to see this is that the real return on reserves  $i^R - \pi$  affects only the liquidity-management curve (18) but not the capital-structure curve (20), whereas the opposite is true for the bond-reserve ratio  $b$ . We now discuss the policy tools in turn. We pay particular attention to another key feature of the model: the magnitude of policy effects depends on financial structure.

### 5.1 The real return on reserves

Changes in the real return on reserves capture two policy shifts that may at first sight appear quite different. Before the financial crisis, most countries had the nominal reserve rate fixed at zero and real returns differed only because of inflation. In this context, a higher growth rate of government liabilities increases the inflation rate and lowers the real return on reserves. More recently, many central banks operate in an environment of abundant reserves and actively change the nominal reserve rate while committing to a stable path of inflation.

Our framework emphasizes the common denominator of these the two policy changes: a lower real return on reserves lowers banks' return on assets and thereby makes liquidity more costly. Since reserves are effectively taxed more, banks respond by adjusting their leverage and portfolio choice. In particular, banks would like to maintain a return  $\delta$  on equity. With a lower return on assets, they can still achieve that return on equity by increasing leverage, or decreasing the collateral ratio  $\kappa$ . Mechanically, the liquidity-management curve shifts down, as in Figure 4, where the shifted curve is shown as a dashed line.

Since a lower real return on reserves reduces banks' desire for collateral, they want fewer reser-



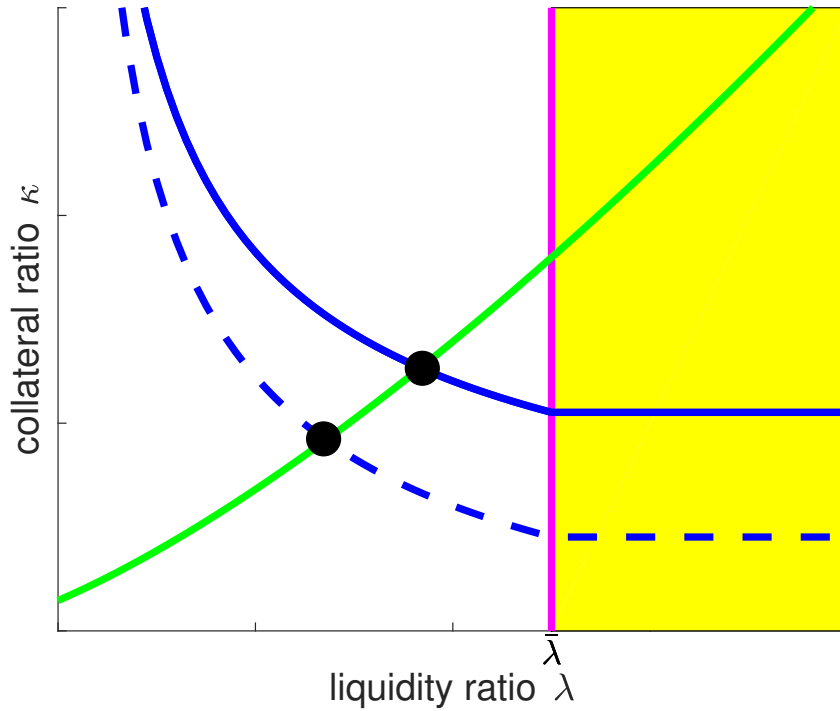


Figure 4: Faster growth rate of nominal liabilities when reserves are scarce.

ves and hence reduce the liquidity ratio – mechanically, we move to a new equilibrium along the upward sloping capital-structure curve. A lower return on reserves thus reduces both  $\lambda$  and  $\kappa$ . The magnitude of the change in  $\lambda$  depends on the slope of the capital-structure curve, or the balance sheet link between collateral and liquidity ratios. If banks hold more nominal collateral, or there is less efficient netting, then the capital-structure curve is steeper and  $\lambda$  falls by less.

From the deposit rate equation (21) the cost of liquidity to households declines – when a key input to the production of liquidity services is taxed less, the banking system passes along the lower cost to its customers. The effect does not depend on whether or not reserves are abundant, or on why the real return on reserves changes: what matters is only that (i) policy affects banks’ desire to hold collateral and (ii) collateral and liquidity ratios are connected through the balance sheet.

We now apply the basic logic of bank adjustment to two concrete policy experiments: faster growth of government liabilities in a scarce reserve regime, and higher interest on reserves with abundant reserves. We show that changes in the real return on reserves have intuitive effects on interest rates and inflation: expansionary policy (such as faster money growth) lowers interest rates and is inflationary in the short run, whereas contractionary policy (such as a higher nominal rate on reserves) does the opposite.

#### *Scarce reserves and higher growth of nominal liabilities*

Figure 4 illustrates the effect of a lower return on reserves in the traditional policy environment. Suppose we start from an initial equilibrium with, say,  $i^R = 0$  and scarce reserves, and the government announces faster growth of nominal liabilities. The capital-structure curve remains unchanged, whereas the liquidity-management curve shifts down. The new equilibrium has lower  $\lambda$  and  $\kappa$ , and hence lower real reserve holdings, more overnight borrowing by banks and higher bank leverage.

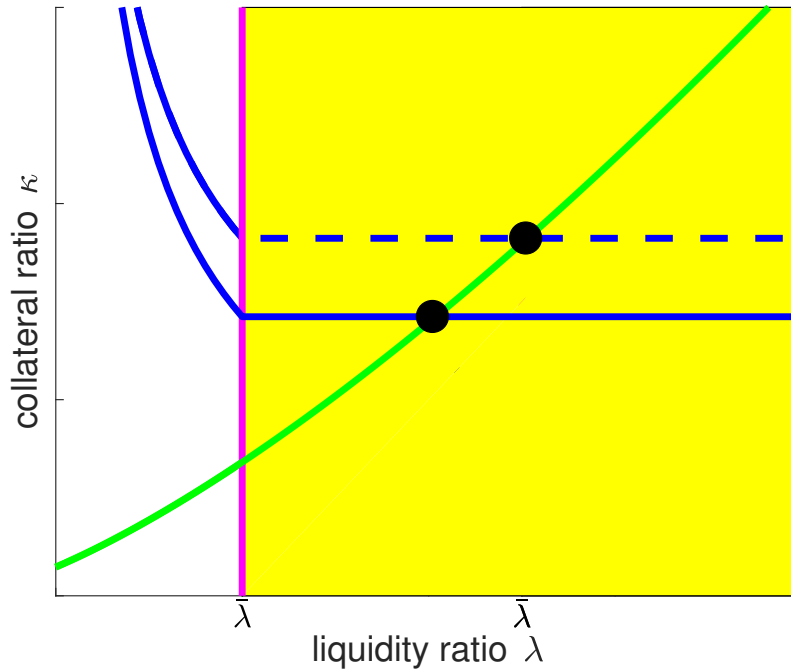


Figure 5: Higher interest on reserves  $i_R$  when reserves are abundant

The policy shift has short and long run effects on inflation. Higher exogenous growth of government liabilities generates higher inflation in the new steady state. In addition, banks respond to the announcement by increasing the money multiplier, creating additional inflation in the short run. This response effect depends crucially on financial structure: it is smaller the more nominal assets bank use to back inside money. Higher inflation also has permanent effects on asset prices – in particular the real overnight rate falls and the price of trees increases. The reason is that changes to the real rate on reserves – effectively, a higher tax on bank assets – makes other collateral more scarce and increases its price.

*Abundant reserves and higher nominal rate on reserves*

Consider now an initial equilibrium with abundant reserves. Suppose that the central bank *increases* the nominal interest rate on reserves, but that the government continues to commit to the same constant growth rate  $\pi$  of nominal liabilities. We now obtain an increase in the real return on reserves, so the mechanical effect is the opposite of the above: as the real rate on reserves increases, the liquidity-management curve shifts up, as in Figure 5.

As the opportunity cost of holding reserves falls, banks can maintain the same return on equity with lower leverage or a higher collateral ratio, and they achieve this by increasing their liquidity ratio. The money multiplier therefore falls, and inflation is lower in the short run. Again, financial structure is important here: a higher share of nominal collateral implies a weaker response of inflation to changes in the reserve rate. After one period, inflation resumes at its old rate in the new steady state. As banks become better collateralized, trees and bonds become less valuable as collateral so the price of trees falls and the real overnight rate increases.

We emphasize that the effects of monetary policy on real asset prices are *permanent* – they result from comparative statics across steady states. This is in contrast to many models with sticky prices or segmented markets, where “liquidity effects” on the real interest rate are temporary phenomena.

Permanent effects arise in our model because the opportunity costs of holding reserves lead banks to change the way they produce inside money, with effects on the cost of leverage and the value of collateral.

### *Negative interest rates on reserves*

Our framework does not require that the nominal or real interest rate on reserves be positive. This is due to our technological assumption that banks rely on reserves for payment and always hold reserves in equilibrium. In particular, banks cannot handle payments or hold collateral by converting reserves into currency. We believe this is a sensible assumption as long as nominal rates on reserves are not too low. If reserves are indeed valuable, then the government – which provides an asset with superior liquidity and collateral properties – thereby has the power to tax a share of bank assets.

If the government chooses negative rates, it makes reserves less attractive and banks will try to economize on reserves by choosing a lower liquidity ratio. Reversing the shift of the liquidity-management curve in Figure 5, the government can in principle lower interest rates so as to move all the way into the scarce reserve region. This policy lowers real rates and props up the value of assets held by banks. At the same time, it creates inflationary pressure as it lowers the liquidity ratio and increases the money multiplier.

## **5.2 The collateral mix**

The second policy tool of the government is changing the mix of collateral available to banks with open market trades. In a traditional open market purchase, the government buys back bonds from banks in exchange for reserves. In our framework, such a trade alters the balance sheet link between collateral and liquidity ratios: as bonds are no longer available to back inside money, it takes more reserves to achieve any given collateral ratio. Mechanically, the capital-structure curve shifts to the right.

The effect of traditional open market policy on bank portfolios depends crucially on whether reserves are scarce or abundant. With scarce reserves, banks with higher liquidity ratios have to borrow overnight less often and therefore choose lower collateral ratios as well. We thus move to a new equilibrium along the downward sloping portion of the liquidity-management curve, as in Figure 6. In contrast, when reserves are abundant the collateral ratio remains fixed and banks absorb the additional reserves to increase the liquidity ratio  $\lambda$ .

More formally, consider a comparative static that increases reserves and offsets this change by an equal change in bonds: we move from an initial equilibrium with  $(B_0^g, M_0)$  to a new equilibrium with  $(\tilde{B}_0^g, \tilde{M}_0)$ , where  $\tilde{M}_0 - M_0 = B_0^g - \tilde{B}_0^g > 0$ . As before, reserves and bonds then grow at the same rate  $\pi$  throughout. For graphical analysis, this expansionary open-market operation can be summarized by a decrease in the bond-reserve ratio from  $b$  to  $\tilde{b}$  which shifts the capital-structure curve to the right by (20). There is no effect on the liquidity-management curve which does not depend on  $b$ .

If reserves are abundant in the initial equilibrium, then the policy has no effect on the collateral ratio, asset prices and inflation. With abundant reserves, both reserves and short bonds are equally valued by banks as collateral only; there is no liquidity benefit from reserves. A trade that leaves total government liabilities  $M_t + B_t$  unchanged must therefore be irrelevant. This result is consistent with analysis in existing monetary models – here it is derived in a two layer system where the demand

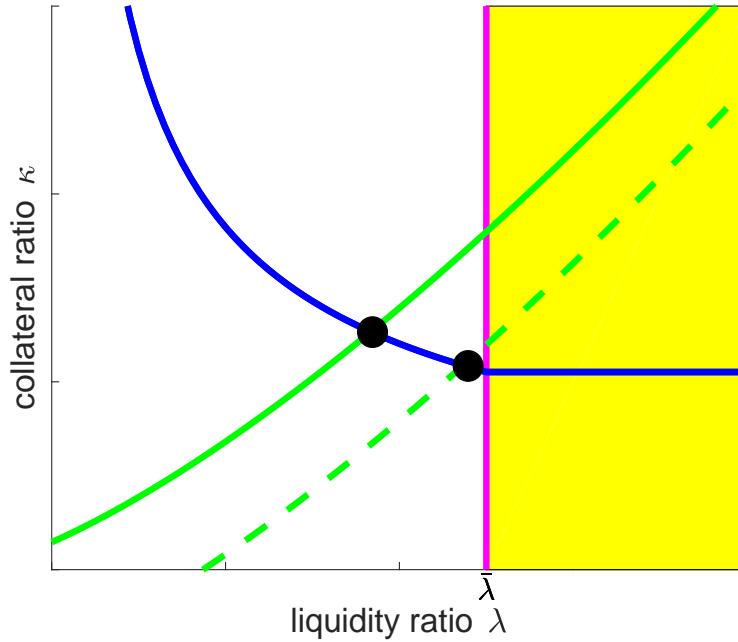


Figure 6: Open market operations when reserves are scarce

for outside money comes from banks.

The result follows from (20) and (22). By construction, the policy maintains the same nominal path of government liabilities, that is  $\tilde{M}_t(1 + \tilde{b}) = M_t(1 + b)$ . Moreover, banks prefer to maintain the same *real* value of government liabilities, since they want to keep leverage constant. Indeed, in the abundant reserve region the liquidity-management curve is flat and  $f(\lambda) = 0$  so that  $\tilde{\lambda}(1 + \tilde{b}) = \lambda(1 + b)$ . From (22), the higher supply of outside money is therefore exactly offset by a drop in the money multiplier. Constant bank leverage further implies that asset prices do not move.

In contrast, when reserves are scarce as in Figure 6, then a purchase of bonds with reserves is inflationary in the short run and permanently lowers the real interest rate. The short run inflation response is subtle since the policy shift again both increases the quantity of outside money and lowers the money multiplier. However, with scarce reserves, banks with higher liquidity ratios choose to reduce the collateral ratio. Lower demand for collateral reduces the real value of government liabilities. The money multiplier thus falls by less than the growth rate of outside money and the price level increases in the short run.

