

# Financial Intermediation and Government Debt Default \*

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## Abstract

Recent empirical evidence suggests that the major cost of sovereign default lies in its effects on the domestic financial system, rather than any external sanctions. To explore this link, we extend the banking model of Gertler and Karadi (2011) to allow banks to hold risky long-term government debt, and then incorporate bank runs following Gertler and Kiyotaki (2013). Banks face an agency problem that limits their ability to issue loans. The government may default on its debt, and the default probability depends on a fiscal limit that measures the government's ability to payoff its debt. In this way we can analyse the interactions between banking and sovereign debt crises. By allowing for distortionary taxes, sticky prices and a debt maturity structure, we also have a rich model of monetary and fiscal policy interactions and can explore how these policies influence developments in the financial and fiscal spheres. We find that without bank runs, sovereign default can reduce investment by a substantial margin and keep capital at a low level for a prolonged period of time; sovereign default risk by itself, however, has a small impact on the economy. On the other hand, if bank runs are possible, sovereign default risk, even if a default does not materialise, is stagflationary and has pronounced and negative impact on the economy.

Keywords: Financial Intermediation, Fiscal sustainability, Sovereign debt default, Fiscal limit, Banking Crisis

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# 1 Introduction

Recent evidence on the costs of sovereign default casts doubt on the traditional view that such costs are due to external sanctions such as exclusion from international financial markets or trade; instead it suggests that the main costs result from the disruption of the domestic financial system which typically holds significant shares of domestic government bonds, see Panizza, Sturzenegger, and Zettelmeyer (2009), among others. Moreover, recent developments in Europe provide clear evidence on the two-way risk spillovers between banking and sovereign debt crises. Figure 1 shows that the sovereign spreads and bank credit default swap (CDS) spreads are highly correlated in Italy and Ireland.

In this paper we explore the joint determination of banking and sovereign debt crises in a New Keynesian model, augmented with a banking sector (as in Gertler and Karadi (2011)) as well as a government facing the possibility of defaulting on its debt. Banks take deposits from households, lend to private sector, and also hold long-term government bonds. Due to an agency problem, whereby they may divert a share of their assets, the banks face an incentive compatibility constraint which limits their ability to issue loans. In addition, we extend the baseline model to incorporate bank runs following Gertler and Kiyotaki (2013). Balance sheet conditions in the banking sector not only affect the cost of raising capital and bonds for private firms and government, but also affect whether bank runs occur.

The contribution of our paper to the literature largely lies in the fiscal and monetary dimension. Firstly, we allow for the possibility that the government may default on its debt. The default process follows Bi (2012) and Davig, Leeper, and Walker (2010), where there is a partial default on outstanding government debt whenever the debt level breaches a ‘fiscal limit.’ Sovereign default risk can affect the banks’ balance sheets and give rise to a sustained reduction in the banks’ loan activities. Deteriorating balance sheet in the banking sector raises the borrowing costs for private firms and government, and further exacerbates sovereign default risk. Similarly, a banking crisis can also raise the possibility of sovereign default. Therefore, our model can capture the kind of prolonged and costly twin banking/sovereign-default crises often observed in the data. Secondly, we allow the maturity structure of government debt to lie in between the polar cases of one-period debt and infinitely lived consols, which can have profound implications on how monetary policy affects the real market value of government debt. Thirdly, taxes are distortionary rather than lump-sum.

We solve the full nonlinear model.<sup>1</sup> This is particularly important as the costs and like-

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<sup>1</sup>Gertler and Karadi (2011) and Gertler and Kiyotaki (2013) focus on the linearised dynamics around a deterministic steady-state.

likelihood of respective sovereign default and banking crises depend nonlinearly on government indebtedness and banks' net worth. The very rapid evolution of these variables following the financial crisis of 2008 reveals that the kinds of episodes we are interested in cannot be modeled as linear movements around a deterministic steady-state.

In the baseline model without bank runs, sovereign default can reduce net worth, investment, and capital by a substantial margin, and keep them at a lower level for a prolonged period of time. This is due to the amplification channel – sovereign default tightens the credit constraint and raises borrowing costs; the subsequent lower demand for capital reduces Tobin's  $Q$ , shrinks the banks' balance sheets, and further constrains the banks' ability to make loans to private sector firms. If sovereign default does not actually materialize, however, sovereign default risk itself has a small impact on the economy when its effects are transmitted through banking sector. This is because, if the banks are lucky enough not to experience a default on their holdings of government debt, the risk premia earned on government bonds facilitates the banks' accumulation of net worth which relaxes the credit constraint.

The conclusion is much different once bank runs are possible. Sovereign default risk, even if the default is never actually realized, can be stagflationary. Effectively, the sovereign default risk reduces bond prices and leads to a tightening of the credit constraint, raising the banks' reliance on deposits (or equivalently increases leverage and reduces net worth). Higher leverage and lower net worth prompts the above average rate of liquidation in the financial sector. The new banks created to replace the liquidated banks start life with a lower level of net worth and thereby give rise to a prolonged reduction in the financial intermediaries' aggregate net worth. It reduces the net worth by more than 15%, investment by 2.5%, triples the reduction in capital, and almost doubles the output loss. At the same time, inflation is higher because of the higher taxes associated with higher debt under the assumed fiscal rule.

## 1.1 Contacts with the Literature:

Our focus on the links between sovereign default and financial intermediation reflects recent empirical evidence on the costs of default. Borensztein and Panizza (2008) identify four types of cost associated with sovereign default: (1) reputational costs (including the full or partial exclusion from international financial markets), (2) the costs of being excluded from international trade, (3) the costs to the domestic economy through the holdings of government debt by domestic institutions and individuals, and (4) the political costs to policy makers of presiding over a default. While early theories of sovereign default have typically adopted one of the first two forms of default cost, the empirical evidence on their

operation has been weak. Panizza, Sturzenegger, and Zettelmeyer (2009) survey the evidence and find that countries do not suffer sustained costs of default either through exclusion from capital markets or from other external sanctions - typically countries regain access to capital markets shortly after the default process has concluded, and do not suffer significant after effects in terms of increased credit spreads or reduced trade. Moreover, the period over which such affects may be identified has been falling over time and did not appear to exist in any meaningful way in the series of defaults observed between 1998 and 2005, beginning with the Russian default in August of 1998. In fact, Yeyati and Panizza (2011) find that the output costs of a default typically occur prior to the default, and the default event itself marks the beginning of the recovery in a sample of emerging market economies.

Panizza, Sturzenegger, and Zettelmeyer (2009) argue that the domestic costs of default are the main source of the costs of sovereign default. This is supported by the analysis in Reinhart and Rogoff (2009) (Chapters 8 and 9) which finds that the output costs of domestic default are significantly higher than for an external default. Moreover, they find that the puzzling observation of external defaults at relatively low levels of external debt can be explained by the simultaneously high levels of domestic debt that exist at the time of the joint default. Of particular importance in explaining these results are the links between sovereign default and the health of the domestic financial system which will often be a major holder of the government debt. Borensztein and Panizza (2008) find that a sovereign default significantly raises the probability of a banking crises (although the converse is not true). De Paoli, Hoggarth, and Saporta (2011) also find that it is rare for a sovereign default to exist in isolation, and that when observed in combination with a banking crisis the economic costs and duration of such twin crises are much higher. Sandleris (2012) also concludes that the main costs of default lie in its effects “on domestic agents’ balance sheets and expectations.” Similarly, Laeven and Valencia (2013) find the costs of a twin banking/sovereign debt crisis to be much higher than either individual crisis in isolation. They also find that the fiscal consequences of a banking crisis are greater in advanced economies, such that what begins as a banking crises in such economies could potentially become associated with a sovereign debt crisis, despite the earlier findings of Borensztein and Panizza (2008). Sosa-Padilla (2012) argues that the empirical evidence supports the hypothesis that the main costs of sovereign default arise from the impact on the balance sheets of the banks holding that debt, which then results in a sharp contraction in corporate lending leading and to a decline in output.

Reinhart and Rogoff (2009) (chapter 7) detail that between 40 and 80% of government debt in their sample covering the period 1900-2007 is domestic, with the share of domestic debt rising for more advanced economies. Uhlig (2013) points out that stress tests of the

Eurozone banking system reveal that the vast majority of banks' holdings of government debt come from their own government – in 2011, 77% of government debt held by Italian banks was Italian, with comparable figures of 82% in Spain and 86% in Portugal. For these reasons we focus on the links between domestic sovereign default and financial intermediation.

A closely related paper is Bocola (2014), who also extends the banking model of Gertler and Karadi (2011) by allowing sovereign default. In order to estimate the nonlinear model using a particle filter, Bocola (2014) simplifies the sovereign default scheme, assuming the sovereign default probability is exogenous and unrelated to the state of the economy. Our paper complements Bocola (2014) but differs in the following ways. First, the default probability depends on the level of government debt in our model. The higher the government liability, the more likely the government may default. This assumption better captures data, and more importantly, it allows the two-way spillover effects between banking/sovereign-debt crises. Second, we model Diamond and Dybvig (1983) style bank runs following Gertler and Kiyotaki (2013), which effectively amounts to allowing for a downsizing of the financial sector through a forced firesale of assets as a financial crisis emerges. We find that without bank runs, sovereign default risk itself has a small impact on the economy. On the other hand, if bank runs are possible, sovereign default risk, even if default doesn't occur, is stagflationary and can have dramatic, negative and prolonged impact on the economy. Last, our model incorporates sticky prices and distortionary taxes, which provides a rich framework for relevant debates on monetary and fiscal policies.

## 2 The Baseline Model

This section outlines the model which extends Gertler and Karadi (2011, 2013) to allow for distortionary taxation, a richer government debt maturity structure, and sovereign default.

