Self-fulfilling Fire Sales: Fragility of Collateralised Short-term Debt Markets

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Abstract

This paper shows that collateralised short-term debt, although optimal to reduce borrower moral hazard, can lead to systemic runs in the debt markets and create endogenous aggregate risk. This is because of a feedback loop between the risk-taking behavior of borrowers (e.g. shadow banks) and the expected price of seized collateral in the secondary market. When the fire-sale price of collateral is expected to be low, lenders demand more collateral (margin) and higher debt yields (repo rates), making it more attractive for borrowers to engage in risk-taking ex-ante (due to limited liability). The riskier pool of projects will lead to more liquidation ex-post and hence more seized collateral to be sold off, justifying the expectation of low fire-sale price. I show that a government commitment to engage in asset purchases in a crisis can improve welfare, and that a ban on the exemption from automatic stay in repo finance can worsen borrower moral hazard and lead to more fire sales.

Keywords: Collateral, Self-fulfilling Fire sales, Repo run, Moral hazard, Optimal contract

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1 Introduction

Financial firms' reliance on collateralised short-term funding such as repurchase agreements (repo) is considered as both a prominent feature and a source of fragility in the modern financial system[1]. These secured loans in this so-called 'shadow banking' system are usually automatically rolled over by creditors in normal times. Yet, the recent crisis has shown that these funding markets can exhibit a 'systemic runs' phenomenon whereby creditors collectively demand tougher borrowing terms or withdraw funding, causing significant distress to the firms and leading to sizeable liquidation of collateral assets at a discount; this phenomenon is commonly known as fire sales[2].

The apparent 'systemic runs' in certain collateralised debt markets however cannot be readily explained by classical bank run models such as Diamond and Dybvig (1983) because the nature of bank debt is different. The first-come-first-served nature of deposit contracts which motivates depositors to front-run each other is absent in repo contracts, for example. As Gorton (2012, p.2) concisely points out:

> '...we know that crises are exits from bank debt... In this form of money (repo), each "depositor" receives a bond as collateral. There is no common pool of assets on which bank debt holders have a claim. So, strategic considerations about coordinating with other agents do not arise. This is a challenge for theory and raises issues concerning notions of liquidity and collateral, and generally of the design of trading securities – private money.'

This paper can be viewed as a response to the above challenge and proposes a new form of coordination failure between *firms* at the ex-ante contracting stage, due to a feedback between the risk-taking incentives of firms and the fire sales of collateral. Under certain conditions, self-fulfilling fire sales and 'systemic runs' can arise.

[1] Adrian and Shin (2011) call this a 'Market-based financial system'. See Brunnermeier (2009) and Krishnamurthy (2010b) for detailed reports on the use of repo and asset-backed commercial paper and how these markets collapsed in the Global Financial Crisis of 2007-2009.

[2] Shleifer and Vishny (2011) survey fire sales in finance and macroeconomic literature. Empirically, He et al. (2010) show that 2007Q4 to 2009Q1, hedge funds and broker dealers reduced holdings of securitised assets by \$800 billion; these assets were mostly absorbed by commercial banks (\$550 bn) and the government (\$350 bn). In terms of liabilities, repo finance shrank by \$1.5 trillion.

I present a three-date, competitive, general equilibrium model of a continuum of firms, each matched with a creditor, and an outside collateral buyer. Each firm is endowed with a divisible asset-in-place which pays a risky dividend at $t = 2$. This asset can be used as collateral to finance an independent, illiquid investment project which becomes successful with some probability and pays a verifiable cash flow at $t = 2$ [3]. Firms are subject to moral hazard problems that at $t = 0$, after borrowing bilaterally from its creditor, each firm privately chooses the success probability of its project by incurring an non-pecuniary effort cost[4]. Pledging collateral to creditors lowers debt yields and thus mitigate firms' incentives to shirk, or equivalently to take on excessive project default risk as in the classic Stiglitz and Weiss (1981) manner. As the creditor is averse to the systematic risk associated with the collateral dividend, effectively values the collateral less than the firm and the collateral buyer, she will seize and liquidate the collateral in a secondary market at $t = 1$ when she knows her firm is insolvent. Finally, the outside collateral buyer is competitive yet capital constrained, hence the market-clearing price of the collateral decreases in the amount of collateral liquidated.