Graphically, consider moving horizontally at the original  $\kappa$  to the new capital-structure curve. Such a move would increase the liquidity ratio to keep the real value of government liabilities  $\lambda(1 + b)$  constant – this is what happens with abundant reserves. In the scarce reserve region, however, the liquidity-management curve slopes down, so that the new equilibrium liquidity ratio is lower. In particular, the percentage change in  $\lambda$  is now less than the change in outside money growth given by the policy.

### 5.3 The role of the nominal interest rate

Some central banks conduct monetary policy by following a nominal interest rate rule. In practice, the rule is typically implemented by open-market policy. For example, during the scarce reserves

regime in place in the US until 2008, the New York Fed’s trading desk bought and sold bonds of various maturities in exchange for reserves in order to move the overnight interest rate (the Federal Funds rate) close to the Fed’s target. More recently, as reserves have become abundant, the Fed Funds rate and the interest rate on reserves have been essentially the same, and the policy lever is the interest rate on reserves. It is then tempting to simply transfer existing analysis of interest-rate rules to the abundant reserves environment even though the policy implementation is different.

In many monetary models, the details of how the central bank implements the interest-rate rule are indeed irrelevant – the nominal interest rate alone summarizes the stance of monetary policy. In particular, many models use households’ optimal choice between currency and short-term bonds to derive optimal real balances as a function of the nominal interest rates and consumption. At the same time, intertemporal asset pricing equations – and possibly price setting equations – imply a path for inflation. The path for the money supply can then be inferred ex post so as to generate the implied path for real balances, but is often omitted from the analysis altogether. In particular, it does not matter whether policy is implemented with open-market purchases or interest on reserves.

In our model, policy cannot be summarized by the nominal interest rate alone. As discussed above, policy matters in two ways. Policy can either change the real return on reserves or the collateral mix between reserves and government bonds, which matters as long as reserves are scarce. Both policy actions affect the overnight nominal interest rate. We now show that, with scarce reserves, the same nominal interest rate can be achieved with many combinations of interest on reserves and open-market purchases that have different implications for real interest rates and inflation. Moreover, we show that the interest rate on reserves is not sufficient to characterize policy with abundant reserves.

*Interest rate policy with scarce reserves*

Consider first the case of scarce reserves. We start from an initial equilibrium in that region; it is generated by initial parameters  $i^R, \pi$  and  $b$  and implies some initial overnight rate. Holding fixed  $i^R$ , we now choose a new target overnight rate that is below the initial overnight rate (but still above the reserve rate) and ask how  $\pi$  and  $b$  can change to implement it. To proceed graphically, we combine the first-order condition for overnight lending (16) and the liquidity-management curve (18) to trace out all equilibrium pairs  $(\lambda, \kappa)$  that are consistent with the target spread  $i - i^R$ :

$$i - i^R = (1 - G(\lambda)) (mc(\kappa) - mb(\kappa)). \tag{25}$$

This spread is the opportunity cost of holding reserves rather than lending overnight. It must be equal to the liquidity benefit of reserves on the right hand side since the collateral benefits of the two assets are the same.

The curve in  $(\lambda, \kappa)$  plane described by equation (25) is displayed as a red line in Figure 7. It has four key properties. First, it is downward sloping, much like the liquidity-management curve with scarce reserves: at a given spread, banks with higher liquidity ratios choose lower collateral ratios since they run out of reserves less often. Second, the red curve never enters the abundant reserves region: as reserves become more abundant, collateral must fall to maintain a positive spread.

Third, the red curve lies above the liquidity-management curve at the initial equilibrium. This is because the new target overnight rate and hence the new target spread are below the initial overnight rate and spread, respectively. For the old target spread, the red line would pass through the initial equilibrium. To lower spread, a given liquidity ratio  $\lambda$  must be associated with more collateral. Finally, we note that the red curve is independent of  $\pi$  and  $b$ , the two parameters describing policy

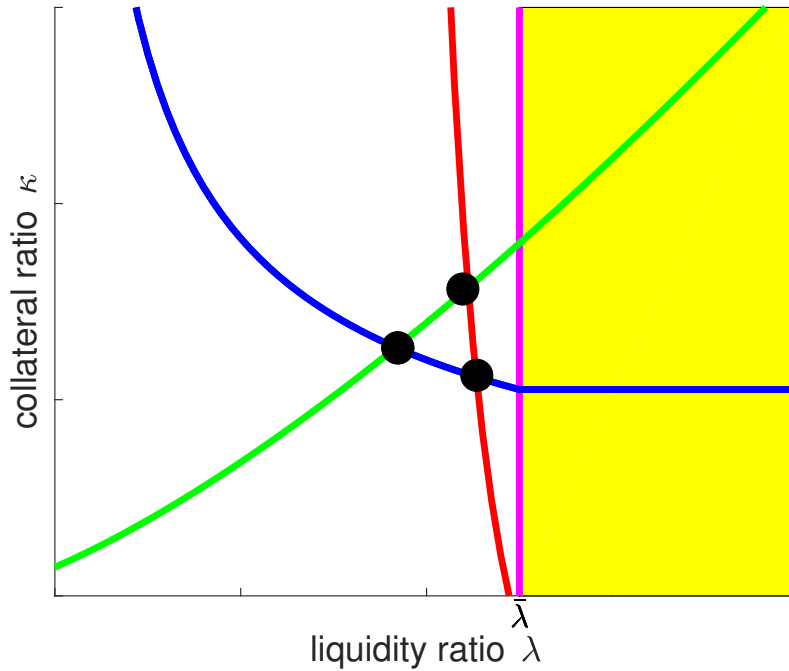


Figure 7: The red curve is equation (25)

with scarce reserves.

How can the government change policy to move to the new lower overnight rate, that is, to shift equilibrium onto the red line? The new equilibrium pair  $(\lambda, \kappa)$  must lie on the capital structure and liquidity-management curves. The policy analysis above thus suggests two simple options. First, the government could announce a lower growth rate of nominal liabilities  $\pi$ , and thereby shift the liquidity-management curve up until all three curves intersect. Second, the government could purchase bonds in the open market and thereby shift the capital-structure curve to the right until all three curves intersect.

Both policies produce the same change in the overnight nominal rate. At the same time, they have very different implications for the real interest rate and inflation. As discussed in the previous section, lower growth of government liabilities lowers inflation and increases the real interest rate. In contrast, open market purchases of bonds increase inflation and lower the real interest rate. With open market purchases, the inflation response is short term only, so that the new lower nominal interest target is achieved via a lower real rate. In contrast, with lower outside money growth the higher real rate is offset by lower inflation.

In addition to the two extreme policies just sketched, many other policies are also consistent with the new target nominal overnight rate. Indeed, we can combine open-market purchases with announcement of future growth of liabilities: we then shift both curves at once, rather than one at a time as for the extreme policies. The only requirement on the shifts is that the new equilibrium ends up on the curve  $\kappa$  described by equation (25). In particular, the same nominal interest rate is compatible with an entire range of bank liquidity ratios in equilibrium.

A key difference between our model and other models of scarce outside money is that the *only* medium of exchange for bank customers is inside money produced by banks. Outside money – here reserves – is only one input into the production of inside money. In particular banks also use

government bonds as collateral to back inside money. As a result, the spread between the overnight rate and the reserve rate measures the scarcity of reserves for banks; it does not measure the scarcity of inside money in the economy as a whole. In particular, there is not a unique amount of reserves implied by a given volume of transactions and a spread.

#### *Interest rate policy with abundant reserves*

Consider policy in the abundant reserve regime. Can the government describe policy only by the single interest rate  $i = i^R$ ? Equilibrium is described by (16) and (20), which determine  $\lambda$  and  $\kappa$  for a given real return on reserves. As a result, a nominal reserve rate alone cannot pin down bank liquidity and collateral. Similarly, a feedback rule that relates the rate on reserve to inflation, for example  $i = g(\pi)$ , does not uniquely determine  $\lambda$ ,  $\kappa$ , inflation and the real interest rate. This is true even if we directly select a rule for the real rate as a function of  $\pi$ , thus eliminating possible multiplicity coming from the shape of  $g$  that has been discussed in the literature.

Our model differs from other monetary models in what happens once outside money becomes abundant. Consider first models with bonds and currency only. At the zero lower bound in such models, bonds and outside money become perfect substitutes to bank customers, so the medium of exchange (currency) loses its liquidity benefit. In the current model, bank customers hold neither bonds nor outside money – both are held only by banks. Equating  $i$  and  $i^R$  makes bonds and outside money perfect substitutes for banks, but does not remove the liquidity benefit of the medium of exchange, inside money produced by banks.

There are also models in which reserves, bonds and currency coexist. In such models,  $i = i^R > 0$  makes bonds and reserves perfect substitutes. At the same time, currency remains a scarce medium of exchange that is relevant for some transactions. The reserve rate represents the spread between reserves and currency; it measures the scarcity of currency and relates the demand for real balances to real variables such as consumption. The tradeoff between currency and reserves is what enables those models to work with interest rate rules in the usual way even when reserves are abundant.

## 5.4 Optimal policy

The optimal payment system minimizes the loss of resources due to leverage. Our technological assumptions say that a given volume of transactions requires  $T$  a fixed amount of deposits supplied by banks. The question for society is how those deposits should be backed. Total consumption lost every period in steady state can be expressed as a function of banks' collateral and liquidity ratios:

$$c_g \left( \frac{\Omega}{(1+b)\lambda} \right) (1+b)\lambda T + c(\kappa) T (1+f(\lambda)). \quad (26)$$

Provided that the leverage cost of the government slopes up fast enough, the indifference curves are upward sloping and convex, as shown in Figure 8.

We consider the best steady state equilibrium that the government can select by choice of its two policy instruments, the collateral mix represented by  $b$  and the real return on reserves  $i_R - \pi$ , which determines the opportunity costs of holding reserves. The optimal policy problem is to choose those instruments together with  $\lambda$  and  $\kappa$  to minimize losses (26) subject to the capital-structure curve (20) and the liquidity-management curve (18).

If the government can freely choose the ratio of bonds to reserves, it is optimal to set  $b = 0$ . Indeed, while bonds and reserves provide the same collateral services, reserves also provide liquidity

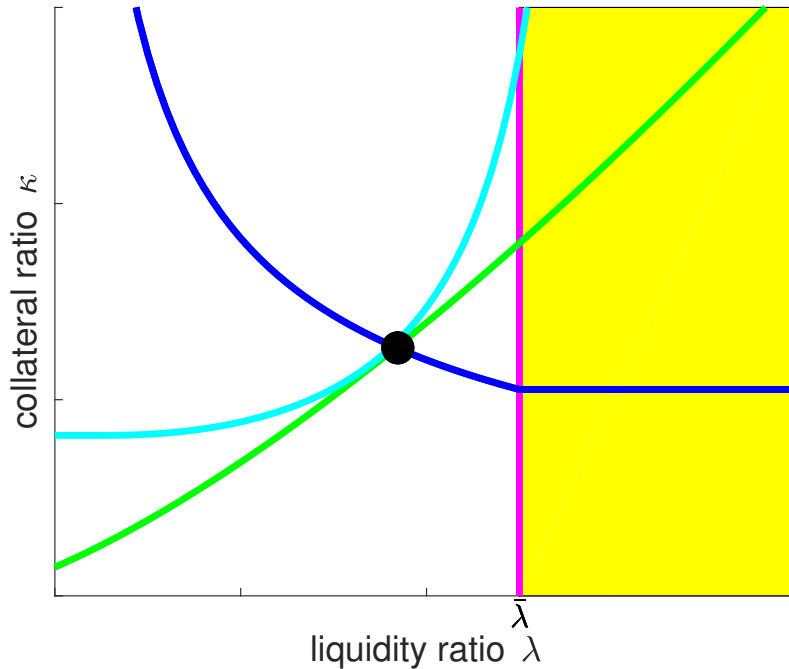


Figure 8: The cyan line is an indifference curve implied by equation (26)

services, which lowers the need for interbank borrowing and hence costly bank leverage. Since the model focuses on the provision of payment services, there is no benefit of government bonds per se, nothing is lost by just issuing reserves. More generally, the way fiscal policy is conducted independently of monetary policy may imply that there is a constraint on  $b$ . We can then view the welfare costs as a function in  $\lambda$  and  $\kappa$  with  $b$  a fixed parameter.<sup>8</sup>

The return on reserves directly affects neither the welfare cost nor the capital-structure curve. We can therefore find the optimal solution in two steps. We first find a point  $(\lambda, \kappa)$  on the capital-structure curve (for given  $b$ ) that minimizes (26). The optimal real return on reserves is then whatever return shifts the liquidity-management curve so that the equilibrium occurs precisely at that optimal point. If the indifference curves are convex and the capital-structure curve is curved less – which is a reasonable assumption if the effects of interbank credit are relatively small – then we obtain an interior solution as shown in Figure 8.

Should reserves be abundant? The figure suggests that this is not necessarily the case. Indeed, if the government leverage cost curve slopes upward very steeply, then it may be optimal to run a system with scarce reserves, in which real government leverage is much lower than the debt required to run the payment system. It is better to have banks rely on other collateral in order to back inside money. However, if the government can borrow cheaply at will, so that its leverage cost is close to zero, then it makes sense to move towards narrow banking where reserves make up the lion's share of bank portfolios.

<sup>8</sup>Alternatively, we could capture fiscal policy by a given real amount of bonds, say  $b$ . The welfare cost can then be written with total debt equal to  $m + b$  as opposed to  $m(1 + B_t/M_t)$ . The basic tradeoff remains the same.



## 6 Securities markets and the payment system

In this section we consider the interplay between securities markets and the payment system. We maintain throughout our focus on steady states. We first introduce uncertainty premia, the key source of fluctuations in asset prices. This allows us to discuss the effect of uncertainty on the supply of inside money as well as unconventional monetary policy – the government buys trees that carry uncertainty premia. These questions can be studied even if the only link between tree (that is, securities) markets and the payment system is that banks invest in trees.