### 2.1 Financial Intermediaries

Consider a representative financial intermediary  $j$ . It collects nominal deposits from households, purchases nominal government bond, makes loans to private sector, and also accumulates nominal net worth. To simplify the notation, nominal deposit, government bond and net worth are re-written in real term – at the end of period  $t$ ,  $N_{jt}$  is the net worth in real term,  $B_{jt}$  are the deposits they receive from households in real term,  $K_{jt}$  are their claims on non-financial firms, and  $D_{jt}$  are their holdings of long term government bonds in real term.  $Q_t^k$  is the relative price of claims on non-financial firms, and  $Q_t^d$  is the price of the

government bond. Therefore their balance sheet equating assets to liabilities is given by,

$$Q_t^k K_{jt} + Q_t^d D_{jt} = N_{jt} + B_{jt} \quad (1)$$

$K_{jt}$  and  $D_{jt+1}$  are effectively equity and debt of the financial intermediary.  $R_{t+1}^k$  is the stochastic return to equity at time  $t + 1$ , and  $R_{t+1}^d$  the return on long-term government bonds. Household deposits earn a gross real return of  $R_{t+1}^b$ . The intermediary net worth evolves according to,

$$\begin{aligned} N_{jt+1} &= R_{t+1}^k Q_t^k K_{jt} + R_{t+1}^d Q_t^d D_{jt} - R_{t+1}^b B_{jt} \\ &= (R_{t+1}^k - R_{t+1}^b) Q_t^k K_{jt} + (R_{t+1}^d - R_{t+1}^b) Q_t^d D_{jt} + R_{t+1}^b N_{jt} \end{aligned} \quad (2)$$

Since the bankers will not lend at a discounted return less than the discounted cost of borrowing, for the intermediary to operate the following conditions must hold,

$$E_t \Lambda_{t,t+1+i} (R_{t+i+1}^k - R_{t+i+1}^b) \geq 0 \quad (3)$$

$$E_t \Lambda_{t,t+1+i} (R_{t+i+1}^d - R_{t+i+1}^b) \geq 0 \quad (4)$$

where  $\Lambda_{t,t+1+i} = \beta^{i+1} \frac{u_c(t+i+1)}{u_c(t)}$  is the stochastic discount factor.

Since she earns a return in excess of the household's return on deposits, the banker should accumulate assets until she is randomly ejected from the market. Therefore the banker's objective is to maximize terminal wealth,

$$V_{jt} = \max E_t \Lambda_{t,t+1} ((1 - \theta_{t+1}) N_{jt+1} + \theta_{t+1} V_{jt+1})$$

A banker stays banker next period with probability of  $\theta_{t+1}$ . In the baseline model without bank runs, the probability is fixed at  $\theta$ , and therefore the average survival time for a banker in any given period is  $1/(1 - \theta)$ . In the extended model with bank runs (see below), the probability would be time-varying, depending on the bank's net worth and leverage.

**Agency Problem:** The friction in financial intermediation is that bankers can divert a fraction,  $\lambda$ , of available funds from the projects they have invested in and transfer them back to their household. If this happens, the depositors can force the bank into liquidation, but are assumed to only be able to recover the remaining  $1 - \lambda$  of assets. Following Gertler and Karadi (2013), we assume that a similar problem exists for banks' holding of long-term government debt, although the problem is possibly less severe. The financial intermediary may divert a fraction,  $\eta\lambda$ , of their holdings of long-term government debt, where  $0 < \eta \leq 1$ .

Therefore depositors will only be prepared to supply funds to the banker if the following incentive compatibility constraint holds,

$$V_{jt} \geq \lambda Q_t^k K_{jt} + \eta \lambda Q_t^d D_{jt}. \quad (5)$$

It implies that the banker prefers to remain in business than syphon off a fraction  $\lambda$  of private assets and  $\eta\lambda$  of their portfolio of government bonds.

For a given net worth  $N_{jt}$ , the bank chooses  $K_{jt}$  and  $D_{jt}$  to maximize the wealth  $V_{jt}$ . Since both the objective and constraints are constant returns to scale, the bank's optimization problem is essentially to maximize the franchise value per unit of net worth,  $\frac{V_{jt}}{N_{jt}}$ , which can be interpreted as the bank's "Tobin's Q," see Gertler and Kiyotaki (2013). Let  $V_{jt} = f_t N_{jt}$ , and  $\mu_t$  be the Lagrangian multiplier associated with the incentive compatibility constraint, equation (5). The optimization conditions are,

$$(K_{jt}) \quad E_t \beta \frac{u_c(t+1)}{u_c(t)} (1 - \theta_{t+1} + \theta_{t+1} f_{t+1}) (R_{kt+1} - R_{t+1}^b) = \mu_t \lambda \quad (6)$$

$$(D_{jt}) \quad E_t \beta \frac{u_c(t+1)}{u_c(t)} (1 - \theta_{t+1} + \theta_{t+1} f_{t+1}) (R_{dt+1} - R_{t+1}^b) = \eta \mu_t \lambda \quad (7)$$

$$(N_{jt}) \quad E_t \beta \frac{u_c(t+1)}{u_c(t)} (1 - \theta_{t+1} + \theta_{t+1} f_{t+1}) R_{t+1}^b + \mu_t f_t = f_t \quad (8)$$

$$(\mu_t) \quad \mu_t (f_t N_t - \lambda (Q_t^k k_t + \eta Q_t^d D_t)) = 0 \quad (9)$$

Without the agency problem, the incentive constraint wouldn't bind, the associated Lagrangian multiplier  $\mu_t$  is zero, and the model collapses to the standard model without banking sector. With the agency problem, on the other hand, a positive  $\mu_t$  provides a wedge between the expected return on bankers' assets and liabilities.

At the aggregate level, denote  $\phi_t^k = \frac{Q_t^k K_t}{N_t}$  and  $\phi_t^d = \frac{Q_t^d D_t}{N_t}$ , then the net worth growth can be defined as,

$$\rho_{t+1}^N = (R_{t+1}^k - R_{t+1}^b) \phi_t^k + (R_{t+1}^d - R_{t+1}^b) \phi_t^d + R_{t+1}^b$$

To track the evolution of aggregate net worth, we need to examine the net worth of both surviving and new bankers,

$$N_t = N_{et} + N_{nt} \quad (10)$$

The net worth of surviving bankers is given by,

$$N_{et} = \theta_t \rho_t^N N_{t-1} \quad (11)$$

New bankers receive start-up funds from their families. Following Gertler and Karadi (2011),

we assume that the funds given to new bankers is,

$$N_{nt} = \omega(Q_t^k K_{t-1} + Q_t^d D_{t-1}) \quad (12)$$

where  $\omega$  is chosen to pin down the steady state leverage ratio  $\phi^k + \phi^d$ . Aggregate net worth evolves as,

$$N_t = \theta_t \rho_t^N N_{t-1} + \omega(Q_t^k K_{t-1} + Q_t^d D_{t-1}) \quad (13)$$

**Alternative Interpretation of the Agency Problem:** The agency problem is motivated as bankers' ability to divert funds for their own use. It creates an incentive compatibility constraint for banks, as depositors would be unwilling to deposit funds were this constraint not to be satisfied. We can alternatively motivate the constraint as arising from capital regulation so that the value of the bank must exceed a share of its assets,  $\lambda$ . The government debt, however, is assumed to have higher quality and valued more in the capital regulation. The banks' holdings of debt may enter that portfolio with a weight,  $\eta$ , less than one.

$$V_{jt} \geq \lambda Q_t^k K_{jt} + \eta \lambda Q_t^d D_{jt} \quad (14)$$

Under this interpretation a failure to change  $\eta$  when government debt becomes risky amounts to a regulatory mis-pricing of the risk of bank balance sheet. Siekmann (2011) identifies a major component of the Eurozone crisis as being the holdings of risky sovereign debt by Eurozone banks, possibly because of a regulatory system that treated all Eurozone sovereign debt equally.

## 2.2 Households

Households, of size 1, contain two types of member - a share of  $f$  are bankers/financial intermediaries and  $1 - f$  are workers. Bankers stay bankers from one period to the next with probability,  $\theta_t$ . This stops them accumulating sufficiently high net worth to avoid the agency cost.  $(1 - \theta_t)f$  bankers become workers each period, and the same number of workers become bankers so that the number of bankers remains fixed.

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \quad (15)$$

$$s.t. \quad c_t = w_t n_t + \Upsilon_t + R_t^b B_{t-1} - B_t + z_t \quad (16)$$

where  $w_t$  are real wages, and  $\Upsilon_t$  are the profit payoffs from both financial and non-financial firms after giving new bankers start-up funds. Deposits earn a gross real rate of return of  $R_t$



between  $t - 1$  and  $t$ . Households also receive lump-sum transfers  $z_t$  from government. The first-order conditions are,

$$-\frac{u_n(t)}{u_c(t)} = w_t \quad (17)$$

$$\beta E_t R_{t+1}^b \frac{u_c(t+1)}{u_c(t)} = 1 \quad (18)$$

## 2.3 Firms

**Final Goods Firms** Competitive final goods firms buy the differentiated products provided by intermediate goods producers to construct consumption aggregates, which have the CES form,

$$y_t = \left( \int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad (19)$$

Cost minimisation for final goods producers results in the demand curve for intermediate good  $i$ ,

$$y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\epsilon} y_t \quad (20)$$

**Intermediate Goods Firms** The imperfectly competitive intermediate goods firms enjoy some monopoly power in producing a differentiated product, and face a downward sloping demand curve. They also face Rotemberg adjustment costs in changing prices of the form,  $\frac{\psi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} \frac{1}{\pi} - 1 \right)^2 Y_t$ , such that large price changes in excess of steady-state inflation rates are particularly costly. The quadratic price adjustment cost renders the firm's problem dynamic,

$$\max \sum_{t=0}^{\infty} \Lambda_{t,t+i} \left( p_t(i) y_t(i) (1 - \tau_t) - P_t P_{mt} y_t(i) - \frac{\psi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} \frac{1}{\pi} - 1 \right)^2 P_t y_t \right) \quad (21)$$

$$s.t. \quad y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\epsilon} y_t \quad (22)$$

where  $P_{mt}$  is the marginal cost, and  $\tau_t$  is a tax on sales revenue. The first-order condition is

$$(1 - \epsilon)(1 - \tau_t) + \epsilon P_{mt} - \psi \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} + \beta \psi E_t \frac{u_c(t+1)}{u_c(t)} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{y_{t+1}}{y_t} = 0$$

It represents the non-linear New Keynesian Phillips curve (NKPC) under Rotemberg pricing, which would, upon linearization, correspond to the standard NKPC under Calvo (1983) pricing.