The key novelty of this paper is the feedback between the firms' moral hazard problems and the equilibrium collateral liquidation values which generates a self-fulfilling fire sales phenomenon. When agents expect a lower liquidation value ex-post, creditors require a higher debt yield to break-even. Firms then have to pledge more collateral, or initial margins, in order to maintain incentives; when there is not enough collateral, they engage in more risk-taking. In aggregate, both more pledged collateral and more defaults of firms lead to more collateral being liquidated in the market, resulting in a larger fire-sale discount ex-post. Thus the anticipation of fire sales *causes* fire sales. Figure 1 summarises the phenomenon of *self-fulfilling fire sales* .

The above feedback can be strong enough to produce multiple rational expectation

[3]The model can be seen as a general equilibrium extension of the borrowers with non-project-related collateral model in Tirole (2006, Section 4.3.5) with multiple risk-taking choices and a market for collateral.

[4]The firms' moral hazard problem can also be modelled as risk-shifting as in Jensen and Meckling (1976) with an assumption that the cash flow difference between risky and safe project in the case of success is non-verifiable. In that case collateralised debt will also emerge as the optimal contract. See Acharya and Vishwanathan (2011).

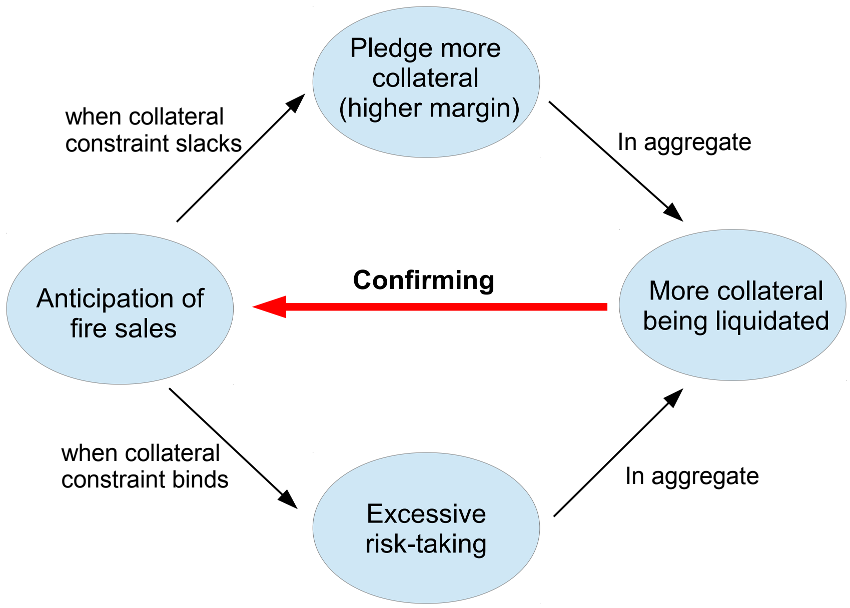


Figure 1: The self-fulfilling fire sales mechanism

equilibria with different collateral liquidation values. There are two (co-existing) channels through which multiple equilibria can arise. First, as discussed above, there exists a threshold of liquidation value below which there is not sufficient collateral to prevent risk-taking. When the equilibrium liquidation value is just above this threshold, a pessimistic expectation of a liquidation value below this threshold triggers firms' risk-taking, which creates a discrete jump in the amount of collateral liquidated as more firms default ex-post, pushing the market-clearing collateral liquidation value below this threshold. I call this the *risk-taking channel*.