We then extend the model to introduce two additional links: banks lend overnight to institutional investors and institutional investors use inside money to trade assets. To clarify the effect of each link in isolation, we introduce two types of asset management firms: *carry traders* buy trees on margin, whereas *active traders* face liquidity constraints for some asset purchases. Both types of firms otherwise work like banks: they are competitive firms owned by households that have access to a subset of trees and maximize shareholder value.

### 6.1 Introducing uncertainty and collateral quality

We capture a change in uncertainty as a change in beliefs about asset payoffs: we assume that households behave *as if* tree dividends  $x$  are permanently lower by  $s$  percent from the next date on. Actual tree payoffs remain constant throughout. One way to think about these beliefs is that households are simply pessimistic. Our preferred interpretation is ambiguity aversion: households contemplate a range of models for payoffs, and evaluate consumption plans using the worst case model. In either case, the key effect of pessimistic valuation is to generate premia on assets: an observer (such as an econometrician measuring the equity premium) will observe low prices relative to payoffs and hence high average returns.

To define equilibrium, we must take a stand not only on beliefs about exogenous variables, but also about endogenous variables such as the nominal price level and asset prices. We follow Ilut and Schneider (2015) who also capture the presence of uncertainty with low subjective mean beliefs about exogenous variables: beliefs about endogenous variables follow from agents' knowledge of the structure of the economy. In particular, agents know the policy rule of the government and that banks maximize shareholder value given the households' discount factor. Households' worst case beliefs thus also affect bank decisions; shareholder value (5) is replaced by its worst case expectation.

Appendix A.5 characterizes steady state equilibrium with uncertainty. Mechanically, agents live in a steady state with all variables constant, yet act *as if* the economy is on a transition path to a worst case steady state with lower tree payoffs in the future. In general, dynamics are characterized by first finding the transition path to derive the law of motion for endogenous variables, and then combining that law of motion with the true dynamics of the exogenous variables. We show that in the present model, the transition path converges to the worst case steady state after one period, and that the bank ratios in the actual and the worst case steady state coincide. These properties allow simple graphical analysis, as in the case without uncertainty.

#### *Uncertainty premia on assets*

Asset prices reflect uncertainty in two ways. First, agents act *as if* payoffs will be lower and hence value trees less. At the same time, however, they discount future payoffs at a lower rate. This is because they understand the quantity equations and hence fear higher inflation. Indeed, while the government commits to a growth rate  $g$  of nominal liabilities, agents act as if output will

fall. Real returns on nominal assets thus reflect *worst case* inflation  $\tilde{\pi}$  which is higher than actual inflation. In particular, since the nominal rate on reserves is fixed, the cost of liquidity next period is perceived to be high.

It is helpful to introduce notation to compare the worst case loss of tree payoffs to the worst case drop in output. A drop in tree payoff of  $s$  percent implies that agents expect output to be permanently lower by  $\tau s$  percent, where  $\tau = x/(\Omega + x)$  is the share of tree payoffs in output. We assume that  $\tau s$  is a small decimal number, whereas  $s$  itself need not be. We thus allow for large losses on trees, but keep the expected loss of output on the order of a typical growth rate. This approach is designed to focus on large disruptions to the financial sector that are not accompanied by extreme drops in real activity.

We summarize the equilibrium effect of agents' concern with high liquidity cost by the effective discount rate

$$\hat{\delta} = \delta - (\tilde{\gamma} - \gamma) = \delta + g - \tilde{\pi} = \delta + \tau s.$$

Here  $\tilde{\gamma}$  is the worst case log marginal utility of wealth next period. Worst case liquidity costs  $\tilde{\gamma}$  are higher than current liquidity costs  $\gamma$  if worst case inflation  $\tilde{\pi}$  is higher than the growth rate of nominal liabilities  $g$ . The latter is driven by the fear of lower tree payoffs, and the effect is smaller if tree payoffs are a smaller share of output.

The presence of uncertainty generates premia on assets to compensate investors. If a tree was held by households, its steady state price of a tree held by households would be

$$\frac{Q}{P} = u(s) \frac{x}{\delta}; \quad u(s) = \frac{1-s}{1-\tau s} \quad (27)$$

The factor  $u$  reflects compensation for uncertainty. If  $s = 0$ , then  $u(s) = 1$ , and the price is the present value  $x/\delta$ . The same result obtains if  $\tau = 1$  if all output comes from trees, then the cash flow and discount rate effects on asset prices exactly offset. In the interesting case where tree payoffs represent some intermediate share of output,  $u$  is strictly between zero and uncertainty lowers prices.

To see how uncertainty generates premia on assets, consider an econometrician who observes tree prices as well as payoffs. The return on trees measured in steady state is

$$\frac{Q/P + x}{Q/P} = 1 + \delta/u(s).$$

As payoff uncertainty  $s$  increases,  $u$  declines and the return on the tree increases to compensate investors. If there was also a second "safe" tree held by households that earns exactly the discount rate, the econometrician would measure an equity premium on the uncertain tree. In terms of comparative statics, an increase in uncertainty captured by an increase in  $s$  leads to higher premia and lower prices.

#### *Uncertainty and collateral quality*

It is natural to assume that uncertain trees are also worse collateral. To capture this effect, we make the weight that trees receive in the aggregation of collateral explicitly a decreasing function  $\rho(s)$  of payoff uncertainty  $s$ . A change in uncertainty thus has two effects on banks' tree portfolios. There is a direct effect on prices that lowers the total value of collateral available to banks. In addition, uncertainty makes trees worse collateral per dollar of funds invested in them.

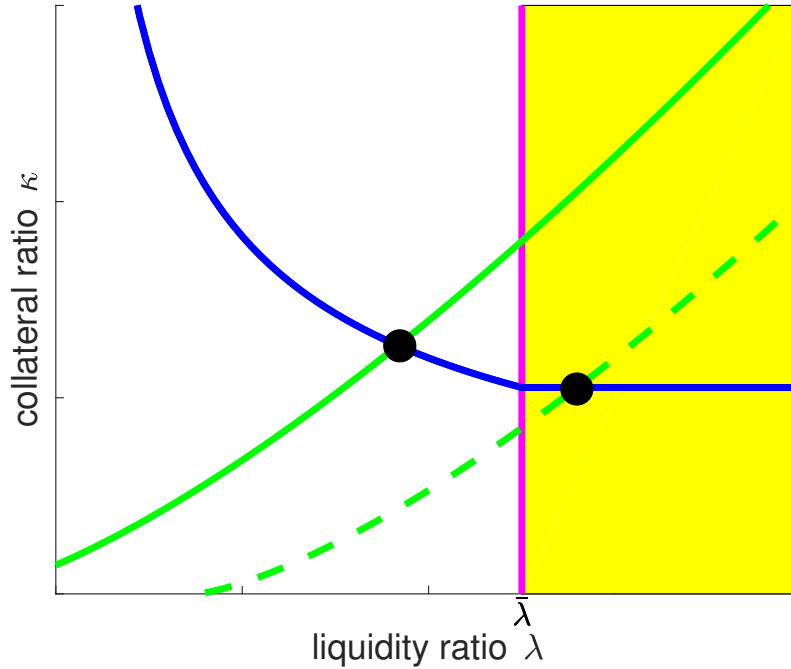


Figure 9: An increase in uncertainty shifts the solid green capital structure curve to the right, resulting in the dotted line.

Since uncertain trees still provide some collateral benefits, banks continue to hold all trees in an equilibrium with uncertainty. From the first order condition for trees – derived in the appendix – the value of a tree held by banks is

$$v(\kappa; s) = \frac{u(s)x}{(\delta - \rho(s) \text{mb}(\kappa))T} \quad (28)$$

Comparing to the frictionless price in (27), an increase in uncertainty lowers the price more since it must compensate banks not only for a lower expected payoff, but also for lower collateral quality.

#### *Characterizing equilibrium with uncertainty*

We can study equilibria with uncertainty using the same graphical analysis as in the previous section. The appendix shows that we only need to replace the value of trees  $v(\kappa)$  in the capital-structure curve (20) by the new valuation formula  $v(\kappa; s)$  from (28). The liquidity-management curve and the determination of the price level are not affected by changes in  $s$ . The richer model allows us to discuss the effects of an increase in uncertainty on asset prices and inflation; this is taken up in the next section.

## 6.2 An increase in uncertainty and policy responses

How does the payment system respond to an increase in uncertainty? An increase in  $s$  lowers the value of uncertain trees (28) and hence the amount of collateral banks can use to back deposits. Since less other collateral is available, it takes a larger liquidity ratio to achieve any given collateral ratio  $\kappa$  – the capital structure moves to the right as in Figure 9. As long as reserves are scarce, banks hold more liquidity and thus choose lower collateral ratios.

The spillover from asset markets to the payment system via bank balance sheets thus leads to

deflation as the money multiplier declines. At the same time, the scarcity of collateral pushes the real interest rate down. If the uncertainty shock is sufficiently strong, the economy can move all the way into the abundant reserve region where the overnight interbank market shuts down. We can thus arrive at abundance of reserves even if there if policy (described the standard central bank tools  $i_R - \pi$  and  $b$ ) does not change.

An increase in uncertainty is an attractive candidate for a shock that could have occurred at the beginning of the recent financial crisis. It is consistent with an increase in asset premia, a drop in uncertain asset prices, a decline in the overnight interest rate all the way to the reserve rate as well as a decline in bank collateral and an effective shutdown of interbank Federal Funds lending. However, we did not see a large deflation – after an initial small drop in late 2008 the price level remained quite stable over time.

#### *An expansion of government liabilities*

According to our model, a candidate for the absence of strong deflation during the financial crisis is an increase in government liabilities. Suppose that the Treasury issues a lot of new debt, some of which is then purchased by the central bank so as to keep the ratio  $b$  of debt held by the public to reserves constant. Suppose further that this is perceived as a one time change, with a stable path of nominal liabilities thereafter. In terms of the model, this policy corresponds to a one time increase in the outstanding nominal quantity of government liabilities.

Bank portfolios do not react to a one time increase in government liabilities – our two curves do not shift. An increase in government liabilities is neutral: the only response is that the price level increases proportionately to maintain the same real value of government liabilities. The real interest rate and leverage do not change, and the economy remains in the abundant reserves regime. A joint increase in uncertainty and government debt can thus move the economy into a period of abundant reserves with low asset prices, collateral and real rates, but stable inflation.

#### *Unconventional monetary policy*

An alternative response by central banks to a decline in asset values has been to purchase low quality collateral, such as risky mortgage backed securities. We now consider what happens when the government purchases risky trees. We set up an experiment analogously to the open-market purchase above. We start from an initial equilibrium with abundant reserves  $M_0$ , price level  $P_0$  and collateral ratio  $\kappa$ .

We assume that the government injects reserves to purchase all trees from banks' balance sheet, that is, new reserves are chosen such that, at the new equilibrium with reserves  $\tilde{M}_0$ , price level  $\tilde{P}_0$  and collateral ratio  $\tilde{\kappa}$ , we have

$$\tilde{M}_0 - M_0 = \frac{\tilde{P}_0 u(s) x}{\delta - \rho(s) \text{mb}(\tilde{\kappa})}.$$

The effect of the purchase are displayed in Figure 10. As trees are removed from bank balance sheets, the capital-structure curve moves to the right.

After the additional injection, reserves continue to be abundant, so the policy has no effect on the collateral ratio. However, the policy does increase the price level and thus counteracts the deflationary effect of higher uncertainty. Indeed, collateral in the new equilibrium is  $\tilde{M}_0/\tilde{P}_0$  which equals the real value of old reserves  $M_0/\tilde{P}_0$  plus the full real value of trees. In contrast, collateral in the initial equilibrium was given by the real value of reserves  $M_0/P_0$  as well as the value of trees

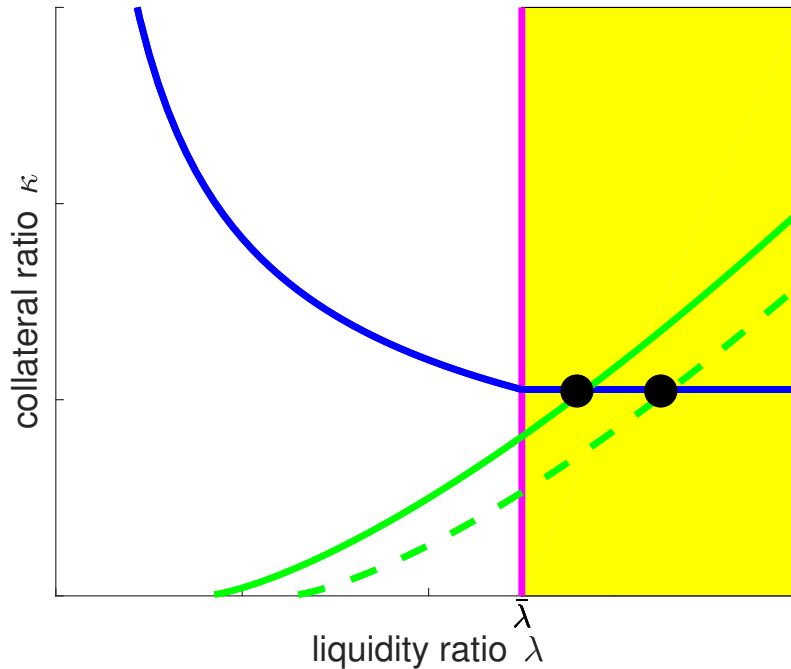


Figure 10: Central bank purchase of trees when reserves are abundant.

multiplied by the collateral quality weight  $\rho(s) < 1$ . Since the collateral ratio is the same in the two equilibria, it must be that  $\tilde{P}_0 > P_0$ .

Unconventional policy thus works by replacing low quality real collateral on bank-balance sheets with high quality nominal collateral. Since reserves continue to be abundant and the real return on reserves stays the same, this does not actually lead to an increase in real collateral. However, backed by the new reserves, banks provide more inside money, which is inflationary in the short run. Compared with an outright increase in reserves, the inflationary effect of tree purchases is smaller since at the same trees are removed from the collateral pool.