The firm finances its capital by obtaining funds from banks, and faces the following cost

minimization problem,

$$\min w_t L_t + R_{kt} k_{t-1} Q_{t-1}^k - Q_t^k k_{t-1} + \delta k_{t-1} \quad (23)$$

$$s.t. \quad y_t = A_t (k_{t-1})^\alpha L_t^{1-\alpha} \quad (24)$$

where  $A_t$  is the productivity level and is fixed at the steady state throughout the paper for now. Let  $P_{mt}$  be the Lagrangian multiplier associated with the production function, which is also the real marginal cost. The first-order conditions are,

$$w_t = P_{mt} \frac{y_t (1 - \alpha)}{L_t} \quad (25)$$

$$R_{kt} = \frac{P_{mt} \alpha y_t + (Q_t^k - \delta) k_{t-1}}{Q_{t-1}^k k_{t-1}} \quad (26)$$

**Capital Firm** At the end of period  $t$ , the capital producing firms buy capital from intermediate goods firms, repair depreciated capital, and build new capital. The value of new capital is  $Q_t^k$ , but the cost of replacing worn-out capital is 1. Firms maximize profits subject to capital adjustment costs,

$$\begin{aligned} \max \quad & (Q_t^k - 1) I_t^n - F\left(\frac{I_t^n}{K_{t-1}}\right) K_{t-1} \\ s.t. \quad & K_{t+1} = K_t + I_{t+1}^n \end{aligned}$$

where the net investment is  $I_t^n = I_t - \delta K_{t-1}$ . The first-order condition for the capital producing firms is,

$$Q_t^k - 1 = F'\left(\frac{I_t^n}{K_{t-1}}\right) \quad (27)$$

We follow Christiano, Eichenbaum, and Trabandt (2014) in adopting an adjustment costs function of the following form,

$$F(x) = \frac{1}{2} \left( \exp(\sqrt{\mu^k} x) + \exp(-\sqrt{\mu^k} x) - 2 \right) \quad (28)$$

which has the steady state properties that  $F(0) = 0, F'(0) = 0$  and  $F''(0) = \mu^k$ . The first-order conditions then become,

$$Q_t^k - 1 = F'_t \quad (29)$$

$$F'_t = \frac{\sqrt{\mu^k}}{2} \left( \exp\left(\sqrt{\mu^k} \frac{I_t^n}{k_{t-1}}\right) - \exp\left(-\sqrt{\mu^k} \frac{I_t^n}{k_{t-1}}\right) \right) \quad (30)$$

## 2.4 Resource Constraint and Government Policy

The economy-wide resource constraint is,<sup>2</sup>

$$y_t = c_t + K_t - K_{t-1} + \delta K_{t-1} + g_t$$

where  $g_t$  is the government spending and is fixed at its steady state for now. Capital evolves according to,

$$K_{t+1} = K_t + I_{t+1}^n \quad (31)$$

**Policy** Monetary policy follows the Taylor rule,

$$\frac{i_t}{i} = \left( \frac{\pi_t}{\pi} \right)^{k_\pi}$$

The risk-free real rate at time  $t + 1$ ,  $R_{t+1}$ , depends on the nominal interest rate at time  $t$  and the realized inflation at  $t + 1$ ,  $i_t = R_{t+1}\pi_{t+1}$ . In the baseline model without bank runs, the risk-free real rate is the same as the deposit rate,  $R_{t+1} = R_{t+1}^b$ , which implies that household deposit rate isn't indexed to inflation.

The government issues a long-term bond  $D_t$ . Following Chatterjee and Eyigungor (2013), a fraction  $(1 - \rho_d)$  of bond matures at each period, and the government pays back the principal to bond holders; for the rest of bond  $\rho_d$ , the government pays the coupon of 1 dollar, and the bond holders receives the principal in the future. The maturity structure can also be interpreted in an alternative way – the government issues perpetuities  $d_t$  with coupons that decay exponentially, and  $D_t$  is the discounted sum of all existing perpetuities. A bond issued at period  $t$  pays  $\rho_d^{k-1}$  at period  $t + k$ , and  $\rho_d \in [0, 1]$  is the coupon decay factor that parameterizes the average maturity of the bond portfolio. The two interpretations are mathematically equivalent.

$$z_t + g_t - \tau_t y_t + (1 + \rho_d Q_t^d)(1 - \Delta_t) \frac{D_{t-1}}{\pi_t} = Q_t^d D_t \quad (32)$$

$\Delta_t$  is the haircut at time  $t$ , see further discussion below.  $\tau_t y_t$  are the tax revenues raised by distortionary sales tax, where the tax rate responds to the real debt level,

$$\frac{\tau_t}{\tau} = \left( \frac{(1 - \Delta_t) D_{t-1}}{D} \right)^{\gamma_d} \quad (33)$$

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<sup>2</sup>The price and the capital adjustment costs are assumed to be returned to household in a lump-sum way in order to simplify the solution procedure. This assumption does not change the qualitative results.

**Default** The default scheme at each period depends on an effective fiscal limit ( $D_t^*$ ). If current outstanding debt obligations are below the effective fiscal limit, then the government repays its liabilities; otherwise, the government partially defaults on its obligations by a fixed share of  $\Delta$ . The amount of unpaid bonds in any period ( $\Delta_t$ ), which we call the default rate, is summarized by

$$\Delta_t = \begin{cases} 0 & \text{if } D_{t-1} < D_t^* \\ \Delta & \text{if } D_{t-1} \geq D_t^* \end{cases}$$

The effective fiscal limit ( $D_t^*$ ) is stochastic and drawn from an exogenous distribution,  $D_t^* \sim \mathcal{D}^*$ . Bi (2012) shows that a similar rational expectations model can give rise to an endogenous distribution of the fiscal limit that is strongly nonlinear — once the default probability begins to rise, it does so rapidly. To capture this strong nonlinearity, we follow Davig, Leeper, and Walker (2010) and model the cumulative density function of the fiscal limit distribution as a logistical function with parameters  $\eta_1$  and  $\eta_2$  dictating its shape,

$$p_t^d \equiv P(D_{t-1} \geq D_t^*) = \frac{\exp(\eta_1^d + \eta_2^d D_{t-1})}{1 + \exp(\eta_1^d + \eta_2^d D_{t-1})}, \quad (34)$$

where  $p_t^d$  is the default probability at time  $t$  that is associated with the debt level of  $D_{t-1}$ . The higher the government liability, the higher the default probability.

Also, the realized return on the government debt at time  $t$  is defined as,

$$R_t^d = (1 - \Delta_t) \frac{1 + \rho_d Q_t^d}{Q_{t-1}^d \pi_t}$$

Therefore  $Q_t^d D_t$  is the real value of government debt.

### 3 Model with Bank Runs

In this section, we expand the baseline model to introduce the possibility of bank runs. This is a stylized device used to capture the downsizing of the financial sector that occurs in a financial crisis. In the potential presence of bank runs, households face a risk when depositing money with a bank. Accordingly their first-order condition for holding deposits is given by,

$$\beta E_t R_{t+1}^b (1 - \Delta_{t+1}^b) \frac{u_c(t+1)}{u_c(t)} = 1 \quad (35)$$

where  $\Delta_t^b$  is the loss in deposits in the event of a bank run and is zero otherwise. For the risk free rate, the first-order condition remains,

$$\beta E_t R_{t+1} \frac{u_c(t+1)}{u_c(t)} = 1$$

The financial intermediaries now must pay deposit rates which rise as the risk of bank runs rise. We also assume that in the event of a bank run, the bank is liquidated such that the bank survival probability becomes time varying. When runs take place, banks are randomly attacked - there is nothing an individual bank can do to prevent a bank run - and it is a random event from the point of view of an individual bank, but endogenously determined in aggregate. In order to do this, we make the following assumptions: although banks have positive net worth in the absence of a run, they are forced to sell their assets at fire-sale prices in a run, and the prices are sufficiently low so that the expectation of a run becomes self-fulfilling. However, when the assets are transferred to households at these prices, they are immediately resold at normal prices. Therefore the effective net worth transferred to households in the event of a bank run is equivalent to the net worth that would be transferred if the bank had died of natural causes. In other words, the households make gains from reselling the fire-sale assets at full prices which offset the losses the bank suffered. Similarly, any reduction in deposit interest due to a bank run is passed back to households in the form of the net worth of the liquidated bank. Although the banks owners do not really fear a run, it forces them to liquidate prematurely, suffer higher interest rates on deposits, and therefore reduces financial intermediaries' ability to accumulate net worth prior to liquidation.