Self-fulfilling fire sales can also arise purely from firms' margin decisions. A lower expected collateral liquidation value requires firms to pledge more collateral; as such, in aggregate more collateral is supplied in the market even when firms' default risks remain unchanged. If the market-clearing price function is sensitive enough in the relevant range, multiple equilibria emerge through this *margin channel*.

To the best of my knowledge, this self-fulfilling fragility due to the feedback between endogenous risk-taking and collateral fire sales in the *absence* of aggregate shock has not been documented in the literature previously[5]. This mechanism generates a *systemic run* phenomenon in the collateralised debt market which is different from classic bank run and financial market run models. The source of fragility in this paper stems from a coordination failure between firms with their ex-ante risk-taking and collateral margin decisions, as opposed to depositors' withdrawal decision within a bank or traders' asset liquidation decision in a market at the interim date under a *de facto* sequential service constraint[6]. The coordination failure here operates through the two channels described above: higher default risk or a higher initial margin chosen by an individual firm increases the expected amount of collateral liquidated in the market ex-post. Due to the limited liquidity in the secondary market, this extra supply of collateral marginally lowers the liquidation value, which in turn tightens other firms' ex-ante incentive constraints under rational expectation, requiring them to pledge more collateral or take on excessive risk. As a result, in general equilibrium, firms' risk-taking and margin decisions become *strategic complements* due to the joint effect of the firms' incentive constraints and the fire-sale externality in the collateral market.

While the model applies to any situation with multiple borrowing firms and a illiquid collateral market in general, the opaque operations of financial firms such as hedge funds and their reliance on collateralised borrowing make risk-taking concern particularly relevant[7]. In addition, the substantial and contemporaneous increase in debt yields, borrowers' counter-party risk and collateral spreads during the recent crisis in the wholesale funding

[5]The margin channel here is similar to the margin spiral in Brunnermeier and Pedersen (2009) while in their model the margins are exogenous and the seed of fragility is an unanticipated, large aggregate shock on asset values.

[6]For instance, in Morris and Shin (2004) a market maker executes sellers' aggregate sell orders sequentially at decreasing prices and a seller's place in the queue for execution is randomly distributed. He and Xiong (2012) provides a recent dynamic bank run type model with coordination failure of roll-over decisions among asset-backed commercial paper holders.

[7]For evidence regarding risk-taking behavior of other financial firms, Becker and Ivashina (2013) and Kacperczyk and Schnabl (2013) document a 'reach-for-yield' phenomenon in insurance companies and money market mutual funds respectively.

markets is consistent with the feedback mechanism between endogenous risk-taking and collateral fire sales in the model[8].

In terms of welfare and policy implications, equilibria with lower collateral liquidation values are less efficient due to firms' inefficient investment decisions, credit rationing, and the inefficient transfer of collateral from firms to creditors. The self-fulfilling nature of the fragility suggests that central banks can *reduce* firms' risk-taking incentives and make the financial system more robust through an ex-ante commitment to intervening in the collateral market, which is opposite to the collective moral hazard concern of bailout and government intervention as noted in Acharya (2009) and Farhi and Tirole (2012). Policies such as asset price guarantee can eliminate the agents' pessimistic expectations and thus the inefficient equilibria. This is in line with the idea that central banks should act as a 'Market Maker of Last Resort' (Buiter and Sibert (2007)) to safeguard the proper functioning of certain key collateral and wholesale funding markets[9].

I conclude the paper with a discussion of the potential unintended consequences of policies to limit post-default fire sales. In the U.S. when firms file for bankruptcy, a provision known as 'automatic stay' prevents creditors from demanding repayments. Repo contracts in practice are usually exempted from automatic stay so that repo lenders can immediately access the collateral. Critics of exemption from automatic stay have argued that it has precipitated the fire sales of collateral during a crisis. While this paper also features potential disorderly fire sales, I find that the ban of stay-exemption may backfire. This is because without stay-exemption, defaulted firms can renegotiate with creditors ex-post to lower the promised repayment amount of collateral by threatening to file for bankruptcy and delay the transfer of collateral. As the creditors value the immediate access and liquidation of the collateral, they will accept the offer. This renegotiation problem thus reduces the amount of *credibly* pledgeable collateral and worsens the firms' ex-ante moral hazard

[8] Gorton and Metrick (2012) and Covitz et al. (2013) find significant spikes and volatility in repo rates and ABCP yields in private-label asset-backed-securities markets during the recent crises which correlate positively with proxies for counter-party risks such as the LIBOR-OIS spread.