Tree purchases by the central bank have two additional, more subtle, effects. First, it makes the capital-structure curve steeper. The slope in turn matters for the inflation response to changes in the interest rate on reserves: indeed, the steeper the capital-structure curve, the less does an increase in the return on reserves push the price level down. Second, removing trees from bank balance sheets reduces banks' exposure to further shocks to asset quality. In particular, suppose that after all trees have been bought by the government, the uncertainty shock is reversed and asset prices increase. The payment system would not react to this shock as trees no longer serve as collateral to produce inside money. The economy would remain in an abundant reserves environment even though the asset-market turbulence that has sent it there in the first place has actually subsided.

### 6.3 Carry traders

So far, the effect of asset values on bank balance sheets is direct: it requires bank investment in trees. In this section, we introduce institutional investors who borrow short-term from banks in order to invest in trees. We call these investors “carry traders” – they do not actively trade trees but roll over their debt. This creates an additional link between asset markets and banks that operates even if banks only engage in short-term lending. Monetary policy can affect carry traders'

funding cost.

Carry traders are competitive firms that issue equity, borrow overnight. They invest in a special set of trees that banks do not have access to. We denote the fruit from carry trade trees by  $x^*$ . Like banks, carry traders face leverage costs, captured by a decreasing convex function  $c^*$  that could be different from the cost function  $c_b$  assumed for banks.<sup>9</sup> The collateral ratio of a carry trader is defined the market value of his tree holdings divided by overnight credit  $F_{t+1}^*$ :

$$\kappa_t^* = \frac{Q_t^* \theta_t^*}{F_{t+1}^*}.$$

Carry traders' marginal collateral benefit is denoted  $mb^*(\kappa)$  and their marginal cost of leverage is  $mc^*(\kappa)$ , respectively.

We assume further that carry traders are more optimistic about the payoff of trees than households: they perceive uncertainty  $s^* < s$  whereas households perceive  $s$ . The idea here is that the firm employs specialized employees who households trust to make asset-management decisions. As a result, the spread relevant for investment in carry trader trees — indirectly through investment by carry traders — carries the lower uncertainty premium  $s^*$ .

#### *Optimal investment and borrowing*

Carry traders' first-order condition for overnight borrowing resembles that of banks in (A.8), except that it does not provide liquidity benefits: the return on equity must be smaller than the real overnight rate plus the marginal cost of leverage. We focus on steady states only and drop time subscripts. Since we already know that the real rate is lower than  $\delta$  in equilibrium, it is always optimal for carry traders to borrow and we directly write the condition as an equality:

$$\delta = i - \pi + mc^*(\kappa^*). \quad (29)$$

Carry traders' collateral ratio is higher in equilibrium when interest rates are high.

Like banks, carry traders hold all trees accessible to them. This is due not only to the collateral benefit conveyed by trees, but also to carry traders' relative optimism. From carry traders' first order condition, the value of trees held by carry traders

$$v^*(\kappa^*, s^*) = \frac{u^* x^*}{\delta - mb^*(\kappa^*)}; \quad u^* = \frac{1 - s^*}{1 - s_y}$$

where  $s_y$  is the worst case loss of output; it depends on the weighted average of losses on the different trees in the economy. When interest rates or uncertainty is low, carry traders apply a lower effective discount rate to trees, which results in higher tree prices.

The amount of carry trader borrowing in steady state equilibrium is

$$F^* = v^*(\kappa^*, s^*) / \kappa^* \quad (30)$$

Lower interest rates increase both leverage and the value of collateral, and therefore increase borrowing. Moreover, an increase in uncertainty (that is, an increase in  $s$ ) lowers collateral values and borrowing.

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<sup>9</sup>We do not consider welfare effects of leverage for carry traders – instead we focus on the positive implications of margin trading. We thus assume for simplicity that leverage costs of carry traders are paid lump sum to households so that they have no impact on welfare.

Our graphical analysis of equilibrium remains qualitatively similar when carry traders are added to the model. The only change is that carry trader borrowing now enters on the asset side of the banking sector. We can thus add in the numerator of the collateral ratio (20) a term  $B^*(\kappa)$  that expresses carry trader borrowing as a function of bank collateral. We obtain the function  $B^*$  by substituting for  $\kappa^*$  in (30) from (29) and then substituting for the interest rate from the bank first-order condition from (A.13). The function  $B^*$  is decreasing: if banks have more collateral, the interest is higher and carry traders borrow less.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with carry traders. Suppose first that, starting from an equilibrium with scarce reserves, there is an increase in uncertainty. The new effect is that, as carry traders value trees less, they demand fewer loans from banks. This lowers bank collateral and shifts the capital-structure curve to the right. The liquidity-management curve does not change. In the new equilibrium, bank collateral is even lower and the interest rate is lower, as is the price level. The deflationary effect therefore amplifies the increase in uncertainty about bank trees considered earlier. The additional prediction is that we should see a decline in funding of institutional investors with short-term credit from payment intermediaries, such as a decline in repo extended by money-market mutual funds to broker-dealers.

It is also interesting to reconsider the effect of monetary policy. Suppose policy engineers a change in the mix of bank assets or their value that lowers the real overnight interest rate. Carry traders borrow more and bid up the prices of the trees they invest in. As one segment of the tree market thus increases in value, the aggregate value of trees also rises: there is a tree market boom. Importantly, this is not a real interest rate effect: the discount rate of households, which is used to value trees held by households, is unchanged. The effect comes solely from the effect of monetary policy on the overnight rate and hence on carry traders' funding costs.

## 6.4 Active traders

Carry traders provide collateral to banks and thus affect the supply of inside money. We now introduce another group of institutional investors with a demand for inside money. "Active traders" periodically rebalance their portfolios as their view of tree payoffs changes, and they require cash to pay for new trees. The money demand of active traders counteracts the supply side effects due to changes in the collateral values or the borrowing by carry traders. At the same time, monetary policy also affects their funding costs.

Active traders are competitive firms that issue equity and have money as well as a special set of trees denoted  $\hat{\Theta}$ . There are many active traders and each is optimistic about one particular "favorite" tree: the trader perceives uncertainty  $\hat{s} < s$  about this tree. In contrast, households and other active traders perceive uncertainty  $s$ . Active traders also perceive uncertainty  $s$  about all other trees. Every period, the identity of the favorite tree within the subset  $\hat{\Theta}$  changes to some other tree in the subset.

To generate a need for inside money, we assume that active traders must pay for new tree purchases with prearranged payment instruments or intraday credit. Active trader  $i$  faces the liquidity constraint

$$\int_{\hat{\Theta}} Q_t^j \hat{\theta}_{i,j,t} dj = I_{i,t} + (\hat{D}_{i,t} + \hat{L}_{i,t}),$$

where  $I_i$  is the intraday credit position,  $\hat{D}_i$  are deposits that the fund keeps at its bank together with credit lines  $\hat{L}_{i,t}$ .

Like a bank, active trader  $i$  faces a limit on intraday credit

$$I_{i,t} \leq \hat{\gamma}(\hat{D}_{i,t} + \hat{L}_{i,t}),$$

where  $\hat{\gamma}$  is a parameter that governs netting in tree transactions. It is generally different from the parameter  $\gamma$  that governs netting among banks, since it captures netting by a clearing and settlement system for the securities that active traders invest in.

Active traders choose money, trees and their shareholder payout. We focus on equilibria in which every active trader always holds only his favorite tree – we can assume that the perceived uncertainty on other trees is high enough. Since money is costly – the real rate on deposits is below the discount rate – active traders hold as little money as necessary in order to purchase the entire outstanding amount of their new favorite tree in case the identity of their favorite tree changes. It follows that the intraday credit limit binds in equilibrium, a form of “cash-in-the-market pricing”.

### *Optimal investment and deposits*

Much like households, active traders equate their marginal liquidity benefit to the marginal cost of money holdings, given by the the opportunity cost of deposits  $\hat{\delta}_t - (i_t^D - \pi_{t+1})$  or equivalently the interest rate on credit lines  $i_t^L$ . The liquidity benefit in turn is due to traders’ ability to invest in their favorite tree, which carries a return that compensates them for cost of liquidity. As in the previous section, tree prices also reflect uncertainty about output and hence inflation. It is again convenient to denote the worst loss of output by  $s_y$  – it depends on the weighted average of losses on the different trees in the economy.

The steady state price of trees held by active traders can then be written as

$$\hat{v} = \hat{u} \frac{\hat{x}}{\delta + \frac{i^L}{1+\hat{\gamma}}}; \quad \hat{u} := \frac{1 - \hat{s}}{1 - s_y}.$$

Here the first factor is again compensation for uncertainty – it takes the same form as in (27) and is less than one if the expected loss in payoff from active trader trees is larger than the expected drop in output. The second term shows that prices reflect traders’ need for inside money: prices are higher if the bank customers’ cost of liquidity  $i^L$  is lower and when netting is more efficient (higher  $\hat{\gamma}$ ).

Equilibrium money holdings by active traders are proportional to the market value of active traders’ favorite trees:

$$\hat{D} + \hat{L} = \frac{\hat{v}}{1 + \hat{\gamma}}.$$

The money demand by active traders is interest elastic, in contrast to the inelastic demand from households. This is a stark way to capture the idea that financial institutions responds more strongly to changes in liquidity costs.

Since the household and active trader sector differ in their money demand, the share of inside money used in the goods versus the asset market changes over time. We define active traders’ money share as

$$\alpha = \frac{\hat{D} + \hat{L}}{D + L} = \frac{\hat{v}/(1 + \hat{\gamma})}{\Omega + x + \hat{v}/(1 + \hat{\gamma})}.$$



If bank customers' liquidity becomes cheaper, the value of active traders' trees increases and their share of the total supply of inside money goes up. In equilibrium, the share is an increasing function  $\hat{\alpha}(\lambda, \kappa)$  of the two bank ratios  $\lambda$  and  $\kappa$ , since both lower the cost of bank customers' liquidity.

### *Equilibrium with active traders*

We focus on local changes to equilibria with abundant reserves. Our graphical analysis of equilibrium bank ratios remains qualitatively similar when active traders are added to the model. The price level is determined by

$$P_0 = \frac{M}{\lambda Y} (1 - \hat{\alpha}(\lambda, \kappa)).$$

The key difference here is the presence of active traders' share, which works like velocity. If the cost of liquidity is lower, then active traders absorb more inside money. As less money is used in the goods market, the price level declines.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with active traders. Suppose first that there is an increase in uncertainty. As active traders value trees less, they demand less money. This increases the collateral ratio of banks and shifts the capital-structure curve to the left. The liquidity-management curve does not change. In the new equilibrium, bank collateral and the interest rate are higher, as is the price level. In other words, active traders are a force that generate the opposite response to a change in uncertainty from banks and carry traders. Since in the typical economy all traders are present to some extent, we can conclude that their relative strength is important. An additional prediction is that we should see a decline in money – either deposits or credit lines – provided to institutional investors.

We can also reconsider the effect of monetary policy. Suppose once more that policy lowers the real overnight interest rate. The opportunity cost of holding deposits falls and active traders demand more money. At the same time, they bid up the prices of the trees they invest in. Again a segment of the tree market increases in value and the aggregate value of trees also rises: there is a tree market boom. Again the effect is not due a change in the discount rate, but instead a change in the funding cost: here it affects active traders' strategy which requires money in order to wait for trading opportunities.

## 7 Related literature

In this section we discuss how our results relate to existing work in monetary economics.

### *Balance sheet effects and government liabilities*

In our model, welfare costs derive from "balance sheet effects" and policy matters by changing the asset mix in the economy. This theme is familiar from other work on unconventional monetary policy. Several papers study setups where banks are important to channel funds to certain borrowers. By purchasing the bonds of these borrowers, policy can effectively substitute public credit when weak balance sheets constrain private credit (e.g. Cúrdia and Woodford 2010, Christiano and Ikeda 2011, Gertler and Karadi 2011, Gertler, Kiyotaki and Queralto 2012).<sup>10</sup> Our model differs from this literature in how banks add value – their special ability is not lending, but the handling of payment instructions.

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<sup>10</sup>Buera and Nicolini (2014) also consider the effect on monetary policy on balance sheets in a model of entrepreneurs who face collateral and cash-in-advance constraints.

Since the price level depends on the supply of payment instruments, shocks to bank assets have deflationary effects in our model. If all payment instruments are taken to be deposits, we obtain a collapse of the money multiplier along the lines of Friedman and Schwartz (1963). Brunnermeier and Sannikov (2016) also consider the link between asset values and the supply of inside money by banks. In their model, banks' special ability is to build diversified portfolios and deposits are a perfect substitute to outside money as a store of value. In contrast, in our model inside money is a medium of exchange for bank customers, and outside money works like an intermediate good for producing inside money, rather than a substitute.

#### *Asset pricing and money*

In our model, collateral benefits generate market segmentation. We thus arrive endogenously at "intermediary asset pricing" equations that are reminiscent of those in He and Krishnamurthy (2013) or Bocola (2016). Unlike our banks, banks in those models are investors with limited net worth who are assumed to hold some assets because they have special investment abilities.

The interaction of liquidity and collateral benefits in our model also generates permanent liquidity effects. In contrast, the literature on monetary policy with partially segmented asset markets (for example, Lucas 1990, Alvarez, Atkeson and Kehoe 2002) obtains temporary liquidity effects; collateral benefits play no role there.

Asset values in our model depend on the cost of inside money to institutional investors. A permanent effect of monetary policy on asset values also obtains in Lagos and Zhang (2014) where the inflation tax discourages trade between heterogeneous investors; this alters which investor prices assets in equilibrium. The effect we derive is different because the cost of liquidity to bank customers is not captured by the inflation tax; instead the cost of inside money depends on banks' cost of leverage. In particular, our mechanism is also operative when reserves are abundant.