At each period, there are three possible outcomes for banks,

1. naturally die with probability of  $1 - \bar{\theta}$ ,

$$\text{pay to depositors: } R_t^b B_{jt-1} \quad (36)$$

$$\text{transfer to HH: } R_t^k Q_{t-1}^k K_{jt-1} + R_t^d Q_{t-1}^d D_{jt-1} - R_t^b B_{jt-1} \quad (37)$$

2. bank run with probability of  $\bar{\theta} - \theta_t$ ,

$$\text{pay to depositors: } R_t^b (1 - \Delta_t^b) B_{jt-1} \quad (38)$$

$$\text{transfer to HH: } R_t^k Q_{t-1}^k K_{jt-1} + R_t^d Q_{t-1}^d D_{jt-1} - R_t^b (1 - \Delta_t^b) B_{jt-1} \quad (39)$$

3. no bank run with probability of  $\theta_t$ ,

$$\text{pay to depositors: } R_t^b B_{jt-1} \quad (40)$$

$$\text{accumulate net worth: } N_{jt} = R_t^k Q_{t-1}^k K_{jt-1} + R_t^d Q_{t-1}^d D_{jt-1} - R_t^b B_{jt-1} \quad (41)$$

Each bank's objective is to maximize terminal wealth,

$$\begin{aligned} V_{jt} &= \max E_t \Lambda_{t,t+1} ((1 - \theta) N_{jt+1}|_{nr} + (\theta - \theta_{t+1}) N_{jt+1}|_{run} + \theta_{t+1} V_{jt+1}) \\ \text{s.t. } V_{jt} &\geq \lambda Q_t^k K_{jt} + \eta \lambda Q_t^d D_{jt} \\ N_{jt+1}|_{nr} &= R_{t+1}^k Q_t^k K_{jt} + R_{t+1}^d Q_t^d D_{jt} - R_{t+1}^b B_{jt} \\ N_{jt+1}|_{run} &= R_{t+1}^k Q_t^k K_{jt} + R_{t+1}^d Q_t^d D_{jt} - R_{t+1}^b (1 - \Delta_{t+1}^b) B_{jt} \end{aligned}$$

where  $N_{jt+1}|_{nr}$  is the net worth transferred from the bank to household next period if no bank run occurs, and  $N_{jt+1}|_{run}$  is the net worth transferred if there is a bank run. The first-order conditions are,

$$\begin{aligned} (K_{jt}) \quad E_t \beta \frac{u_c(t+1)}{u_c(t)} & \left( (1 - \theta_{t+1} + \theta_{t+1} f_{t+1}) (R_{t+1}^k - R_{t+1}^b) + (\bar{\theta} - \theta_{t+1}) \Delta_{t+1}^b R_{t+1}^b \right) = \mu_t \lambda \\ (D_{jt}) \quad E_t \beta \frac{u_c(t+1)}{u_c(t)} & \left( (1 - \theta_{t+1} + \theta_{t+1} f_{t+1}) (R_{t+1}^d - R_{t+1}^b) + (\bar{\theta} - \theta_{t+1}) \Delta_{t+1}^b R_{t+1}^b \right) = \eta \mu_t \lambda \\ (N_{jt}) \quad E_t \beta \frac{u_c(t+1)}{u_c(t)} & (1 - \theta_{t+1} + \theta_{t+1} f_{t+1} - (\bar{\theta} - \theta_{t+1}) \Delta_{t+1}^b) R_{t+1}^b + \mu_t f_t = f_t \\ (\mu_t) \quad \mu_t (f_t N_t^b - \lambda (Q_t^k K_t^b + \eta Q_t^d D_t^b)) & = 0 \end{aligned}$$

At the aggregate level, net worth evolves in the same way as in the baseline model, because banks experiencing a run don't survive to the next period such that realised haircuts on deposits will not directly affect the evolution of the net worth of surviving banks. Nevertheless, the premium paid on deposits as a result of this risk, in conjunction of lower survival rate, will slow the accumulation of net worth.

$$N_t = \theta_t (R_{kt} Q_{t-1}^k K_{t-1} + R_{dt} Q_{t-1}^d D_{t-1} - R_t^b B_{t-1}) + \omega (Q_t^k K_{t-1} + Q_t^d D_{t-1})$$

The bank survival rate is then assumed to be decreasing in the amount of deposits banks held. The intuition is that since,

$$B_{t-1} = (\phi_{t-1}^k + \phi_{t-1}^d - 1) N_{t-1}$$

a survival rate that is decreasing in deposits is essentially increasing in net worth and de-

creasing in leverage. The higher the bank net worth, the lower the bank leverage, the higher the survival rate. We assume it takes the following function form,

$$\theta_t = \frac{\exp(\eta_1^b - \eta_2^b B_{t-1})}{1 + \exp(\eta_1^b - \eta_2^b B_{t-1})} (\bar{\theta} - \theta_{min}) + \theta_{min} \quad (42)$$

The function is calibrated such that  $\theta(B_{t-1} = -\infty) = \bar{\theta}$ ,  $\theta(B_{t-1} = \infty) = \theta_{min}$ , and also  $\theta(B_{t-1} = \bar{B}) = \bar{\theta}$ .

## 4 Calibration

The model is calibrated similarly to Gertler and Karadi (2011, 2013), who in turn take their standard parameters from the estimates of Primiceri, Schaumburg, and Tambalotti (2006).<sup>3</sup> We assume a debt to GDP ratio of 49%, a steady-state leverage ratio of 4 and a banker survival rate of 0.972. We also assume that the agency problem associated with holding long-term bonds is half that associated with private loans, and that the premium on bank loans is 100 basis points (on an annualised basis), and 50 basis points on long-term government debt. This implies that the proportional transfer to new bankers is  $\omega$  is 0.0015 and the extent of the agency problem,  $\lambda$  is 0.3504. The fiscal limit rule is specified such that the default probability is 0.1% when the debt level is 0.1% higher than the steady state level, and is 99% when the debt level is 10% higher than the steady state level.<sup>4</sup> The haircut on government bond is calibrated to 0.08 at quarterly frequency. The survival rate rule in the bank run model is specified such that the rate is  $\bar{\theta}$  when deposits are 0.1% higher than their steady state level, and is 5% lower than  $\bar{\theta}$  when deposits are 2% higher. The haircut on deposits is set to zero at the moment, implying bank runs only reduce bank survival rate but don't affect deposit rates. Table 1 shows the calibration.

## 5 Results: Baseline Model

In this section, we discuss the decision rules and impulse responses in the baseline model without bank runs.

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<sup>3</sup>The estimate of the Calvo probability of price change is 0.221 in Primiceri, Schaumburg, and Tambalotti (2006). We use a price adjustment cost parameter of 49.64 to match the slope of the linearized Phillips curve given other parameter values.

<sup>4</sup>The calibrations on fiscal limits and bank run probability are very preliminary at this stage. A more realistic calibration, however, is unlikely to change the key message from this paper.

parameters		
$\beta$	discount rate	0.995
$\mu^k$	capital adjustment cost para	71.2
$\alpha$	capital ratio	0.33
$g/y$	government spending-output ratio	0.15
$z/y$	government transfers-output ratio	0.1
$\pi$	steady-state inflation	1
$\psi$	price adjustment cost para	49.64
$\theta$	banker survival rate	0.972
$\kappa_\pi$	taylor coefficient	1.5
$\gamma_d$	tax response coefficient	1
$1/\zeta_l$	inverse of Frisch	0.276
$\phi$	leverage ratio	4
$\frac{Q^d D}{4y}$	Annualised Govt Debt to GDP	0.49
$\rho_d$	maturity of bonds	$1 - \frac{1}{8}$
$\eta$	weight on government bond	0.5
$\epsilon^z$	shock standard deviation	0.03

Table 1: **Calibrated Parameters.**

## 5.1 Decision Rules

Figure 2 compares the decision rules across different transfer states. The top left panel shows how the decision rules for capital  $K_t = f(\mathbf{S}_t)$  depend on the existing government debt level  $D_{t-1}$  at different levels of transfers  $z_t$ , while the other states  $(K_{t-1}, B_{t-1}, i_{t-1})$  are at their steady states. There are two channels – firstly, a higher debt level raises the banks’ assets and improves their balance sheets, which would raise the amount of loans banks make to firms; secondly, a higher debt level can also raise default probability and therefore reduce the level of capital. That is why the capital decision rule is nonlinear with respect to the existing debt level. Also, everything else being equal, higher transfers reduce the quantity of loans banks make to firms.

The middle left panel illustrates a similar message. If the default probability is trivial, the more debt banks hold, the less binding the constraint becomes; if the default probability is nontrivial, on the other hand, a higher debt level tightens the credit constraint and raises the Lagrange multiplier.

The three panels in the middle column show the decision rules for labor supply, output, and real marginal cost. They follow a similar pattern. For the real marginal cost  $P_{mt}$ , as debt levels raise tax rates, pre-tax marginal cost falls, and that is why the decision rule for  $P_m$  decreases with respect to the existing debt levels. As the default probability rises, tax rates are expected to fall and the marginal cost starts to rise again due to sticky prices.

The top and middle panels in the right column show how inflation and expected inflation depend on the existing debt level at different levels of transfers, everything else being at



the steady states. Sovereign default, when it occurs, is contractionary. Therefore, a higher default probability reduces inflation expectations. On the other hand, a higher debt today and therefore higher taxes today raise inflation. That is why the inflation decision rules have a kink as the default probability becomes nontrivial.

## 5.2 Sovereign Crises

**Default Scenario** Figure 3 shows how sovereign default affects the private sector through banks. Transfers stay at a high level (9% higher than the steady state) for 12 quarters, debt rises (5% higher than the steady state at the peak), and the default probability becomes nontrivial. The bond price, reflecting the markets' willingness to hold government bonds, is an inverse image of the default probability. It decreases as the default probability rises, and drops by a maximum of 6%. Higher transfers raise taxes, which reduce labor supply and investment. The paths for Tobin's Q and net worth follow a similar pattern due to the amplification channel – a higher default probability tightens up the credit constraint and raises the borrowing cost; a lower demand for capital reduces Tobin's Q, shrinks the banks' balance sheet, and further constrains the banks' ability to make loans to private firms.

In this particular example the government defaults when the default probability is 0.25. Following the haircut on government debt, the bond price sharply returns to the steady state level as the default probability becomes trivial again. Labor supply rises because of lower taxes, and the inflation rate drops. The private sector, on the other hand, recovers much more slowly. It takes another 30 quarters before net worth returns to its steady state level. Due to the sluggish recovery in investment, capital stays at a low level for a prolonged period of time. After the default, output rises in the short run (driven by labor supply), but drops and stays at a lower level in the long run (driven by the level of capital).

**Risk Scenario** Figure 4 shows the impulse responses with sovereign risk but without actual default. The transfer path is the same as figure 3, as it stays at a high level (9% higher than the steady state) for 12 quarters. Debt rises (5% higher than the steady state at the peak), and the default probability becomes nontrivial (peaks at 0.3). As a result, the market value of government debt  $Q_t^d D_t$ , plotted as red dashed line in the debt panel, stays lower than the debt level  $D_t$  for a prolonged period. The paths for investment, Tobin's Q, and net worth follow a similar pattern due to the amplification channel.