[9] In a 'longer term' model with endogenous production of collateral, this asset price guarantee policy could encourage the over production of collateral with deteriorating quality. The usual moral hazard concern of government guarantee will hence kick in again.

problem. In short, limiting post-default fire sales can exacerbate the ex-ante risk-taking problem, leading to more pre-default fire sales and dry-ups of some low quality collateral markets.

Related Literature My paper first relates to the recent literature on the fragility of collateralised debt market. Martin et al. (2014) build an infinite-horizon Diamond-Dybvig model with an asset market and characterise liquidity, collateral, and asset liquidation constraints under which banks can ward off an unexpected systemic run by depositors in all banks in the steady-state. Their fragility hence stems from the sequential-service constraint faced by the depositors and an unanticipated aggregate shock to collateral value. In contrast I show the anticipation of fire sales can interact with firms' moral hazard problems and cause fragility.

Models on the use of collateral to mitigate borrowers' moral hazard and adverse selection problems go back to Chan and Thakor (1987) and Besanko and Thakor (1987). See Coco (2000) for a survey. The main difference in my model is that I allow endogenous collateral fire sale discount to study the feedback between firms' moral hazard problems and collateral fire sales. Hombert (2009) also studies a similar feedback but he assumes the solvency of firms are publicly observed so that firms with successful projects can expand and purchase collateral from insolvent firms. In contrast to this paper, he shows that fire sales discourage risk-taking. I assume that solvency of a firm is only observed by its creditor with limited capital thus it is difficult for solvent firms to expand at interim. This adverse selection in the market is arguably more natural for opaque financial firms.

My paper belongs to the self-fulfilling financial crisis literature. Malherbe (2014) shows how liquidity dry-up due to adverse selection can arise from ex-ante self-insurance motives of liquidity hoarding. **?** shows in a bad aggregate state, systemic failure in the banking system can arise because banks scramble for deposits by raising interest rate which in turns causes more bank failures and further liquidity shortage. In a financial market run context, Morris and Shin (2004) shows how loss-limit constraints on traders' position can trigger coordinated liquidation[10]. My paper contributes to the above literature by highlighting a

[10] For demand-deposit based bank runs models, see Diamond and Dybvig (1983) Rochet and Vives (2004), Goldstein and Pauzner (2005).

new type of coordination failure from firms' investment and contracting decisions.

The negative feedback spiral in this paper is similar in spirit to the ones in the literature on asset pricing with constraints. For example, Brunnermeier and Pedersen (2009) and Danielsson et al. (2011) show the existence of an amplifying feedback loop between anticipated and realised asset price volatility when financial institutions operate under a Value-at-Risk constraint. Gromb and Vayanos (2002) and Vayanos (2004) study models with limits to arbitrage due to margin and agency constraint. Building on these insights, Krishnamurthy (2010a) also proposes an asset price guarantee policy to stabilise the asset market. Most of these papers take the constraints as given and focus on the asset pricing and portfolio allocation implications when an exogenous aggregate shock hits. This paper in contrast endogenises the collateralised debt contracts and margin constraints, and the source of risk comes from the endogenous risk-taking of firms.