#### *Bank liquidity management and monetary policy*

With scarce reserves, bank liquidity management matters for asset valuation, policy impact and welfare in our model. The liquidity management problem arises because banks cannot perfectly insure against liquidity shocks due to customer payment instructions, as in Bhattacharya and Gale (1987).<sup>11</sup> Recent work has discussed the interaction of monetary policy and liquidity management with scarce versus abundant reserves (for example, Whitesell 2006, Keister, Monnet and McAndrews 2008)<sup>12</sup>. While these papers consider more detail that is useful to understand the cross section of banks, our stylized model tries to capture the main tradeoff and its interaction with other features of the economy.

Several papers have incorporated bank liquidity management into DSGE models. Cúrdia and Woodford (2011) study optimal monetary policy in a New Keynesian model.<sup>13</sup> In their setup,

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<sup>11</sup>A related literature asks whether government supplied liquidity is useful when firms cannot perfectly insure shocks to investment opportunities (for example, Woodford 1990, Holmstrom and Tirole 1998). An alternative approach to bank liquidity, following Diamond and Dybvig (1983), considers optimal contracts offered by banks to endusers. This approach typically abstracts from interbank transactions; the focus is instead on optimal dependence of contracts on enduser liquidity needs, given information problems as well as the scope for multiple equilibria that include bank runs.

<sup>12</sup>Similar tradeoffs have been developed in the literature on the dynamics of the Federal Funds market (e.g. Ho and Saunders 1985, Hamilton 1996, Afonso and Lagos 2015.)

<sup>13</sup>Other notions of bank liquidity have also been explored in DSGE settings. Gertler and Kiyotaki (2010) consider a model in which bank borrowing not only depends on bank net worth but also is fragile and subject to runs. Del Negro, Eggertsson, Ferrero and Kiyotaki (2013) study a model in which assets become illiquid in the sense of being

reserve policy can be stated in terms of a rule for the overnight interest rate and the reserves rate; there is no need to formulate policy in terms of the quantity of reserves. Our setup is different because of market segmentation: the nominal interest rate is not directly connected to a household marginal rate of substitution, but rather to bank leverage. Rules for interest rates are then not enough to characterize the behavior of inflation – the supply of nominal government liabilities is also relevant.

Bianchi and Bigio (2014) study a quantitative model in which banks have a special ability to lend and face a perfectly elastic demand for debt as well as idiosyncratic liquidity shocks. Monetary policy changes the tradeoff between reserves and interbank credit and hence the willingness of banks to make loans. In contrast, the demand for inside money in our model comes from its role as a medium of exchange for goods and securities; monetary policy affects the cost of money holdings to bank customers, not only to banks.

In Drechsler, Savov and Schnabl (2016), banks are investors with relatively low risk aversion who issue debt subject to aggregate liquidity shocks. Monetary policy changes the cost of self-insurance via reserves and thereby affects banks' willingness to take leveraged positions in risky assets as well as the risk premium on those assets. In our model, monetary policy affects not only the funding cost of banks, but also that of banks' institutional clients; the two channels have opposite effects on uncertainty premia.

The role of interest on reserves as a policy tool has recently received renewed attention. A number of papers ask when the price level remains determinate (Sargent and Wallace 1985, Hornstein 2010, Ennis 2014). Woodford (2012) and Ireland (2014) consider macroeconomic effects of interest on reserves in a New Keynesian framework. Kashyap and Stein (2012) consider a model with a financial sector; they emphasize the presence of quantity and price tools for macroeconomic and financial stability, respectively.

#### *Multiple media of exchange and liquidity premia*

While our model allows for both deposits and credit lines in the bank-customer layer, we assume that those instruments are perfect substitutes. An interesting related literature asks which instruments are used in which transactions. In particular, Telyukova and Wright (2008) consider a model in which both credit and money are used and explain apparently puzzling cost differences with convenience yields. Lucas and Nicolini (2015) distinguish currency and interest bearing accounts and show that a model that makes this distinction can better explain the relationship between interest rates and payment instruments. Nosal and Rocheteau (2011) survey models of payment systems.

In our model, asset values reflect collateral benefits to banks and hence indirectly benefits to the payments system. Moreover, government policy can matter by changing the scarcity of collateral that effectively backs inside money. Similar themes appear in "new monetarist" models with multiple media of exchange.<sup>14</sup> In models based on Lagos and Wright (2005), assets that are useful in decentralized exchange earn lower returns. Several papers have recently studied collateralized IOUs as media of exchange, following Kiyotaki and Moore (2005).

For example, in Williamson (2012, 2014) some payments are made with claims on bank portfolios that contain money, government bonds or private assets; banks moreover provide insurance to individuals against liquidity shocks. Rocheteau, Wright and Xiao (2015) consider payment via

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harder to sell, as in Kiyotaki and Moore (2008).

<sup>14</sup>See Lagos, Rocheteau and Wright (2014) for a recent survey.

money or government bonds (or, equivalently in their setup, IOUs secured by bonds). These models give rise to regimes of scarcity or abundance for each medium of exchange. The real effects of scarcity can be different for, say, bonds and money because money is used to purchase a different set of goods.

While we also study how policy affects the scarcity of different assets like bonds and reserves, the mechanisms we emphasize as well as our welfare conclusions differ in important ways.<sup>15</sup> Indeed, in our model only one medium of exchange helps in bank-customer transactions: inside money supplied by banks. Since any bank commitment is costly, inside money is never abundant and collateral to back it is always scarce – only the degree of scarcity changes and affects welfare. In contrast, reserves can be scarce or abundant depending on their role in bank liquidity management.

In our two layer setup, whether scarcity affects asset prices or welfare thus depends crucially on features of the banking system. For example, scarcity of bonds has different effects from scarcity of reserves because reserves change banks' liquidity management problem and the leverage costs of tapping the overnight market. In addition, the price level in our model is related to the supply of inside money by banks and hence the nominal collateral that banks hold. For example, the quantity of nominal collateral available to banks shapes the price level response to policy.

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<sup>15</sup>In addition, the new monetarist literature considers explicit models of decentralized exchange, rather than reduced form liquidity constraints as we do. At the same time, Andolfatto and Williamson (2015) consider a cash-in-advance model with bonds and money and show that several key effects from the more complex papers can be seen already there.

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# A Appendix

In this appendix we provide all the equations to accompany the analysis in the text. Section A.1 derives banks' first-order conditions. Section A.2 derives a system of equations characterizing equilibrium, Section A.3 considers its steady state and Section A.4 provides conditions such that transition to a steady state happens in one period. Section A.5 introduces uncertainty and Section A.6 adds active traders.

## A.1 Bank optimization

This section studies the optimal choice of banks. We work through the extended model that allows for credit lines and interbank netting. The liquidity constraint is

$$\tilde{\lambda}_t(D_t + L_t) \leq (1 + \gamma)(M_t + F_{t+1}), \quad (\text{A.1})$$

where  $F_{t+1} \geq 0$  is overnight borrowing and  $\gamma \geq 0$  is the parameter that captures the efficiency of netting arrangements among banks.

Banks maximize (5) subject to (A.1), (11).and the budget constraint

$$\begin{aligned} P_t y_t^b &= i_{t-1}^L L_t + M_t (1 + i_{t-1}^R) - M_{t+1} - D_t (1 + i_{t-1}^D) + D_{t+1} \\ &+ (B_t - F_t) (1 + i_{t-1}) - (B_{t+1} - F_{t+1}) + ((Q_t + P_t x_t) \theta_{t-1} - Q_t \theta_t) \\ &- e^{\pi_t} c(\kappa_{t-1}) (\sigma(D_t + L_t) + F_t) + D_t^b (1 + i_{t-1}^D) - D_{t+1}^b. \end{aligned} \quad (\text{A.2})$$

The only difference to (9) is that we have added a term reflecting the fee income from credit lines to the first line.

The definition of the collateral ratio is

$$\kappa_t := \frac{M_{t+1} + \rho Q_t \theta_t + B_{t+1}}{\sigma(D_{t+1} + L_{t+1}) + F_{t+1}}$$

where credit lines are appear in the denominator together with deposits. We allow for a weight  $\sigma$  on payment instruments that could capture the idea that payment instruments may require a different amount of collateral than other debt.

*Paying for leverage costs, marginal leverage costs and marginal collateral benefit*

Consider first banks' choice of deposits at other banks held in order to pay for leverage costs. Deposits held at other banks do not contribute to collateral and hence do not show up in the leverage ratio (10). As long as the interest rate on deposits is below  $\hat{\delta}_t$ , the constraint then binds in equilibrium: banks hold just enough deposits to cover the leverage costs that will accrue next period.

Using the budget constraint and the binding liquidity constraint, the last two terms in the bank budget equation (9) describe the cost of leverage chosen in the previous period and can be written as

$$- e^{\pi_t} (1 + (\hat{\delta}_{t-1} - (i_{t-1}^D - \pi_t))) c(\kappa_{t-1}) (\sigma(D_t + L_t) + F_t). \quad (\text{A.3})$$

Banks' effective cost of leverage also includes the opportunity cost of holding cost of holding deposits at other banks.

*Bank first-order conditions for assets*



Consider the first-order condition for overnight lending. The return on equity must be higher than the marginal benefit:

$$\hat{\delta}_t \geq i_t - \pi_{t+1} - e^{-\hat{\delta}_t} c'_b(\kappa_t) \left\{ 1 + \hat{\delta}_t - (i_t^D - \pi_{t+1}) \right\} \quad (\text{A.4})$$

with equality if the bank lends overnight. In the latter case, low real interest rates imply that overnight credit is costly, so banks optimally choose lower collateral. Put differently, highly levered banks obtain a high benefit from overnight lending as collateral and thus require a lower return on credit.

By our convention on equilibrium rates of return,  $c'(\kappa_t)$  must be a small decimal number – we impose assumptions below to ensure that this is the case. As a result, we can work with the simpler approximate first order condition

$$\hat{\delta}_t \geq i_t - \pi_{t+1} + \text{mb}(\kappa_t)$$

where  $\text{mb}(\kappa_t) := -c'_b(\kappa_t)$  is the marginal benefit of an extra unit of collateral.

The first-order condition for trees is similar. The real rate of return on a tree is  $r_{t+1} = \log(Q_{t+1} + P_{t+1}x_{t+1})/Q_t - \pi_{t+1}$ . Since trees also deliver collateral benefits, we must have

$$\hat{\delta}_t \geq r_{t+1} + \rho \text{mb}(\kappa_t),$$

with equality for trees held by the bank. We have thus derived equation (12) in the text. Since the collateral benefit lowers the return on trees held by the bank, it raises the price of the trees relative to payoff. Indeed, holding fixed the payoff  $Q_{t+1} + P_{t+1}x_{t+1}$ , the price of a tree held by the bank is  $Q_t = (Q_{t+1} + P_{t+1}x_{t+1}) / (\hat{\delta}_t - \rho \text{mb}(\kappa_t))$ .

Reserves differ from overnight lending and trees in that they not only provide returns  $i_t^R - \pi_{t+1}$  and collateral benefits, but also liquidity benefits – they can be used for payments. The liquidity benefit depends on the Lagrange multiplier on the liquidity constraint (6). Writing  $\mu_t$  for that Lagrange multiplier divided by the price level, the first-order condition for reserves is

$$\hat{\delta}_t \geq i_t^R - \pi_{t+1} + \text{mb}(\kappa_t) + (1 + \gamma) e^{-\hat{\delta}_t - \pi_{t+1}} E[\mu_{t+1}]. \quad (\text{A.5})$$

The liquidity benefit (the second term) is higher the higher the expected discounted Lagrange multiplier  $E[\mu_{t+1}]$  and the more payments can be made per dollar of reserves (higher  $\gamma$ ).

*Bank first-order conditions for liabilities & credit lines*

To derive the first order condition for deposits, it is helpful to compute the marginal cost of leverage as the derivative of the discounted effective leverage cost (A.3) with respect to deposits (the numerator in (10))

$$\begin{aligned} & e^{-\hat{\delta}_t} (c_b(\kappa_t) - c'_b(\kappa_t) \kappa_t) \left\{ 1 + \hat{\delta}_{t-1} - (i_t^D - \pi_{t+1}) \right\}, \\ & \approx c_b(\kappa_t) - c'_b(\kappa_t) \kappa_t := \text{mc}(\kappa_t), \end{aligned} \quad (\text{A.6})$$

where the second line uses the fact that all rates of returns are small decimals.

The first-order condition for deposits says that the equity return must be smaller than the sum of the real deposit rate plus the marginal leverage and liquidity costs of deposits

$$\hat{\delta}_t \leq i_t^D - \pi_{t+1} + \text{mc}(\kappa_t) + e^{-\hat{\delta}_t - \pi_{t+1}} (1 + \gamma) E[\mu_{t+1} \tilde{\lambda}_{t+1}], \quad (\text{A.7})$$

with equality if the bank issues deposits. Leverage costs increase with overall leverage through  $\text{mc}(\kappa_t)$  and are also scaled by the parameter  $\sigma$  which makes deposits cheaper than other borrowing. Liquidity costs arise in those states next period when positive liquidity shocks  $\tilde{\lambda}_{t+1} > 0$  coincide with a binding intraday credit limit  $\mu_{t+1} > 0$ .

The first-order condition for overnight borrowing says that the equity return must be smaller than the sum of the real overnight rate plus the marginal leverage cost, *less* the liquidity benefit provided by overnight credit:

$$\hat{\delta}_t \leq i_t - \pi_{t+1} + \text{mc}(\kappa_t) - \mu_t(1 + \gamma). \quad (\text{A.8})$$

For banks that borrow overnight, the condition holds with equality. This can happen because for those banks the intraday credit limit binds and  $\mu_t > 0$ . In contrast, banks with sufficient reserves have  $\mu_t = 0$  and do not borrow.

Finally, consider banks' choice to extend credit lines, a form of implicit liability. The fee earned for extending the line must at least compensate the bank for the leverage cost occurred as well as the expected liquidity costs

$$i_t^L \geq \sigma \text{mc}(\kappa_t) + \bar{v} e^{-\hat{\delta}_t - \pi_{t+1}} E \left[ \mu_{t+1} \tilde{\lambda}_{t+1} \right]. \quad (\text{A.9})$$

As discussed above, we focus on equilibria with  $i_t^D - \pi_{t+1} = \hat{\delta}_t - i_t^L$  so that deposits and credit lines are equivalent from the perspective of households. Comparing (A.7) and (A.9), the two payment instruments are then also equivalent from the perspective of banks.