Once the transfers return to the steady state level, however, net worth, Tobin's Q, and investment increase rapidly and reach levels that are substantially higher than their steady-state levels. This is because, if the government does not actually default, the risk premia earned on government bond subject to default risk helps banks to accumulate net worth.

Also, underlying sovereign risk is inflationary, while realised sovereign default is deflationary as shown in figure 3. The middle bottom panel compares the actual inflation rate (blue solid line) and the expected inflation rate (red dashed line). When the default probability is nontrivial but no default actually occurs, actual inflation is always higher than expected inflation. This is because a higher level of debt raises inflation through expected higher taxes, even though sovereign default is deflationary and a higher default probability reduces expected inflation.

## 6 Sovereign Default Cost

One purpose of this paper is to understand the cost associated with sovereign default and sovereign risk through the banking sector channel. In this section, we compare the impulse responses among the baseline model, the model without sovereign default, and the bank run model.

In figure 5, solid blue lines are for the baseline model (as shown in figure 3), and the dashed red lines are for the same model but without sovereign default risk  $\Delta = 0$ . Both sets of impulse responses are under the same sequence of transfer shocks. A higher default probability and the subsequent sovereign default reduce net worth, investment, and capital by a substantial margin, and keep them at a lower level for a prolonged period of time. The tax relief associated with sovereign default, however, also raises labor supply and therefore output in the short and medium run. This effect is consistent with the empirical observation that the realisation of the sovereign default often heralds the beginning of the economic recovery - see, for example, Yeyati and Panizza (2011).

Figure 6 shows a similar comparison between baseline model and no default model, except that default doesn't occur in the baseline model even though the default probability is nontrivial. In this case, as transfers stay at a high level, default probability rises, and the debt is higher in the baseline model than the no default model, while the bond price is lower. Net worth, investment and Tobin's  $Q$  are slightly lower in the baseline model, but the differences are negligible. Once the transfers return to the steady state level, however, net worth, investment and Tobin's  $Q$  are actually higher in the baseline model, because the risk premia earned on government bonds allows banks to accumulate net worth which relaxes the credit constraint. This comparison suggests that if the sovereign default does not actually occur, sovereign default risk itself has a small impact on the economy when its effects are transmitted through banking sector.

If we allow for the bank run channel, however, the conclusion is very different. Figure 7 adds one more set of impulses responses on top of figure 6 – the dotted-dashed black

lines are for the model with bank runs. With bank runs, sovereign default risk, even if the default is never actually realised, can be stagflationary. Effectively, the sovereign default risk, reduces bond prices and leads to a tightening of the credit constraint. This raises the banks' reliance on deposits (or equivalently increases leverage and reduces net worth) which prompts the above average rate of liquidation in the financial sector. The new banks created to replace the liquidated banks start life with a lower level of net worth and thereby give rise to a prolonged reduction in the financial intermediaries' aggregate net worth. At the peak, it reduces the net worth by more than 15%, investment by 2.5%, triples the reduction in capital, and almost doubles the output loss. Inflation is higher because of the higher taxes associated with higher debt under the assumed fiscal rule.

## 7 Conclusion

In our model, the rich interactions between banks and government can capture the potential risk spillovers between the two sectors. It allows us to analyze the economic costs of such linkages, and see whether our model can fit with the empirical evidence on the costs of sovereign default and of banking crises. For example, we find that for sovereign default risk (which may or may not actually lead to a default) the costs are small unless, either the default is realized or the default risk itself forces an enhanced rate of liquidation of financial intermediaries. Aside from clarifying this aspect of the transmission mechanism, our model provides a framework to address relevant policy debates. For instance, when a worsening fiscal outlook could threaten the stability of the banking system, what role does policy play? Is a monetary policy which seeks to accommodate and offset any emerging risk premia effective in averting a crisis? How effective is an austerity programme aimed at avoiding default and stabilizing debt quickly? How does monetary and fiscal policy influence the response to a shock to banks' net worth (from a source other than sovereign default risk)? Last but not least, Livshits and Schoors (2009) argue that the banking crises that can follow sovereign default may reflect a failure of prudential regulation, in that the banks construct portfolios positively correlated with default when risky government debt is considered to be safe by the regulator. We can analyze the implications of such policy failings on our model by re-interpreting the agency problem facing the bank as a regulatory scheme, and discuss how monetary and fiscal policy affect this risk taking behavior on the part of banks.<sup>5</sup>

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<sup>5</sup>Such an approach complements that in Gertler, Kiyotaki, and Queralto (2012) where banks can choose between financing themselves from short-term deposits or issuing costly outside equity, thereby endogenising the risk facing the bank on the liabilities side of their balance sheet. We would focus on the banks' portfolio decision on the asset side.

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## A Model Summary

Assume  $u(c, L) = \log c - \chi \frac{L^{1+\zeta_t}}{1+\zeta_t}$ ,

$$w_t = \chi c_t L_t^{\zeta_t} \quad (\text{A.1})$$

$$1 = \beta E_t R_{t+1} \frac{c_t}{c_{t+1}} \quad (\text{A.2})$$

$$y_t = A_t (k_{t-1})^\alpha L_t^{1-\alpha} \quad (\text{A.3})$$

$$w_t = P_{mt} \frac{(1-\alpha)y_t}{L_t} \quad (\text{A.4})$$

$$R_{kt} = \frac{P_{mt} \alpha y_t + (Q_t^k - \delta) k_{t-1}}{Q_{t-1}^k k_{t-1}} \quad (\text{A.5})$$

$$Q_t^k - 1 = F_t' \quad (\text{A.6})$$

$$F_t' = \frac{\sqrt{\mu^k}}{2} \left( \exp \left( \sqrt{\mu^k} \left( \frac{I_t}{k_{t-1}} - \delta \right) \right) - \exp \left( -\sqrt{\mu^k} \left( \frac{I_t}{k_{t-1}} - \delta \right) \right) \right) \quad (\text{A.7})$$

$$I_t = k_t - (1-\delta)k_{t-1} \quad (\text{A.8})$$

$$0 = -\psi \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} + (1-\epsilon)(1-\tau_t) + \epsilon P_{mt} + \beta \psi E_t \frac{u_c(t+1)}{u_c(t)} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1} y_{t+1}}{\pi y_t} \quad (\text{A.9})$$

$$y_t = c_t + k_t - (1-\delta)k_{t-1} + g_t \quad (\text{A.10})$$

$$R_{dt} = (1-\Delta_t) \frac{1 + \rho_d Q_t^d}{Q_{t-1}^d \pi_t} \quad (\text{A.11})$$

$$Q_t^d D_t = g_t - \tau_t y_t + (1 + \rho_d Q_t^d)(1 - \Delta_t) \frac{D_{t-1}}{\pi_t} \quad (\text{A.12})$$

$$\frac{\tau_t}{\tau} = \left( \frac{(1-\Delta_t) D_{t-1}}{D} \right)^{\gamma_d} \quad (\text{A.13})$$

$$\frac{i_t}{i} = \left( \frac{\pi_t}{\pi} \right)^{\kappa_\pi} \quad (\text{A.14})$$

$$i_t = R_{t+1} \pi_{t+1} \quad (\text{A.15})$$

$$0 = \mu_t (f_t N_t - \lambda (Q_t^k k_t + \eta Q_t^d D_t)) \quad (\text{A.16})$$

$$f_t = E_t \beta \frac{u_c(t+1)}{u_c(t)} (1 - \theta_{t+1} + \theta_{t+1} f_{t+1} - (\bar{\theta} - \theta_{t+1}) \Delta_{t+1}^b) R_{t+1}^b + \mu_t f_t \quad (\text{A.17})$$

$$N_t = \theta_t (R_t^k Q_{t-1}^k k_{t-1} + R_t^d Q_{t-1}^d D_{t-1} - R_t^b B_{t-1}) + \omega (Q_t^k k_{t-1} + Q_t^d D_{t-1}) \quad (\text{A.18})$$

$$B_t = Q_t^k k_t + Q_t^d D_t - N_t \quad (\text{A.19})$$

$$\mu_t \lambda = E_t \beta \frac{u_c(t+1)}{u_c(t)} ((1 - \theta_{t+1} + \theta_{t+1} f_{t+1})(R_{t+1}^k - R_{t+1}^b) + (\bar{\theta} - \theta_{t+1}) \Delta_{t+1}^b R_{t+1}^b) \quad (\text{A.20})$$

$$\eta \mu_t \lambda = E_t \beta \frac{u_c(t+1)}{u_c(t)} ((1 - \theta_{t+1} + \theta_{t+1} f_{t+1})(R_{t+1}^d - R_{t+1}^b) + (\bar{\theta} - \theta_{t+1}) \Delta_{t+1}^b R_{t+1}^b) \quad (\text{A.21})$$

$$\log \frac{z_t}{z} = \rho_z \frac{z_{t-1}}{z} + \epsilon_t^z \quad (\text{A.22})$$

$$\theta_t = \frac{\exp(\eta_1^b - \eta_2^b B_{t-1})}{1 + \exp(\eta_1^b - \eta_2^b B_{t-1})} (\bar{\theta} - \theta_{min}) + \theta_{min} \quad (\text{A.23})$$

$$\Delta^b = 0 \quad (\text{A.24})$$

$$\Delta_t = \begin{cases} 0 & \text{if } D_{t-1} < D_t^* \\ \Delta & \text{if } D_{t-1} \geq D_t^* \end{cases}$$

$$p_t^d \equiv P(D_{t-1} \geq D_t^*) = \frac{\exp(\eta_1^d + \eta_2^d D_{t-1})}{1 + \exp(\eta_1^d + \eta_2^d D_{t-1})}, \quad (\text{A.25})$$

## B Nonlinear Model Solution

When solving the nonlinear model, the state space is  $\mathbf{S}_t = \{D_{t-1}, K_{t-1}, B_{t-1}, i_{t-1}, z_t\}$ . Define the decision rules for the labor supply as  $L_t = f^L(\mathbf{S}_t)$ , the inflation as  $\pi_t = f^\pi(\mathbf{S}_t)$ , the end-of-period government debt as  $D_t = f^d(\mathbf{S}_t)$ , the real marginal cost as  $P_t^m = f^{pm}(\mathbf{S}_t)$ , and the banks' value function coefficient  $f_t = f^f(\mathbf{S}_t)$ . The decision rules are solved as follows.