This paper also relates to the vast literature on the consequences of short-term debt and asset fire-sales. Diamond and Rajan (2011) demonstrate that distressed banks financed with deposit will gamble for resurrection and take the excessive risk of forced liquidation when an aggregate shock hits in the future. Outside collateral buyers who anticipate this fire sales hoard liquidity for asset purchase, leading to a reduction in lending to real sector. Stein (2012) assumes a 'money-like' premium in lenders' preferences for absolutely safe contract and shows that firms tend to create too much safe asset by excessive short-term borrowing and fails to internalise the fire-sale externality when aggregate shocks hit. Eisenbach (2011) shows that the existence of aggregate uncertainty distorts the disciplining effect of short-term debt, and creates inefficiency in both good and bad states. Acharya et al. (2011) show roll-over risk of short-term debt can cause credit market freeze when bad news hits. My work complements the above literature by showing that the expectation of fire sales can interact with borrowers' risk-taking incentives to generate aggregate risk.

2 Model: feedback between risk-taking and fire sales

In this section I first give an overview of the model. Then I analyse the firm-creditor contracting problem at the initial stage $t = 0$ and describe creditors' liquidation decisions and the collateral market at $t = 1$.

2.1 Overview of the model

Consider a three-date ($t = 0, 1, 2$) model with a continuum of borrowing firms each matched with a corresponding creditor, and a representative outside collateral buyer. There is a storage technology with returns normalised to zero.

Firms and projects Firms are risk-neutral, identical ex-ante, and each has a unit of common asset-in-place (collateral) with no cash and debt. At $t = 0$ each firm has the opportunity to invest in a project which requires an initial investment of \$1 and will return a verifiable cash flow X in the case of success and X_f otherwise at $t = 2$. Without loss of generality I normalise X_f to 0. Firms are subject to a moral hazard effort-provision problem as in Holmström and Tirole (1997). The success probability of the project depends on the unobservable effort exerted by the firm after financing the project. Effectively the firm can choose the success probability of the project $p_1 > p_2 > p_3$ by incurring a private effort cost $c(p_i) \geq 0$. Shirking here is thus interpreted as risk-taking. Project risk is idiosyncratic, and the realisation of projects is therefore independent across firms.

Collateral assets and financing Aside from the investment opportunity, each firm has one divisible unit of asset (e.g. financial securities) which pays a random, non-negative dividend $\tilde{v}$ with expected value v at $t = 2$. The dividend risk is uncorrelated with the project. The asset is also independent of the operation of the project and can be used as collateral for borrowing. I assume this collateral dividend $\tilde{v}$ to be non-verifiable. As such, the firm can effectively choose $k \in [0, 1]$ fraction of the collateral to pledge to the creditor at the ex-ante contracting stage and keep the remaining $(1 - k)$ fraction beyond the creditor's reach. To fix idea, one can think of a shadow bank who can secretly move assets on and off balance sheet unless the assets are explicitly pledged. While the flexibility to choose k is not crucial to the main result of this paper, this allows me to endogenise the optimal amount of pledged collateral, or initial margin, in the financing contract.

Firms borrow in the form of collateralised short-term debt contract. Specifically, a firm borrows \$1 from its creditor and promises to repay r at $t = 1$ and immediately transfer $k \in [0, 1]$ measure of the collateral to the creditor if repayment is demanded at $t = 1$ and the firm fails to repay. This contract resembles a repurchase agreement (repo) as commonly used in practice, r as the repo rates and k as the initial margin. In Section 6, I will discuss the optimality of such a contract and its implementation.

I will make the following assumptions about the net present value and the degree of moral hazard of the project:

Assumption 1 *(NPV and moral hazard intensity of the project) Define* $NPV_i \equiv p_i X - 1 - c(p_i)$, $\Delta p_i \equiv p_i - p_{i+1}$, $\Delta c_i \equiv c(p_i) - c(p_{i+1})$, *and* $A_i \equiv 1 - p_i(X - \frac{\Delta c_i}{\Delta p_i})$ *for* $i = 1, 2$

