

**MARKET RISK MEASUREMENT, BEYOND VALUE AT RISK**

Financial risk management has evolved dramatically over the last few decades. One of the most widespread tools used by financial institutions to measure market risk is value at risk (VaR), which enables firms to obtain a firm-wide view of their overall risks and to allocate capital more efficiently across various business lines. This box places the VaR approach into a broader risk measurement context and compares the metric with alternatives.

VaR summarises in a single number the risk of loss of a portfolio over a defined time horizon and a given confidence level  $\alpha$  so that the probability of exceeding this loss is equal to  $p = 1 - \alpha$ . If it is assumed, for example, that the VaR of a portfolio over a one-week period is equal to -1.5% of its value with a 95% confidence level ( $\alpha$ ), this implies that the investor could expect the portfolio to exceed this loss with a probability of 5% ( $p = 1 - \alpha$ ). VaR depends on two arbitrarily chosen parameters: the confidence level  $\alpha$ , which indicates the probability of an outcome of less than or equal to VaR; and the holding period or the period over which the portfolio's profit or loss is measured.

VaR owes its prominence as a risk measure in the financial markets to several positive characteristics of the metric. It enables risk managers to aggregate the risks of sub-positions into an overall and consistent measure of portfolio risk while simultaneously taking into account the various ways in which different risk factors correlate with each other. It is a holistic measure in that it takes into account all risk factors that affect the portfolio, and a probabilistic measure in that it provides information on the probabilities associated with specified loss amounts.

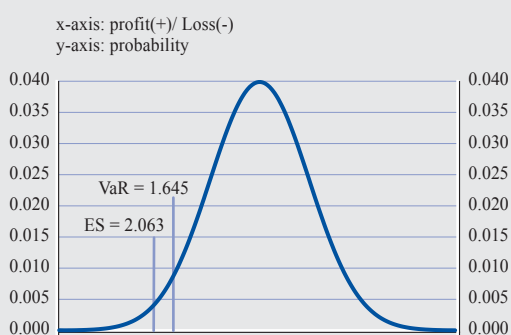
However, VaR has some drawbacks and limitations. One important limitation is that it cannot tell how much an investor can expect to lose should a tail event occur. Instead, it can only provide information on potential losses if the tail event does not occur. This could have undesirable consequences. Two positions may have the same VaR and therefore appear to have the same risk, but in reality they could have very different risk exposures, as one position could potentially lead to a very high loss in the tail.

Partially in response to criticisms of VaR, a newer risk measurement paradigm has emerged, following the theory of coherent risk measures as proposed by Artzner et al. (1999).<sup>1</sup> In contrast to VaR, this approach specifies the properties a risk measure should have in order to be a coherent measure. One important property is subadditivity. This property implies that when individual risks are added, there will be some diversification effects, i.e. the risk of the sum is always less than or equal to the sum of the risks. It turns out that VaR is not subadditive and therefore cannot be regarded as a proper risk measure. Other alternatives in the form of coherent risk measures need to be employed instead.

Expected shortfall (ES) is one such coherent alternative risk measure. It comprises the average of the worst 100(1-  $\alpha$ )% of losses of a portfolio's profit and loss distribution. ES is a superior

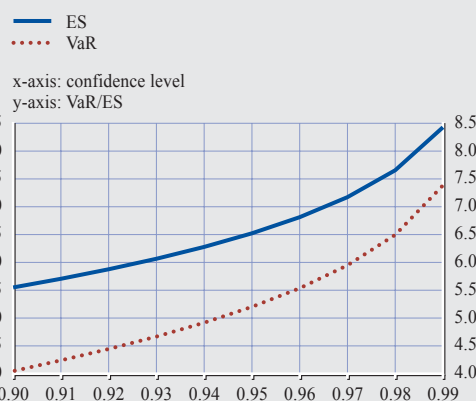
<sup>1</sup> See P. Artzner, F. Delbaen, J.-M. Eber and D. Heath (1999), "Coherent Measures of Risk", *Mathematical Finance*, 9 November, 203-228.