1. Define the grid points by discretizing the state space. Make initial guesses for  $f_0^L$ ,  $f_0^\pi$ ,  $f_0^D$ ,  $f_0^{pm}$  and  $f_0^f$  over the state space.
2. At each grid point, solve the nonlinear model and obtain the updated rules  $f_i^L$ ,  $f_i^\pi$ ,  $f_i^D$ ,  $f_i^{pm}$  and  $f_i^f$  using the given rules  $f_{i-1}^L$ ,  $f_{i-1}^\pi$ ,  $f_{i-1}^D$ ,  $f_{i-1}^{pm}$ , and  $f_{i-1}^f$ :
  - (a) Compute  $y_t, w_t, c_t, K_t, I_t$  for the given state and the function guess using equations (A.3), (A.4), (A.1), (A.10), (A.8), and (A.6).
  - (b) Derive  $Q_t^k, R_t^k Q_{t-1}^k$  using equations (A.7) and (A.5).
  - (c) Update  $i_t$  using equation (A.14).
  - (d) Compute  $\tau_t$  using equation (A.13),  $Q_t^d$  from equation (A.12),  $R_t^d Q_{t-1}^d$  from equation (A.11).
  - (e) Compute  $N_t$  using equation (A.18) and update  $B_t$  using equation (A.19).
  - (f) Use linear interpolation to obtain  $f_{i-1}^L(\mathbf{S}_{t+1})$ ,  $f_{i-1}^\pi(\mathbf{S}_{t+1})$ ,  $f_{i-1}^d(\mathbf{S}_{t+1})$ ,  $f_{i-1}^{pm}(\mathbf{S}_{t+1})$ , and  $f_{i-1}^f(\mathbf{S}_{t+1})$ , and follow the above steps to solve variables at time  $t + 1$ .
  - (g) Use Gauss-Hermite method to compute the expectations in equations (A.2), (A.9), (A.17), (A.20), (A.21), and update the decision rules  $f_i^L$ ,  $f_i^\pi$ ,  $f_i^D$ ,  $f_i^{pm}$  and  $f_i^f$ .
3. Check convergence of the decision rules. If  $|f_i^L - f_{i-1}^L|$ , or  $|f_i^\pi - f_{i-1}^\pi|$ , or  $|f_i^d - f_{i-1}^d|$ , or  $|f_i^{pm} - f_{i-1}^{pm}|$ , or  $|f_i^f - f_{i-1}^f|$  is above the desired tolerance, go back to step 2; otherwise,  $f_i^L$ ,  $f_i^\pi$ ,  $f_i^D$ ,  $f_i^{pm}$  and  $f_i^f$  are the decision rules.

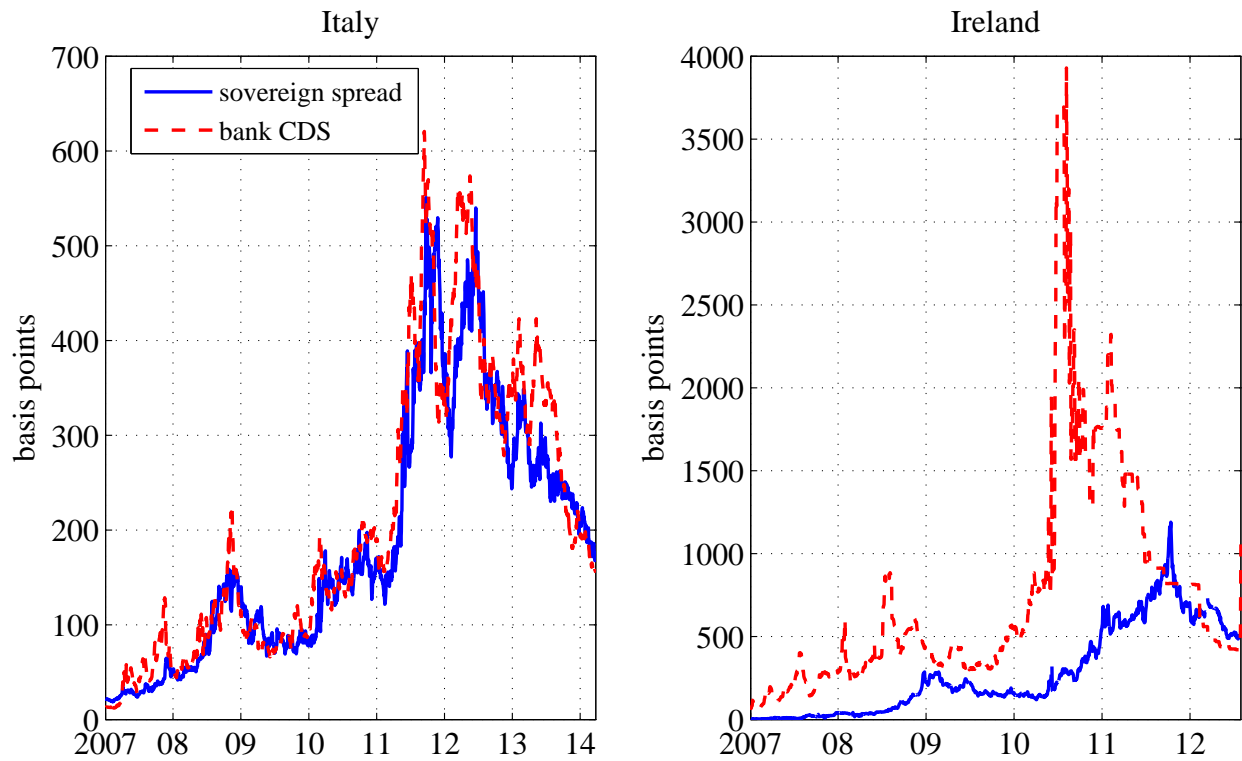


Figure 1: Sovereign vs. bank CDS spread. Left: blue line shows the interest rate spread between 10-year Italian and German government bonds, and the red dashed line shows the average CDS for the two largest Italian banks (UniCredit and Banca Intesa); right: blue line shows the interest rate spread between 10-year Irish and German government bonds, and the red dashed line shows the CDS for the Anglo Irish Bank.



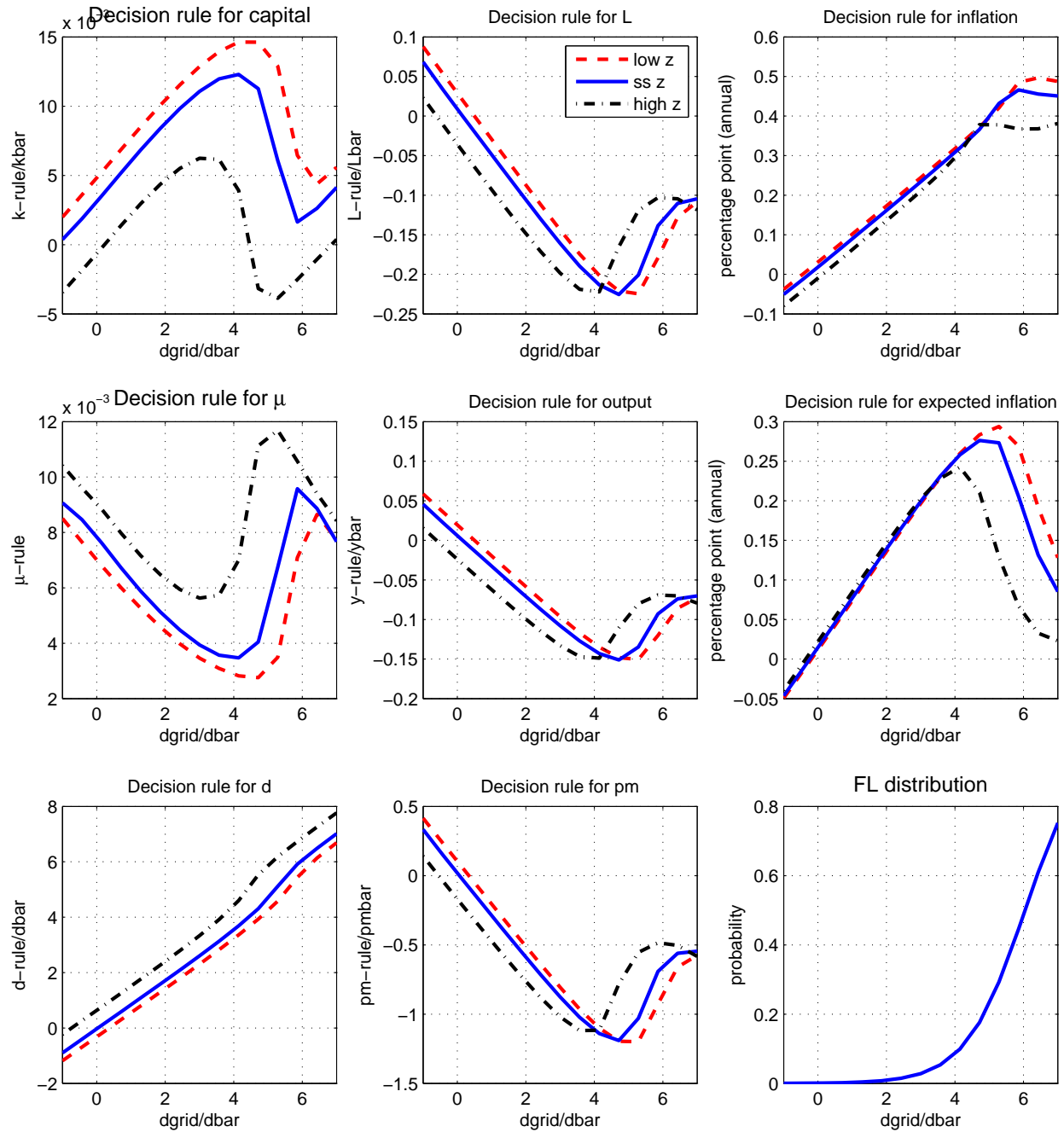


Figure 2: Decision rules in the baseline model without bank runs (deviations from steady states in percentage)