(i) $NPV_1 \geq NPV_2 > 0 > NPV_3$

(ii) $A_1 > A_2 > 0$ *and*

(iii) $(1 - p_1)A_1 \leq (1 - p_2)A_2$

Assumption 1 is there to preserve the efficiency ranking of actions and at the same time allows risk-taking to arise in equilibrium. Assumption 1(i) implies that prudent investment (p_1) is the efficient action but risk-taking (p_2) is also profitable. Part (ii) and (iii) are about the magnitude of the moral hazard problem, i.e. the absolute and relative size of $\frac{\Delta c_i}{\Delta p_i}$. A_i is the value of collateral required to induce action p_i when the firm and creditor value the collateral symmetrically and (ii) implies that the project cannot be funded without collateral as the firm will choose the negative NPV action (p_3) after financing ($A_i > 0$) and more collateral is needed to induce prudent investment $A_1 > A_2$[11]. Finally the collateral is transferred to the creditor when the project fails with probability $(1 - p_i)$ and (iii) implies that the expected value of collateral lost is weakly lower in the case of prudent investment. Although losing the collateral to the creditor in the case of symmetric valuation is costless, (iii) ensures that p_1 is always the preferred and efficient action even if the creditor values the collateral less because p_1 entails a higher NPV and a smaller expected collateral loss.

Creditors' rollover and collateral liquidation decision At $t = 0$ each firm is matched with a creditor who has cash \$1 to lend. After the financing, at $t = 1$ each creditor

[11]To see why the project cannot be funded without external collateral, the minimum repayment to the creditor is $r_i = 1/p_i$ if p_i is chosen. However, the firm would privately choose the negative NPV action p_3 after financing as $p_3(X - r_i) - c_3 > p_i(X - r_i) - c_i$ when $A_i > 0$

receives a private, non-contractible signal about the success or failure of her borrowing firm's project, that is, whether cash flow X or 0 will realise at $t = 2$. I assume the signal is perfect and hence the creditor essentially observes the solvency of the firm she financed. If the project has succeeded, the creditor is willing to roll over her short-term debt to $t = 2$ at the yield r as she knows she will be repaid for sure. When the project fails, the creditor demands repayment and as the insolvent firm cannot repay, the creditor seizes the collateral asset and could potentially sell it on the market[12]. I assume creditors value the collateral less than the firms and the collateral buyer, thus creating a motive for them to sell the collateral at a discount.

Assumption 2 *Creditors' expected utility derived from holding the collateral to $t = 2$ is $\underline{l} \leq v$, i.e. less than the firms' and the collateral buyer's valuation.*

Effectively creditors are averse to the collateral dividend risk and $\underline{l}$ can be understood as their certainty equivalent of the risky dividend. Hence they prefer selling the collateral on the market as long as the market clearing price is above $\underline{l}$. The wedge between the creditors' and the collateral buyer's valuation of the collateral $(v - \underline{l})$ can be motivated by the creditors' lack of expertise in managing the systematic risk associated with the collateral or (indirect) holding cost stemming from tougher regulatory constraints on creditors[13]. As such, from an ex-post perspective, fire sales are an *efficient* transfer of collateral.

I will interpret $\underline{l}$ as the *collateral quality*. For example safe collateral such as U.S. Treasuries will have a high $\underline{l}$ close to v and the creditor can hold such collateral to maturity with minimal cost or limitation. In Section 5, I discuss how collateral quality affects fragility and amplifies risk in the financial system.

The assumptions of a perfect signal and ex-post efficient fire sales shut down other sources of inefficiency stemming from the wrongful liquidations of successful project or a

[12] Rolling over a failed firm and receiving the collateral risky dividend at $t = 2$ is a weakly dominated strategy for the creditor because seizing the collateral at $t = 1$ gives her the option to sell the collateral in the market.

[13] For example, money market mutual funds are typical lenders in the wholesale funding markets and they are subject to the regulation of Rule 2a-7 of the Investment Company Act of 1940 on the amount of holdings of assets with particular rating and maturity. See Kacperczyk and Schnabl (2013).

coordination failure between creditors trying to front-run each other in the collateral market as in Morris and Shin (2004) and Bernardo and Welch (2004). This allows me to focus on the inefficiency of the coordination failure of firms' ex-ante investment and contracting decisions, which is the main result of this paper.