Chart B13.1 Value at Risk (VaR) and expected shortfall (ES)



Source: ECB calculations.  
Note: This assumes a 95% confidence level, a one-day holding period and that the daily profit and loss is distributed as standard normal (i.e. mean 0 and standard deviation 1).

Chart B13.2 Value at Risk (VaR), expected shortfall (ES) and the confidence level



Source: ECB calculations.  
Note: This assumes a ten-day holding period, and that daily profit and loss is distributed as standard normal (i.e. mean 0 and standard deviation 1).

risk measure to VaR because, among other reasons, it produces a measure of what losses can be expected in a bad situation, whereas VaR only provides indicates that the loss will be higher than itself. The ES measure is coherent as it always satisfies subadditivity. Chart B13.1 shows the value of the ES measure and VaR for a return distribution based on a hypothetical stock whose price is normally distributed with mean 0 and standard deviation equal to 1. Chart B13.2 shows that the ES measure, like VaR, tends to rise with the confidence level.

However, the ES measure cannot be considered the “best” coherent risk measure even though its computational ease makes it widely used. In the normal world, investors are risk-averse so that risk aversion is an aspect that should be reflected in the risk measure. A more general coherent risk measure that is capable of capturing the risk aversion profile of investors is called the risk spectrum measure, which comprises the weighted average of the quantiles of the portfolio’s loss distribution.

In this measure the investor needs to define the weighting function of the quantiles of the loss distribution, which weights losses according to their individual risk aversion characteristics. It turns out that the ES measure and VaR are special cases of this generic risk spectrum measure. For instance, the ES gives tail losses an equal weight of  $1/(1-\alpha)$ , and the other quantiles a weight of 0.

To produce a coherent risk spectrum measure, the loss-weighting function must meet a number of conditions. Crucially, the weighting function must give higher losses at least the same weight as lower losses, even though in normal circumstances, i.e. when there is risk aversion, higher losses are likely to be given greater weight. The weights attached to higher losses in spectral risk measures are thus a reflection of the user’s risk aversion, or the rate at which the higher weights rise be related to the degree of risk aversion. To obtain a spectral risk measure, a particular form of the loss-weighting function must be specified. This makes this risk measure not as widespread in use as for example VaR, as each investor would need to use a distinct

weighting function. This drawback makes it impossible for example to use the measure for purposes of comparison across different investors.

The connection between the weighting function and risk aversion makes spectral risk measures a superior alternative to ES if the user is risk-averse, with a weighting function that gives higher losses a higher weight than the ES measure, which gives all losses in the tail region the same weight. However, if the user is risk-neutral, ES represents a superior measure.<sup>2</sup>

Within the coherent risk measure paradigm, scenario analyses represent another coherent risk measure, together with ES and spectral risk measures. The theory of coherent risk measures provides a solid justification for stress-testing. Indeed, the outcome of scenario analysis, i.e. loss estimates with a set of associated probabilities, can be regarded as tail-drawing from the relevant distribution function, and their average value is the ES measure associated with the distribution function. Since ES is a coherent risk measure, the outcomes of scenario analysis are also coherent risk measures. Scenario analysis and stress-testing are increasingly used to handle correlation and path-dependent effects in a portfolio context.

VaR remains the financial community's and banking supervisor's risk measure of choice with regard to market risk measurement. Although VaR is an effective risk measure with several positive characteristics, it has some important drawbacks. Conceptually superior alternatives exist such as ES, spectral risk measures and even scenario analysis. Financial institutions are increasingly incorporating these newer, more coherent risk measures into their risk control frameworks. In fact, it is not difficult to upgrade to an ES measure if a VaR calculation system is already in place. This process is welcome, and should contribute to a more robust and resilient financial system.

<sup>2</sup> See for example H. Grootveld and W. Hallerbach (2004), "Upgrading Value at Risk from Diagnostic Metric to Decision Variable: A Wise Thing to Do?", in: *Risk Measures for the 21st century*, London: John Wiley and Sons.