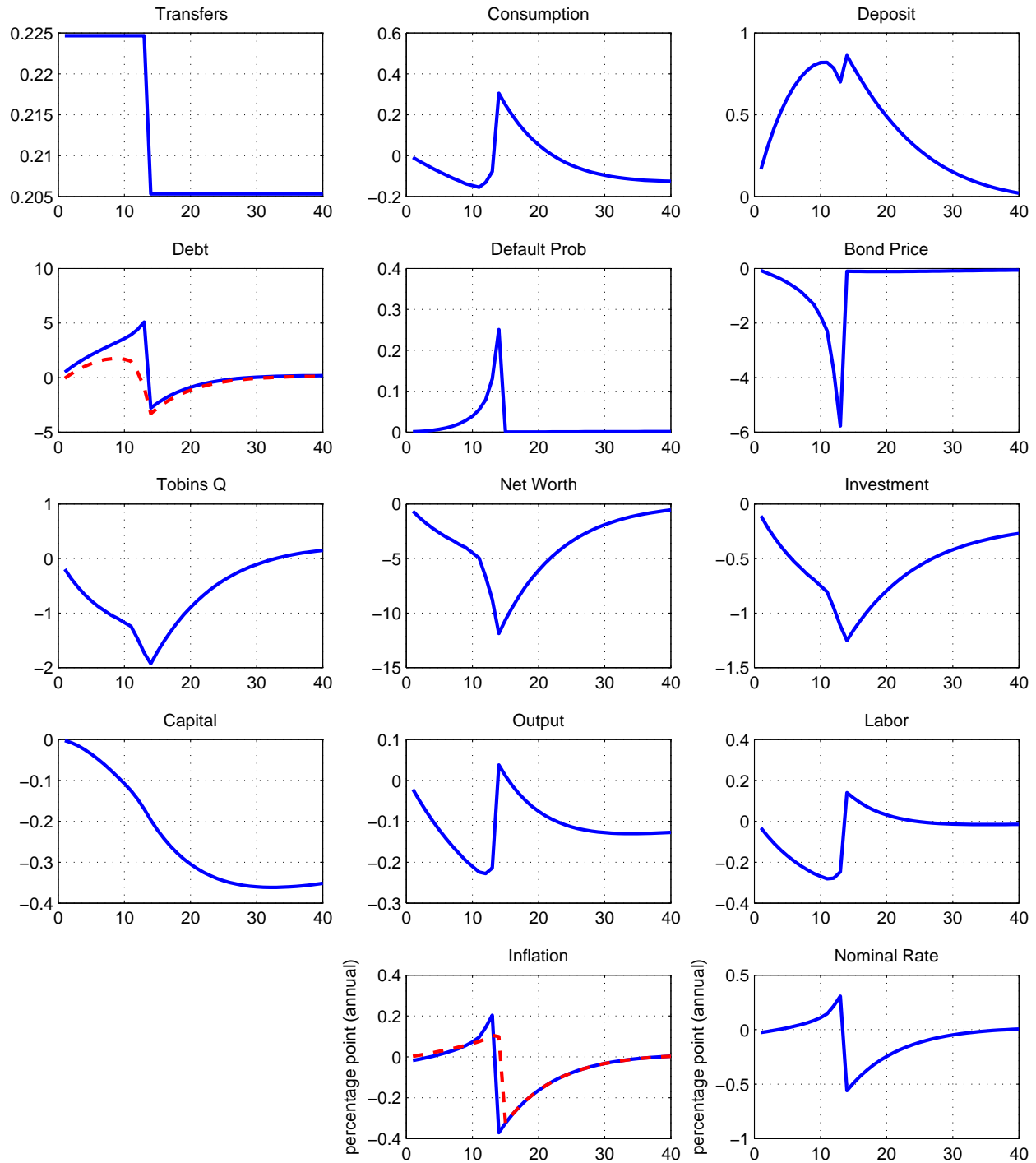


Figure 3: Impulse responses under transfer shocks in the baseline model without bank runs (with default): red dashed line in the debt panel shows the market value of government debt, and that in the inflation panel shows the expected inflation path.

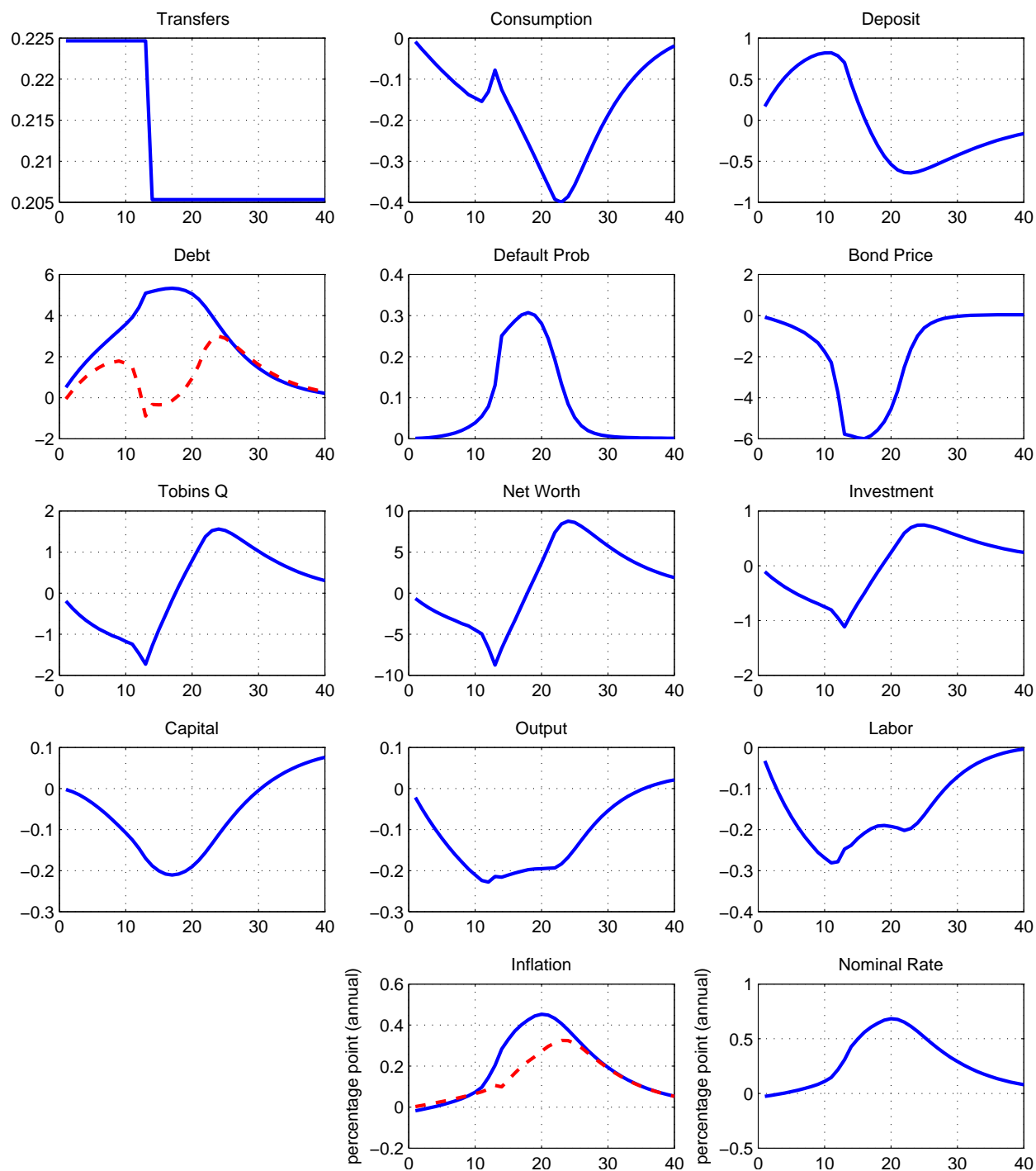


Figure 4: Impulse responses under transfer shocks in the baseline model without bank runs (with no default materializing): red dashed line in the debt panel shows the market value of government debt, and that in the inflation panel shows the expected inflation path.