### *The cross section of banks*

The bank first order conditions imply that all banks choose the same collateral and liquidity ratios in any equilibrium in which banks hold reserves at an interest rate  $i_t^R \leq i_t$ . Indeed, suppose there are two banks such that bank 1 chooses a higher collateral ratio than bank 2. Since  $\text{mb}(\kappa)$  is strictly decreasing, the first order conditions for bonds and trees imply that bank 1 cannot hold either of these assets. Bank 1 must therefore hold reserves. However, if its entire asset portfolio consists of reserves, then its expected liquidity benefit next period is zero – given  $i^R < i$  it should not hold any reserves, a contradiction.

Consider now the choice of liquidity ratios. Since banks choose the same collateral ratios, (A.8) implies that the Lagrange multiplier on the liquidity constraint is the same for all constrained banks and equal to

$$\tilde{\mu}_t = (1 + \gamma)^{-1} \left( \hat{\delta}_t - (i_t - \pi_{t+1} + \text{mc}(\kappa_t)) \right) \quad (\text{A.10})$$

Intuitively, the marginal contribution to leverage costs is the same. Since moreover the distribution of liquidity shocks is iid across banks, the conditional distribution of  $\mu$  one period ahead is also the same: the multiplier is zero with probability  $G(\lambda)$  and equal to  $\tilde{\mu}_{t+1}$  otherwise. Since banks hold reserves, (A.5) holds with equality and implies equal  $\lambda$ s across banks.

## **A.2 Characterizing equilibrium**

In this section, we derive a difference equation that summarizes the equilibrium dynamics. Policy is described by a growth rate for reserves  $g$  and a path for the ratio of bonds to money  $B_t/M_t$ . The endogenous variables are the *(i)* asset prices, that is, interest rates  $i_t$  and  $i_t^D$ , the tree price  $Q_t$ , *(ii)* the nominal price level  $P_t$  and the quantity of payment instruments  $D_t + L_t$ , *(iii)* bank ratios that

are equated across banks in equilibrium, that is, the liquidity ratio  $\lambda_t$  and the collateral ratio  $\kappa_t$  (*iv*) constrained banks' marginal benefit of liquidity  $\tilde{\mu}_t$  and finally the household marginal utility of wealth  $e^{-\gamma t}$  which determines the effective discount rate  $\hat{\delta}_t$ . The initial condition is the liquidity ratio  $\lambda_0$  which implies also the predetermined initial nominal price level.

We note that  $\tilde{\mu}$  as defined in (A.10) represent the liquidity benefit that obtains per real dollar for any bank for which the liquidity constraint binds. In contrast, the symbol  $\mu$  used above in the first order conditions is the Lagrange multiplier of an individual bank (A.5). In the cross section of banks, there are always banks for whom the constraint does not bind and for those banks we have  $\mu = 0$ .

### *Asset pricing*

Consider participation in overnight credit and tree markets. In an equilibrium with positive consumption, banks must supply payment instruments. Since payment instruments entail leverage costs, banks obtain a positive collateral benefit from assets they are eligible to hold. As a result, households do not invest in any asset markets that banks can invest in: at least one bank will bid up the price of any asset accessible to banks until its return is below the discount rate and the asset is unattractive to households.

The collateral ratio is the same for all active banks in equilibrium. The equilibrium rate of return on trees is related to the aggregate collateral ratio by the bank "Euler equation"

$$\hat{\delta}_t = r_{t+1} + \rho \text{mb}(\kappa_t). \quad (\text{A.11})$$

The flip side of lower returns is higher prices. Indeed, let  $x_t^b$  denote the sum of dividends on trees at date  $t$  and let  $v_t^b = Q_t / P_t$  denote the real value of those trees. In equilibrium, the value of trees reflects not only the present value the future payoff on the tree, but also the collateral value:

$$v_t^b = e^{-\hat{\delta}_t} (v_{t+1}^b + x_{t+1}^b + \rho \text{mb}(\kappa_t) v_t^b). \quad (\text{A.12})$$

Using our conventions, pricing effectively works as if payoffs are discounted at the lower rate  $\hat{\delta}_t - \rho \text{mb}(\kappa_t)$ .

### *Liquidity benefits and reserve scarcity*

Bank liquidity management and their activity in the overnight credit market depends crucially on the scarcity of reserves. Indeed, with abundant reserves, the first-order condition for reserves (A.5) implies that the real rate on reserves is equal to  $\hat{\delta}_t - \text{mb}(\kappa_t)$ . From the first-order condition for overnight lending (A.4), this is the same overnight interest rate that obtains if banks lend in the overnight market – if reserves are abundant, they are a perfect substitute to overnight paper and must earn the same interest rate.<sup>16</sup>

The overnight interest rate is connected to collateral holdings through a bank Euler equation analogous to (A.11),

$$\hat{\delta}_t - (i_t - \pi_{t+1}) = \text{mb}(\kappa_t). \quad (\text{A.13})$$

As long as there is some government debt  $B_t$ , this equation holds in any equilibrium, whether reserves are abundant or not. At the same time, the inequality (A.4) implies that households never

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<sup>16</sup>Of course, reserves still flow across banks. In particular an amount of reserves  $\bar{\phi} \bar{v} (D + L) / (1 + \gamma)$  still serves to buffer liquidity shocks. However reserves beyond this amount are equivalent to overnight paper that cannot be used to handle payments instructions.

participate in the overnight market – as with banks trees, banks bid up the price of overnight paper to the point where the asset is unattractive to households. In particular, a regime of abundant reserves has the property that reserves and overnight credit are perfect substitutes from the perspective of banks, but neither asset is ever held by households.

Consider a bank that must borrow at the current date, that is, it receives a liquidity shock  $\tilde{\lambda}_t$  beyond the liquidity ratio  $\lambda_t = M_t(1 + \gamma) / \bar{v}(D_t + L_t)$  from (7). The bank's liquidity benefit from overnight credit follows from (A.4) and (A.8):

$$\tilde{\mu}_t(1 + \gamma) = mc(\kappa_t) - mb(\kappa_t) > 0. \quad (\text{A.14})$$

Tapping the overnight market entails both a leverage cost, and an opportunity cost in terms of foregone collateral value. A banking system with less collateral thus faces a larger penalty of running out of reserves.

The spread between overnight and reserve rate reflects the expected liquidity benefit of holding reserves. Substituting into the first-order condition for reserves, we obtain

$$i_t - i_t^R = (1 - G(\lambda_{t+1})) e^{-\hat{\delta}_t - \pi_{t+1}} \tilde{\mu}_{t+1}(1 + \gamma). \quad (\text{A.15})$$

A liquidity benefit obtains only when the withdrawal shock exceeds  $\lambda_{t+1}$ , that is, with probability  $1 - G(\lambda_{t+1})$ . In this case, a bank holding an extra dollar of reserves saves the excess cost of overnight lending relative to equity.

#### *Liquidity and leverage*

Bank leverage depends on how much of each source of collateral is available, and how much banks must borrow apart from issuing deposits. In particular, interbank credit contributes both to collateral (for lender banks) and raises costs (for borrower banks). Given collateral  $\kappa$  and a liquidity ratio  $\lambda \leq \bar{\lambda}$ , the equation for interbank borrowing (8) delivers the ratio of outstanding interbank credit to transactions

$$\frac{F_t}{D_t + L_t} = \frac{1}{1 + \gamma} \int_{\lambda_t}^{\bar{\lambda}} (\bar{\lambda} - \lambda_t) dG(\tilde{\lambda}) =: \frac{1}{1 + \gamma} f(\lambda_t). \quad (\text{A.16})$$

The function  $f$  is decreasing in  $\lambda_t$ : if interest rates are such that banks hold a lot of reserves, then  $\lambda_t$  is high and banks rarely run out of reserves, so outstanding interbank credit is low. In this sense, reserves and overnight are substitutes in liquidity management.

Collecting promises due to payment instruments, real reserves, trees and interbank credit, as well as collateral in the form of reserves, government debt, trees, and interbank credit, equilibrium collateral satisfies

$$\kappa_t = \frac{M_{t+1} + B_{t+1}^g + P_t \rho v_t + (D_t + L_t) \frac{1}{1 + \gamma} f(\lambda_t)}{\sigma(D_{t+1} + L_{t+1}) + (D_t + L_t) \frac{1}{1 + \gamma} f(\lambda_t)}. \quad (\text{A.17})$$

Holding fixed the value of collateral from outside the banking system – reserves, government debt and trees – scarcity of reserves requires more leverage to support a given quantity of transactions. Indeed, we have assumed a cost function  $c_b$  such that the economy operates in the range  $\kappa^{-1} = \ell \leq 1$ ; the presence of an interbank market adds an equal amount of debt and collateral and hence increases leverage.

#### *The cost of payment instruments*

From banks' first-order condition, the equilibrium rate on credit lines satisfies

$$i_t^L = \sigma \text{mc}(\kappa_t) + e^{-\hat{\delta}_t - \pi_{t+1}} \tilde{\mu}_{t+1} (1 + \gamma) \int_{\lambda_{t+1}}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}). \quad (\text{A.18})$$

The two terms represent a leverage and a liquidity component. When reserves are abundant, we have  $\lambda_t > \bar{\lambda}$  so the liquidity component is zero and the cost of a credit line simply reflects banks' cost of leverage.

When reserves are scarce, banks incur additional costs when they run out of reserves. Those costs are larger if velocity is higher and there is less netting among banks. They further depend on the marginal benefit of liquidity as well as on the liquidity shock the bank receives in the next period. The costs banks incur when providing payment instruments also lower the deposit rate  $i_t^D - \pi_{t+1}$  by the same amount.<sup>17</sup>

Consider now the valuation of payment instruments by the household. If the liquidity constraint binds, the household first order condition for deposits is

$$e^{-\gamma t} = \beta \left( e^{i_t^D - \pi_{t+1}} e^{-\gamma_{t+1}} + e^{-\pi_{t+1}} (1 - e^{-\gamma_{t+1}}) \right)$$

Investing a unit of wealth in deposits delivers not only a real pecuniary return – the first term on the right hand side – but also a liquidity benefit – the second term. The liquidity benefit is positive since the liquidity constraint is binding so that  $e^{-\gamma_{t+1}} < 1$ .

Using the definition of  $\hat{\delta}_t = \delta - (\gamma_{t+1} - \gamma_t)$ , we can write bank customers' liquidity benefit as approximately

$$\gamma_{t+1} = \hat{\delta}_t - (i_t^D - \pi_{t+1}) = i_{t+1}^L \quad (\text{A.19})$$

where we have again used that rates of return are small. The liquidity benefit is thus equated to the household's liquidity cost, represented by the opportunity cost of deposits or the fee on a credit line.

### *Equilibrium*

An equilibrium is characterized by the seven equations (14) and (A.12)-(A.15) and (A.17)-(A.19) together with equation (7) that defines the liquidity ratio as well as the two equations (A.6) that define marginal leverage cost and collateral benefit and the definition of  $\hat{\delta}_t$ . The twelve equations determine twelve endogenous variables  $i, i_L, v_B, P, D + L, \lambda, \kappa, \text{mc}(\kappa), \text{mb}(\kappa), \tilde{\mu}, \hat{\delta}$  and  $\gamma$ . The difference equation characterizing equilibrium has only three state variables: aggregate nominal reserves  $M_t$ , aggregate nominal payment instruments  $D_t + L_t$  and the liquidity ratio  $\lambda$ .

In order to compute equilibrium consumption and welfare at date 0, we need to know additional predetermined variables: initial collateral ratios  $\kappa_{-1}$  and  $\kappa_{-1}^g$  for banks and the government, respectively as well as banks' outstanding overnight borrowing  $F_0$  and lending  $B_0$ . Those variables are needed to compute the real resources  $c_g(\kappa_{-1}^g) M_0/P_{-1}$  and  $c(\kappa_{-1})(D_0 + L_0 + F_0)/P_{-1}$  purchased by the government and banks to pay leverage costs at date 0. At the same time, they do not affect any endogenous variables other than consumption which does not enter the difference equation; as a result there is no need to treat them explicitly as state variables.

<sup>17</sup>The term "deposit rate" should be interpreted broadly here: it is the only cost endusers pay for payments serves in our model, since we do not explicitly model other costs such as account and transaction fees.

Consider equilibria in which reserves are abundant at all times, so  $\lambda_t \geq \bar{\lambda}$  for all  $t$ . The three liquidity management equations (A.14)-(A.16) are then redundant, the variables  $\lambda$  and  $\mu$  can be removed from the system and the overnight interest rate achieves its lower bound  $i = i_R$ . The remaining six equations then determine  $i_L, v, P, \kappa, mc(\kappa)$  and  $mb(\kappa)$ .

*Approximating equilibria with small rates of return*

We have simplified formulas above by assuming that rates of return are small decimals so their products can be ignored. To guarantee that small rates obtain in equilibrium, we scale the leverage cost function and bound the real rate on reserves. Let  $\bar{r}_R$  denote a lower bound on the real rate on reserves, a small decimal number. In what follows, we guarantee this bound by appropriate assumptions on monetary policy.

The lowest possible leverage ratio that can obtain in any equilibrium then satisfies

$$\hat{\delta} - \bar{r}_R = -c'_b(\bar{\kappa}) (1 + c(\bar{\kappa}) - c'(\bar{\kappa}) \bar{\kappa})$$

This collateral ratio corresponds to an equilibrium where the reserve rate hits the bound and reserves are abundant. If reserves were scarce, the benefit of reserves would include liquidity components and the collateral ratio would have to be higher.