Collateral buyer and endogenous fire sale discount The final element in the model is illiquidity in the collateral market. At $t = 1$, there is a competitive risk-neutral outside investor who clears the collateral market. However, he has limited capital in the sense that instead of holding cash to purchase the collateral at $t = 1$, he could have invested in a productive technology with decreasing returns to scale which pays off at $t = 2$. I assume the output of this productive technology is non-verifiable and thus creditors cannot directly lend to the collateral buyer. Similar assumptions of a patient investor or outside liquidity provider can be found in Diamond and Rajan (2011), Stein (2012), and Bolton et al. (2011).

As a result the market-clearing price for the collateral offered by the buyer at $t = 1$, denoted by $L(\phi; \theta)$, decreases in the amount of collateral sold ϕ and increases in the amount of the buyer's available capital θ. Further discussion on the properties and micro-foundation of the function $L(\phi; \theta)$ will be put forward in Section 2.3. The amount of the collateral buyer's capital θ is an exogenous parameter and common knowledge in the model, and is thus *not* a source of aggregate risk.

A time-line summarising the sequence of events is available in the Appendix.

2.2 Firms' investment problems: from fire sales to risk-taking

In this section I analyse the ex-ante contracting problem between a firm and its creditor at $t = 0$ while taking the equilibrium collateral liquidation value l as given. Each firm offers a collateralised short-term debt contract to its creditor to raise \$1 for investing in a project. More specifically, the firm promises to repay a r (or gross debt yields r) and should the creditor demand repayment at $t = 1$ (i.e. does not roll over the debt) but the firm fails to repay, the creditor can seize $k \in [0, 1]$ fraction of the collateral asset. This contract is superior to a long-term debt and demanding repayment dominates rolling over a failed firm because a creditor receiving the collateral at $t = 1$ has the option to sell it on the market for l, potentially higher than the utility $\underline{l}$ derived from holding it and getting the risky dividend at $t = 2$. After signing a contract $\{r, k\}$, the firm privately chooses the success probability of the project to maximise its expected net payoff from investing:

$$p(r,k) \equiv \underset{p \in \{p_1, p_2, p_3\}}{\operatorname{argmax}} \quad p(X - r) - (1 - p)kv - c(p) \tag{1}$$

which is the expected residual cash flow from the project minus the expected loss of collateral and effort cost. The incentive compatible action $p(r, k)$ for a given contract can be expressed as follows:

$$(IC) \quad p(r,k) = \begin{cases} p_1 & \text{for} \quad r \leq \bar{r}_1(k) \\ p_2 & \text{for} \quad r \in (\bar{r}_1(k), \bar{r}_2(k)] \\ p_3 & \text{otherwise} \end{cases} \tag{2}$$

$$\text{where } \bar{r}_i(k) \equiv X - \frac{\Delta c_i}{\Delta p_i} + kv \quad \text{for } i = 1, 2 \tag{3}$$

Equation (2) shows that when the promised repayment r, or debt yields, is higher than certain thresholds $\bar{r}_i(k)$, the firm chooses to take more risk. Pledging more collateral (higher k) increases those thresholds as seen in Equation (3) and thus discourages risk-taking because the firm loses more collateral when the project fails. Note that in equilibrium p_3 could not be chosen as investing is a negative NPV action in that case.

The contract offered has to satisfy the creditor's participation constraint. For a given equilibrium collateral liquidation value l, the creditor accepts the contract when

$$(PC) \quad \hat{p}r + (1 - \hat{p})kl \geq 1 \tag{4}$$

where $\hat{p}$ is the creditor's conjectured project success probability. In the case of failure, the creditor receives measure k of the collateral which is worth $l \in [\underline{l}, v]$ to her in equilibrium.