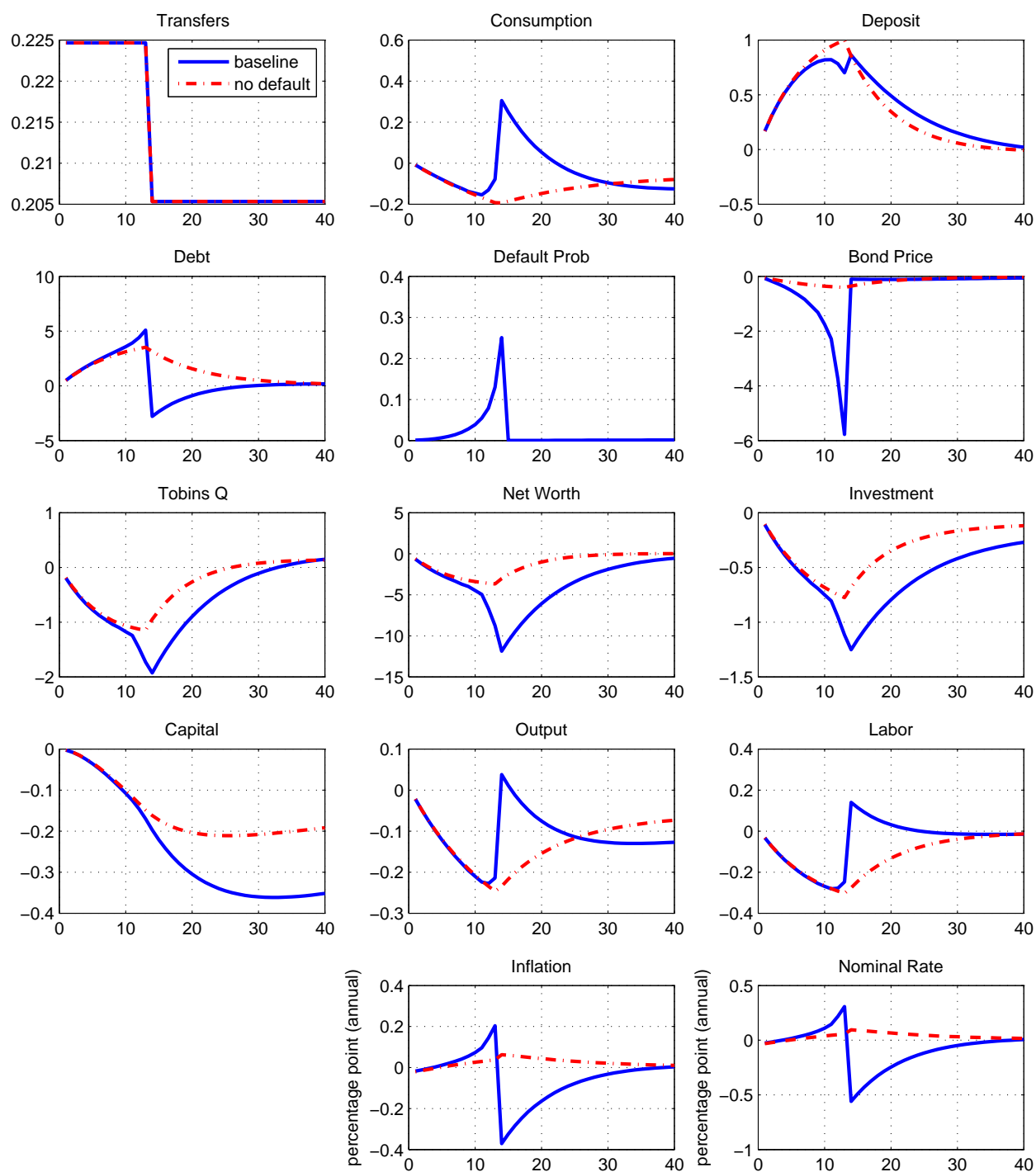


Figure 5: Impulse responses comparisons: baseline model vs. no default model (with default)

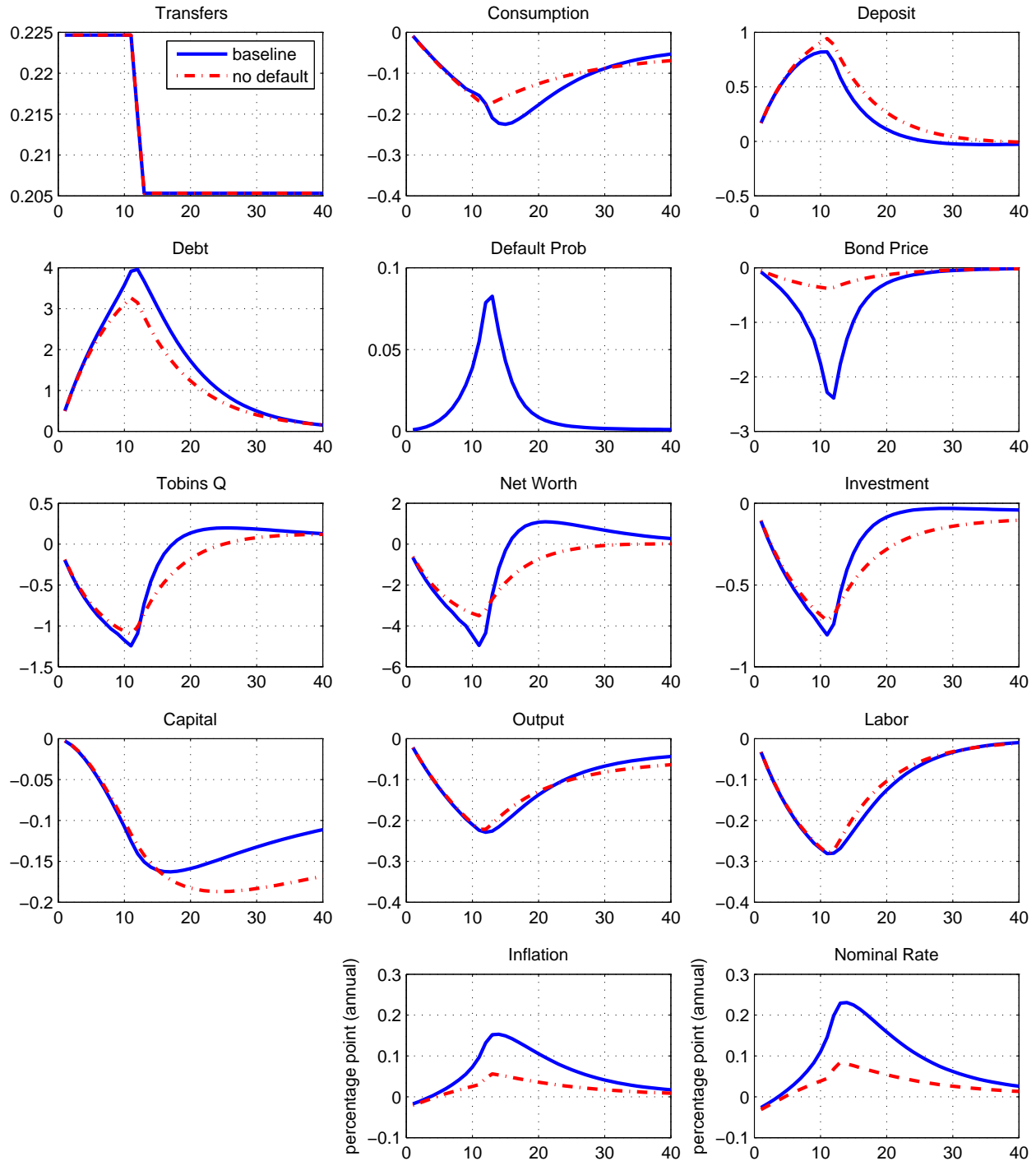


Figure 6: Impulse responses comparisons: baseline model vs. no default model (without default materializing)

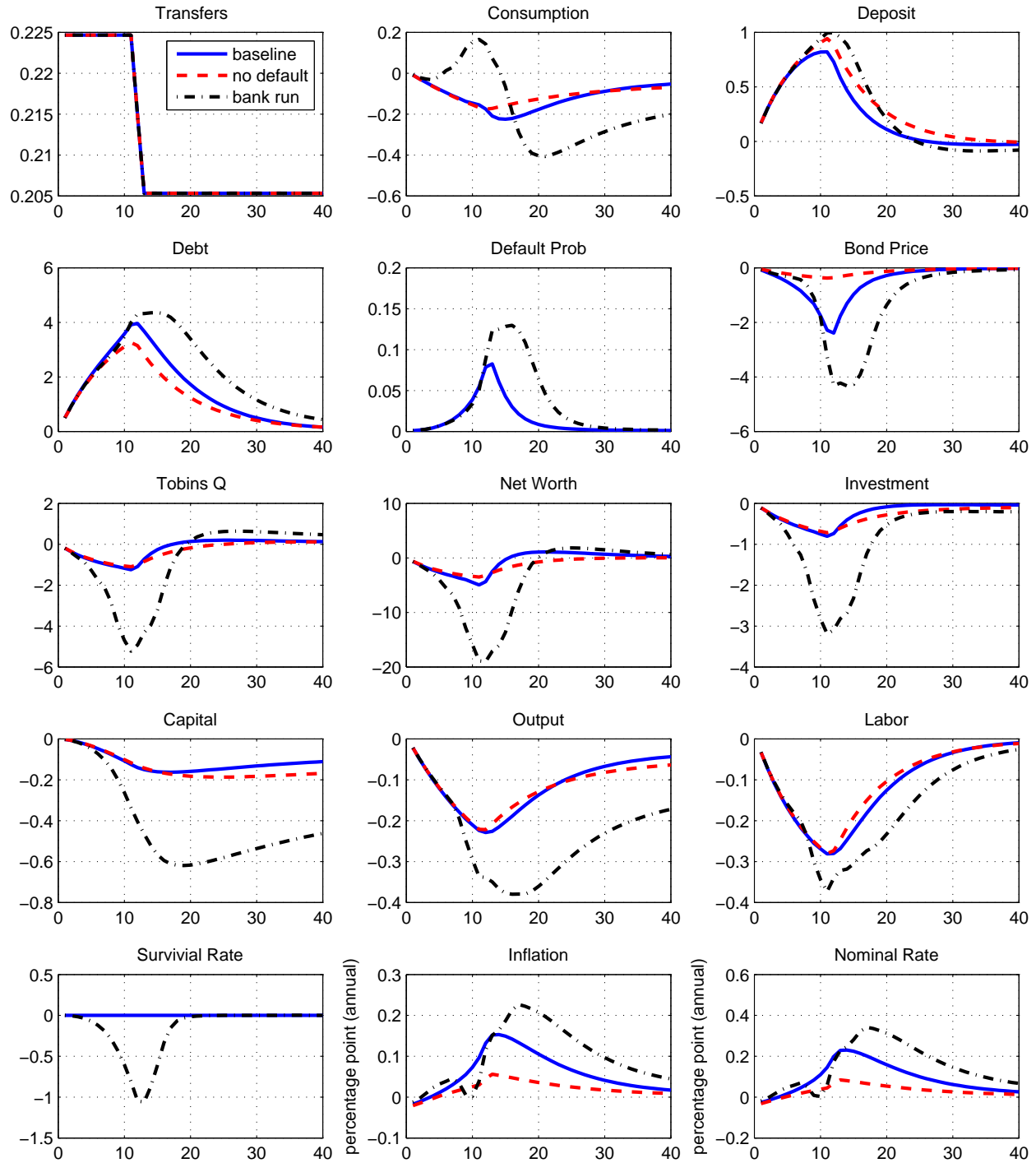


Figure 7: Impulse responses comparisons: baseline model vs. no default model vs. bank run model (without default materializing)