We now assume that the cost function is such that  $\bar{\kappa} > 1$  and  $c(\bar{\kappa}) - c'(\bar{\kappa}) \bar{\kappa}$  is a small decimal number, much like  $\hat{\delta} - \bar{r}_R$ . It follows that the marginal cost of leverage  $mc(\kappa)$  and the marginal benefit of collateral  $mb(\kappa)$  are also small decimal numbers in any possible equilibrium. Moreover, the effect of the interest rate on credit lines on  $mc(\kappa)$  and  $mb(\kappa)$  is second order in any equilibrium. Finally, we can omit the factor  $e^{-\hat{\delta}-\pi}$  on the right hand sides of (A.15) and (A.18). The timing of the liquidity benefit has only a second-order effect.

*Collecting equations*

We restate here the equations characterizing equilibrium, making use of the approximations justified above. It is convenient to organize them in three blocks. The first block describes *bank behavior*: it says how banks respond to market prices in equilibrium by setting their liquidity and collateral ratios. It is given by

$$\begin{aligned} i_t - i_R &= (1 - G(\lambda_{t+1})) \frac{\tilde{\mu}_{t+1}}{1 + \gamma} & (A.20) \\ \hat{\delta}_t - (i_t - \pi_{t+1}) &= mb(\kappa_t) \\ i_t^L &= \sigma mc(\kappa_t) + (1 + \gamma) \tilde{\mu}_{t+1} \int_{\lambda_{t+1}}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}) \\ v_t^b &= \exp\left(-\hat{\delta}_t + \rho mb(\kappa_t)\right) (v_{t+1}^b + x_{t+1}^b). \\ \tilde{\mu}_{t+1} (1 + \gamma) &= mc(\kappa_{t+1}) - mb(\kappa_{t+1}) \end{aligned}$$

Since the bank behavior block describes responses to current conditions, it is entirely forward-looking, that is, it does not involve any state variables.

The second block consists of the evolution of household marginal utility of wealth.

$$\begin{aligned} \gamma_{t+1} &= i_t^L \\ \hat{\delta}_t &= \delta - (\gamma_{t+1} - \gamma_t) & (A.21) \end{aligned}$$

Household discounting uses the rate  $\delta$  unless there is a difference in the cost of liquidity between different dates. In particular, when liquidity is more costly next period then future payoffs are discounted at a lower rate – future consumption is effectively more expensive.

Finally, consider the quantity block that relates bank ratios and the money supply to the price level. Here it is convenient to eliminate inside money ( $D_t + L_t$ ) and the price level so the only state variable is the liquidity ratio  $\lambda_t$ . Denoting the growth rate of reserves by  $g$ , we have

$$\begin{aligned} \kappa_t &= \frac{e^{g_{t+1}} \lambda_t \left(1 + B_{t+1}^g / M_{t+1}\right) \frac{1}{1+\gamma} + \rho \frac{v_t}{\Omega_t + x_t} + \frac{1}{1+\gamma} f(\lambda_t)}{\sigma e^{g_{t+1}} \frac{\lambda_t}{\lambda_{t+1}} + \frac{1}{1+\gamma} f(\lambda_t)} \\ e^{\pi_{t+1}} \frac{\Omega + x_{t+1}}{\Omega + x_t} &= e^g \frac{\lambda_t}{\lambda_{t+1}} \end{aligned} \quad (\text{A.22})$$

Since we have eliminated  $D_t + L_t$  and can write mb and mc just as functions of  $\kappa_t$ , we are left with 9 equations in the 9 variables  $i_t, v_t, i_t^L, \lambda_t, \kappa_t, \tilde{\mu}_t, \pi_t, \hat{\delta}_t$  and  $\gamma_t$ .

### A.3 Steady state

This section derives the equations characterizing steady state. We assume the same exogenous growth rate  $g$  for the nominal quantities  $M_t$  and  $B_t^g$  so the ratio  $B_t^g / M_t$  is constant over time. With constant rates of return, the marginal rate of substitution of wealth across dates equals the discount rate, that is  $\hat{\delta}_t = \delta$ . Moreover, the key ratios chosen by banks, collateral  $\kappa$  and liquidity  $\lambda$ , are constant over time. With a constant money multiplier and output, the price level grows at the rate  $\pi = g$ .

Omitting time subscripts in (A.20) we obtain the steady state relationships between asset prices and bank ratios:

$$\begin{aligned} i - i_R &= (1 - G(\lambda)) \frac{\tilde{\mu}}{1 + \gamma} \\ \delta - (i - \pi) &= \text{mb}(\kappa) \\ i^L &= \sigma \text{mc}(\kappa) + \bar{v} (1 + \gamma) \tilde{\mu} \int_{\lambda}^{\bar{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}) \\ v^b &= \frac{x^b}{\delta - \text{mb}(\kappa)}. \\ \tilde{\mu} (1 + \gamma) &= \text{mc}(\kappa) - \text{mb}(\kappa) \end{aligned} \quad (\text{A.23})$$

To derive the liquidity-management curve (18), we combine the first two equations:

$$\delta - (i^R - \pi) = \text{mb}(\kappa) + (1 - G(\lambda)) (\text{mc}(\kappa) - \text{mb}(\kappa)). \quad (\text{A.24})$$

Banks' opportunity cost of holding reserves (the difference between the return on equity and the real return on reserves) is equated to the sum of the collateral and liquidity benefits the bank earns on reserves. Equation (25) – used in the text to discuss interest rate policy – is the first equation in (A.23).

Consider now the capital-structure curve relationship (20). Dropping time subscripts and substituting for the value of trees from (A.23), the first equation in (A.22) becomes

$$\kappa = \frac{e^g \lambda \left(1 + \frac{B_t^g}{M_t}\right) \frac{1}{1+\gamma} + \rho \frac{x}{(\delta - \rho) \text{mb}(\kappa)(\Omega + x)} + \frac{1}{1+\gamma} f(\lambda)}{\sigma e^g + \frac{1}{1+\gamma} f(\lambda)}. \quad (\text{A.25})$$

Since the growth rate of nominal liabilities  $g$  is a small decimal number, we omit it in the formula in the text. We thus ignore small effects on collateral ratios that derive from the difference in timing between when collateral is measured and when deposits provide services.

Given the solution for the money multiplier  $\lambda$ , the price level evolves in steady state as

$$P_t = \frac{1}{\lambda} \frac{1}{1 + B_t^g/M_t} \frac{1 + \gamma}{\bar{v}} \frac{M_t + B_t^g}{Y}$$

Across steady states, changes in the availability collateral alter the money multiplier and hence the steady state price level. The next section shows that the transition across steady states takes one period so that we can think of the change in price across steady states as a one time inflation response.

## A.4 Transition dynamics

Consider solutions to the difference equation (A.20)-(A.22) for  $t \geq 0$  given initial condition  $\lambda_0$ . We focus on equilibria in which the government commits to a constant growth rate  $g$  of reserves and to a ratio of bonds of reserves  $B/M$  that is fixed from date  $t = 2$  onwards. We show that for any initial conditions close enough to the steady state there exists a bond-money ratio  $B_1/M_1$  chosen at date 0 such that the equilibrium settles down at a steady state from date  $t = 1$  onwards. The additional open market operation is required only when reserves are scarce and is small if outstanding interbank lending is small.

Suppose we have a sequence of variables that enters steady state at  $t = 1$ . In particular, the variables  $v_1$ ,  $\tilde{\mu}_1$ ,  $\lambda_1$  and  $\gamma_1$  are at their steady state values. It follows from the first equation in (A.21) and the first and third equation in (A.20) that  $\kappa$ ,  $i$  and  $i^L$  must achieve their steady state values already at date 0. The steady state interest rate  $i$  and collateral ratio  $\kappa$  thus satisfy the pair of equations

$$\begin{aligned} \delta - (i - g) &= \text{mb}(\kappa) \\ \hat{\delta}_0 - (i - \pi_1) &= \text{mb}(\kappa) \end{aligned} \quad (\text{A.26})$$

where we have used the fact that steady state inflation must be equal to the growth rate of nominal liabilities  $g$ .

It follows that the date 0 effective discount rate can be written as

$$\hat{\delta}_0 = \delta + g - \pi_1 = \delta - \log(\lambda_0/\lambda)$$

where we have used the second equation in (A.22) to substitute for  $\pi_1$ . If the date 0 liquidity ratio is above that in steady state ( $\lambda_0 > \lambda$ ), then the money multiplier will rise from date 0 to date 1 and inflation will be high. As a result, liquidity will be more costly to hold in the future and future payoffs are discounted at a lower rate.



We can further solve for the price of trees

$$v_0 = e^{-(\hat{\delta}_0 - \rho \text{mb}(\kappa))} \frac{x}{1 - e^{-(\delta - \rho \text{mb}(\kappa))}} = \frac{x}{\delta - \text{mb}(\kappa)} \frac{\lambda_0}{\lambda},$$

If the date 0 liquidity ratio is higher than that in steady state, lower discounting increase the value  $v$ . We obtain  $\gamma_0$  from the second equation in (A.21). If inflation is close enough to the steady state inflation rate  $g$ , then we have  $\gamma_0 > 0$ .

We now have a sequence of variables that satisfies all equations except possibly the first equation in (A.22). The variables entering that equation are all pinned down:  $\lambda_0$  is predetermined,  $\kappa_0$  and  $\lambda_1$  are equal to steady state  $\kappa$  and  $\lambda$  and  $v_0$  is as above. We must therefore have

$$\kappa_0 = \frac{e^g \lambda_0 (1 + B_1^g/M_1) \frac{1}{1+\gamma} + \rho \frac{x(1-s)}{\delta - \text{mb}(\kappa)} \frac{1}{\Omega+x} \frac{\lambda_0}{\lambda_1} + \frac{1}{1+\gamma} f(\lambda_0)}{\sigma e^g \frac{\lambda_0}{\lambda_1} + \frac{1}{1+\gamma} f(\lambda_0)}$$

If reserves are abundant then  $f(\lambda_0) = 0$ , the predetermined liquidity ratio  $\lambda_0$  cancels and the equation is equivalent to the steady state relationship (A.25). Since  $\kappa_1$  and  $\lambda_1$  are their steady state values, the equation holds provided that  $B_1/M_1$  is also at its steady state value. Moreover, if reserves are scarce and the initial liquidity ratio  $\lambda_0$  is close enough to the steady state value  $\lambda_1$ , then we can choose  $B_1/M_1$  to satisfy the equation. In general this requires  $B_1/M_1$  different from the steady state value – the government intervention offsets the transition dynamics that would arise from the fact that interbank lending is a predetermined form of collateral. As long as interbank lending is a small share of the balance sheet, the intervention is also small.

We have thus found an equilibrium that transits to steady state in one step. The only endogenous variables that are different from steady state at date 0 are the effective discount rate  $\hat{\delta}_0$  and the log marginal utility of wealth  $\gamma_0$ . Moreover, the inflation rate  $\pi_1$  from date 0 to date 1 produces a change in the price level from the old to the new steady state and is thus different from the inflation rate that obtains from date 2 on.

## A.5 Steady state with uncertainty

We find a steady state with uncertainty in three steps, following Ilut and Schneider (2015). We first determine the "worst case" steady state, that is, the steady state to which the economy would converge were the worst case scenario contemplated by agents to actually occur. Here this worst case scenario is that the payoff of the trees is multiplied by  $1 - s$ . Worst case steady state prices and bank ratios follow from the equations in the previous subsection, with  $x$  replaced by  $x(1 - s)$ . We indicate worst case steady state values by stars. In particular, the worst case steady state ratios  $\lambda^*$  and  $\kappa^*$  are determined by the intersection of the liquidity management and capital-structure curves (A.24)-(A.25), with  $x$  replaced by  $x(1 - s)$ .

The second step characterizes agents' perceptions of the equilibrium dynamics. Agents observe every period the true tree payoff  $x$  which is larger than the worst case steady state payoff. As a result, agents always act *as if* they are on a transition path away from an initial transitory shock  $x$  that is higher than the (perceived) steady state value  $x(1 - s)$ . The actual steady state value of the endogenous state variable  $\lambda$  is such that agents' behavior leaves  $\lambda$  constant over time given their worst case beliefs. Formally, we are looking for a solution to the system (A.20)-(A.22) together with an initial condition for the endogenous state variable  $\lambda$  such that (i) the solution converges to the

worst case steady state, and (ii) the endogenous state variable  $\lambda$  is constant in the first period of the transition path.

The third step then characterizes the actual dynamics by combining the law of motion for the endogenous variables implied by the second step with the actual dynamics of the exogenous variables. Compared to the model studied in Ilut and Schneider (2015), the second step is simple in the sense that the transition path moves to the worst case steady state after one period. As a result, the actual steady state liquidity ratio  $\lambda$  is equal to the worst case ratio  $\lambda^*$ . Moreover, the third step is simple in that the actual evolution of output – the ambiguous exogenous variable – enters only one equation, namely the last equation in (A.22) that relates inflation to output growth.

*Transition to the worst case steady state*

Suppose that the solution in fact settles at the worst case steady state after one period. In particular, from date 1 on, the liquidity ratio is  $\lambda^*$  and the liquidity multipliers are  $\tilde{\mu}^*$  and  $\gamma^*$  remain constant at the worst case steady state values. Given those values, the actual steady state values of  $i, \kappa$  and  $i^L$  must be equal to their worst case steady state counterparts. Indeed, those variables follow from

$$\begin{aligned} i - i_R &= (1 - G(\lambda^*)) \frac{\tilde{\mu}^*}{1 + \gamma}, \\ i^L &= \sigma mc(\kappa) + (1 + \gamma) \tilde{\mu}^* \int_{\lambda^*}^{\tilde{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}), \\ \gamma^* &= i^L. \end{aligned} \tag{A.27}$$

Let  $\tilde{\pi}$  denote the worst case inflation rate expected by agents in the first period as the system transits to the worst case steady state. From the second equation in (A.22), we have

$$\tilde{\pi} = g + \log\left(\frac{\Omega + x}{\Omega + x(1 - s)}\right) + \log(\lambda/\lambda^*).$$

Agents fear higher inflation since they worry about lower growth. Moreover, a higher date 0 liquidity ratio would increase the money multiplier and contribute to further inflation.

Consider the steady state price of a tree. From (A.26) we have that the steady state effective discount rate is  $\hat{\delta} = \delta + g - \tilde{\pi}$ . We can therefore write the price of a tree in the actual steady state and in the worst case steady state as

$$\begin{aligned} v &= e^{-(\hat{\delta} - \rho \text{mb}(\kappa))} (v^* + x(1 - s)) \\ v^* &= e^{-(\delta - \rho \text{mb}(\kappa))} (v^* + x(1 - s)) \end{aligned}$$

The only difference here is the discount rate applied to payoffs between the current and next period.

Combining these equations and the relationship between the discount rate, money growth and worst case inflation, the steady state tree payoff is

$$\begin{aligned} v &= e^{-(\hat{\delta} - \rho \text{mb}(\kappa))} \frac{x(1 - s)}{1 - e^{-(\delta - \rho \text{mb}(\kappa))}} \\ &= \frac{x(1 - s)}{\delta - \text{mb}(\kappa)} \frac{\Omega + x}{\Omega + (1 - s)x} \frac{\lambda}{\lambda^*}, \end{aligned} \tag{A.28}$$

where in the second line we have used the second equation in (A.22) and the fact that  $\delta - mb(\kappa)$  is a small decimal number.

To verify that the actual steady state liquidity ratio is  $\lambda = \lambda^*$ , we now substitute the actual steady state value  $v$  into the first equation of (A.22) for date  $t = 0$ . Given the worst case steady state  $\lambda^*$ , the actual steady state ratios  $\lambda$  and  $\kappa$  are related by

$$\kappa = \frac{e^g \lambda (1 + B^g/M) \frac{1}{1+\gamma} + \rho \frac{x(1-s)}{\delta - mb(\kappa)} \frac{1}{\Omega + x(1-s)} \frac{\lambda}{\lambda^*} + \frac{1}{1+\gamma} f(\lambda)}{\sigma e^g \frac{\lambda}{\lambda^*} + \frac{1}{1+\gamma} f(\lambda)} \quad (\text{A.29})$$

If  $\lambda = \lambda^*$ , then this equation reduces to (A.25), but with the tree payoffs  $x$  replaced by the worst case payoff  $x(1-s)$ . Since  $\kappa = \kappa^*$ , it is therefore satisfied at  $\lambda = \lambda^*$ .

We have now derived a solution to (A.20)-(A.22) with the desired properties, thus verifying our conjecture above. Starting from  $\lambda = \lambda^*$ , a transitory shock that increases all tree payoffs by a factor of  $1/(1-s)$  induces banks to maintain the liquidity ratio constant at  $\lambda^*$ . Intuitively, since output is expected to fall agents worry that inflation is temporarily higher than the growth rate of nominal government liabilities and hence expect a higher cost of liquidity. As a result, future tree payoffs are discounted at a lower rate and the ratio of tree value to output is the same as in the worst case steady state.

#### *Graphical analysis and steady state asset prices*

The graphical analysis of optimal ratios works the same way as in the absence of uncertainty. The CS curve is described by (A.29) with  $\lambda^* = \lambda$ . The liquidity-management curve is still described by (A.24). Given numbers for  $\lambda = \lambda^*$  and  $\kappa = \kappa^*$ , the short interest rate and the rate on credit lines follows from (A.27). Finally, the inflation rate in the actual steady state is  $\pi = g$  from the second equation in (A.22). This rate is lower than the worst case rate of inflation  $\tilde{\pi}$  contemplated by agents. This is because agents fear a drop in output that will translate into higher inflation for given growth of nominal liabilities.

The price of trees follows from (A.28). Using  $\lambda = \lambda^*$  and the definition of  $\tau = x/(\Omega + x)$ , we have

$$v = \frac{x}{\delta - mb(\kappa)} \frac{1-s}{1-\tau s} = u(s) \frac{x}{\delta - mb(\kappa)}$$

We also note that while the realized real interest rate is  $i - g = \delta + mb(\kappa)$  as before, the ex ante real interest rate  $i - \pi_1 = \hat{\delta} + mb(\kappa)$  reflects the lower discount rate induced by output and hence inflation uncertainty.

#### *Welfare*

Consider welfare in the presence of uncertainty. We compute the expected loss due to leverage cost under agents' worst case perception of the future. We thus take into account the fact that agents act *as if* output will be lower starting next period, which also implies lower leverage costs. Let  $\Delta = (\Omega + x(1-s))/(\Omega + x)$  denote the drop in output between dates 0 and 1 under worst case expectations. It is also helpful to introduce notation for the total flow cost of leverage

$$K(Y) = c_g \left( \frac{1+\gamma}{(1+b)} \frac{\Omega}{\lambda Y} \right) \frac{1+\gamma}{(1+b)\lambda} Y + c_b(\kappa) Y \left( \sigma + \frac{f(\lambda)}{1+\gamma} \right)$$

The steady state loss can then be written as

$$K(T) + \frac{\beta}{1-\beta} K(\Delta T)$$

Since the output loss simply reduces transactions proportionally, the tradeoff between government leverage and private sector leverage (or equivalently  $\lambda$  and  $\kappa^{-1}$ ) is qualitatively the same as in the absence of uncertainty. As a result, the graphical analysis of welfare remains the same as in the text.

## A.6 Equilibrium with active traders

Active traders maximize shareholder value, with nominal dividends given by

$$(\hat{Q}_t + \hat{x}_t)\theta_{t-1} - \hat{Q}_t\theta_t - i_{t-1}^L \hat{L}_t - \hat{D}_{t+1} + \hat{D}_t (1 + i_{t-1}^D)$$

where  $\theta_t$  is the number of trees bought at date  $t$  and  $\hat{L}_t$  is the credit line arranged at  $t - 1$  in order to pay for trees at  $t$  and  $\hat{D}_t$  are deposits made at  $t - 1$ .

The first order conditions for active traders can be written as

$$\begin{aligned} i_t^L &= \hat{\delta}_t - (i_t^D - \pi_{t+1}) = \hat{\mu}_{t+1} (1 + \hat{\gamma}) \\ \hat{v}_t (1 + \hat{\mu}_t) &= e^{-\hat{\delta}_t} (\hat{v}_{t+1} + \hat{x}_{t+1} (1 - \hat{s})) \end{aligned} \quad (\text{A.30})$$

where  $\hat{\mu}_t$  is the multiplier on active traders' liquidity constraint multiplied by the price level. The interest rate on credit lines – bank customers' cost of liquidity – is equated to active traders' liquidity benefit. The expected payoffs of active trader trees in the second equation incorporate the worst case payoff on trees.

To capture the quantity of deposits absorbed by active traders, we define their share  $\alpha_t = (\hat{D}_t + \hat{L}_t) / (D_t + L_t)$ . The equilibrium share satisfies

$$\alpha_t = \frac{\hat{v}_t}{\hat{v}_t + (1 + \hat{\gamma}) \bar{v} (\Omega + x_t)} \quad (\text{A.31})$$

Active traders absorb more inside money if the netting system is less efficient (low  $\hat{\gamma}$ ). It is also helpful to denote aggregate goods market transactions by  $Y_t = \Omega + x_t$ .

Since the money held by active traders does not directly affect the price level, the leverage and inflation equations contain only the share of inside money that circulates in the goods market:

$$\begin{aligned} \kappa_t &= \frac{e^{g_{t+1}} \lambda_t (1 + B_{t+1}^g / M_{t+1}) + \rho \bar{v} \frac{v_t^B (1 - \alpha_t)}{Y_t} + \frac{\bar{v}}{1 + \gamma} f(\lambda_t)}{\sigma e^{g_{t+1}} \frac{\lambda_t}{\lambda_{t+1}} + \frac{\bar{v}}{1 + \gamma} f(\lambda_t)} \\ e^{\pi_{t+1}} \frac{Y_{t+1}}{Y_t} &= e^g \frac{\lambda_{t+1}}{\lambda_t} \frac{1 - \alpha_{t+1}}{1 - \alpha_t} \end{aligned} \quad (\text{A.32})$$

The system with active traders is thus given by (A.20), (A.21) and (A.30). Endogenous state variables are now both the banks' liquidity ratio  $\lambda$  and the active trader money share  $\alpha$ .

*Worst case steady state*

To characterize steady state equilibrium, the first step is again to derive the worst case steady state, that is, the steady state if  $\hat{x}_t = x^* = \hat{x}(1 - \hat{s})$  and  $Y_t = Y^*$ , where the worst case output reflects low payoffs on all trees including bank trees and active trader trees. The bank equations (A.23) and therefore the bank ratios  $(\lambda, \kappa)$  still lie on the liquidity-management curve described by (A.24). The steady state liquidity benefit, and money share as well as the price of trees held by active traders satisfy

$$\begin{aligned} i^{L*} &= \hat{\mu}^* (1 + \hat{\gamma}) \\ \hat{v}^* &= \frac{\hat{x} (1 - \hat{s})}{\delta + \hat{\mu}^*} \\ \alpha^* &= \frac{\hat{v}^*}{\hat{v}^* + (1 + \hat{\gamma}) \bar{v} (\Omega + x)} \end{aligned}$$

To describe the capital-structure curve, we first write the cost of bank customers' liquidity and the active trader money share as functions of the bank ratios  $\lambda$  and  $\kappa$ :

$$\begin{aligned} \tilde{i}^L(\lambda^*, \kappa^*) &:= \sigma \text{mc}(\kappa) + \bar{v} (1 + \gamma) (\text{mc}(\kappa^*) - \text{mb}(\kappa^*)) \int_{\lambda^*}^{\tilde{\lambda}} \tilde{\lambda} dG(\tilde{\lambda}) \\ \tilde{\alpha}(\lambda^*, \kappa^*) &:= \frac{\hat{x} (1 - s)}{\hat{x} (1 - s) + \bar{v} (\Omega + x + \hat{x} (1 - \hat{s})) ((1 + \hat{\gamma}) \delta + \tilde{i}^L(\lambda^*, \kappa^*))} \end{aligned}$$

The function  $\tilde{i}^L$  is decreasing in both arguments: higher liquidity or collateral ratios lower bank customers' cost of liquidity. As a result, the active trader money share is increasing in both  $\lambda^*$  and  $\kappa^*$ . In the region of  $(\lambda, \kappa)$ -plane where reserves are abundant both functions are independent of  $\lambda$ .

We find the worst case bank ratios from the intersection of the liquidity-management curve and the new capital-structure curve described by

$$\kappa^* = \frac{e^g \lambda (1 + B^g/M) \frac{\bar{v}}{1+\gamma} + \rho \bar{v} \frac{x^b(1-s)}{\delta - \text{mb}(\kappa)} \frac{1}{Y^*} (1 - \tilde{\alpha}(\lambda^*, \kappa^*)) + \frac{\bar{v}}{1+\gamma} f(\lambda^*)}{\sigma e^g + \frac{\bar{v}}{1+\gamma} f(\lambda^*)} \quad (\text{A.33})$$

The difference to (A.25) is the presence of the factor  $1 - \alpha$  in the numerator. In the region where reserves are abundant,  $\tilde{\alpha}$  is independent of  $\alpha$  and the curve is upward sloping in the  $(\lambda, \kappa)$ -plane without further assumptions. With scarce reserves, the presence of active traders reduced the slope of the curve. To guarantee an upward slope, we need the share of nominal assets sufficiently large or the payoff of active trader small enough relative to output.

*Actual steady state*

The second step in the characterization of equilibrium is to find the transition path from the actual to the worst case steady state. We conjecture again that this transition takes only one period, so that  $\lambda = \lambda^*$  and  $\alpha = \alpha^*$ . Equations (A.27) hold as before and deliver actual steady state  $i$ ,  $\kappa$  and  $i^L$ . To derive the value of bank trees, we apply the same argument as in Section A.5 but use the new equation for inflation – the second equation in (A.32) – to obtain

$$v^b = \frac{x^b (1 - s) Y \lambda (1 - \alpha^*)}{\delta - \text{mb}(\kappa) Y^* \lambda^* (1 - \alpha^*)}$$

Substituting into the first equation in (A.32), the actual steady state ratios  $\lambda$  and  $\alpha$  satisfy

$$\kappa = \frac{e^g \lambda (1 + B^g/M) + \rho \bar{v} \frac{x^b(1-s)}{\delta - mb(\kappa)} \frac{1-\alpha^*}{Y^*} \frac{\lambda}{\lambda^*} + \frac{\bar{v}}{1+\gamma} f(\lambda)}{\sigma e^g \frac{\lambda}{\lambda^*} + \frac{\bar{v}}{1+\gamma} f(\lambda)}$$

The equation is satisfied if  $\lambda = \lambda^*$  since we have  $\alpha^* = \tilde{\alpha}(\lambda^*, \kappa^*)$  and the equation is otherwise identical to (A.33).

It remains to check the second equation in (A.30) as well as (A.31). The actual steady state value of active trader trees follows from (A.31) as

$$\hat{v} = \frac{\alpha^*}{1 - \alpha^*} Y$$

Given this value, we can find the actual steady state multiplier  $\hat{\mu}$  to satisfy (A.30). We have shown that if the initial conditions are given by the worst case steady value  $\alpha^*$  and  $\lambda^*$  and there is a transitory shock that increases tree payoffs, agents respond by choosing again worst case steady state  $\alpha$  and  $\lambda$ .

We summarize the actual steady state for the economy with active traders as follows. Actual inflation is given by the growth rate of nominal government liabilities  $\pi = g$ . Since  $\kappa = \kappa^*$  and  $\lambda = \lambda^*$ , the actual steady state bank ratios are determined from the intersection of the liquidity management and capital-structure curves (A.24) and (A.33). The value of active trader trees and the evolution of the nominal price level are given by

$$\hat{v} = \hat{v}^* \frac{Y}{Y^*} = (1 - \hat{s}) \frac{Y}{Y^*} \frac{\hat{x}}{\delta + \hat{\mu}^*},$$

$$P_t = \frac{1 - \tilde{\alpha}(\lambda, \kappa)}{\lambda} \frac{1}{1 + B_t^g/M_t} \frac{1 + \gamma}{\bar{v}} \frac{M_t + B_t^g}{Y}.$$