Knowing the firm's incentive compatibility constraint, the creditor can rationally anticipate the firm's risk-taking decision by looking at the contractual terms $\{r, k\}$. Thus the creditor's conjectured probability $\hat{p}$ is always correct in equilibrium, i.e.

$$(RE) \quad \hat{p} = p(r, k) \tag{5}$$

Finally, since pledging collateral to invest risks losing the collateral, the firm would choose to undertake the project only if the expected net payoff of investing is positive. This project-taking (PT) constraint can be written as

$$(PT) \quad U(l) \equiv \max_{\{r,k\}} p(r,k)(X-r) - (1-p(r,k))kv - c(p(r,k)) \geq 0 \tag{6}$$

where $U(l)$ is the maximised (indirect) net utility from investing for a given equilibrium collateral liquidation value l when the firm offers the optimal collateralised short-term debt contract $\{r, k\}$.

Formally the firm offers a contract $\{r, k\}$ to the creditor which solves the following optimisation problem:

$$\max_{\{r,k\}} p(X-r) - (1-p)kv - c(p)$$
$$\text{subject to } (IC), (PC), (RE) \text{ and } (PT)$$

and $k \in [0, 1]$ and $r \geq 0$. In the case of no solution, the firm chooses not to invest in any project.

Before proceeding to the firm's optimal investment decision and financing contract, I will first state some parameter assumptions on the expected value of the collateral v and the NPV of risk-taking, to make the analysis interesting. I will discuss the role of these parameter restrictions after the discussion of Proposition 1. Detailed derivations can be found in the Appendix.

Assumption 3 *(Parameter assumptions on v and the NPV of risk-taking)*

(i) $v \in (A_1, \bar{v})$ *where* $\bar{v} = \dfrac{A_1}{1 - [(1-p_1)(NPV_2)]/[(1-p_2)(A_2 + NPV_2)]}$

(ii) $NPV_2 \leq \min\{v - A_2, \dfrac{1-p_2}{p_2} A_2\}$

Proposition 1 *(Fire sales induce a higher margin or more risk-taking) When Assumptions 1 and 3 hold, there exist two critical values l_{CR}, l_{RT} where $0 \leq l_{CR} < l_{RT} < v$ such that for any given equilibrium collateral liquidation value l, the firm's optimal investment decision $p^*(l)$ and the corresponding contract $\{r(l), k(l)\}$ are as follows:*

1. *for* $l \in [l_{RT}, v]$*, the firm invests prudently* $(p^*(l) = p_1)$*and promises debt yield* $r_1(l)$ *and pledges* $k_1(l)$ *fraction of the collateral;*

2. *for* $l \in (l_{CR}, l_{RT})$*, the firm engages in risk-taking and promises debt yield* $r_2(l)$ *and pledges* $k_2(l)$ *fraction of the collateral;*

3. *for* $l = l_{CR}$*, the firm engages in risk-taking with probability* $\lambda \in [0,1]$ *and forgoes the project with probability* $(1-\lambda)$*;*

4. *for* $l < l_{CR}$*, the firm forgoes the investment project* $(p^*(l) = \emptyset)$ *(Credit Rationing)*

The optimal margin and debt yield are

$$k_i(l) = \frac{1 - p_i(X - \Delta c_i/\Delta p_i)}{p_i v + (1-p_i)l} \quad , \quad r_i(l) = \bar{r}_i(k_i(l)) = X - \frac{\Delta c_i}{\Delta p_i} + k_i(l)v \tag{7}$$

and l_{CR} *and* l_{RT} *are implicitly defined in* $U(l_{CR}) = 0$ *and* $k_1(l_{RT}) = 1$*.*
Proof: *See Appendix.*

Proposition 1 demonstrates the first half of the feedback loop in Figure 1: anticipation of a lower collateral liquidation value requires the firm to pledge more collateral or take on excessive risk when there is not enough collateral. The key intuition behind this result is that pledging collateral is costly to the firm but good for incentive and there is a finite amount of collateral. The firm in general can repay the creditor in the form of either collateral or future cash generated from the project, but cash is the preferred option because the creditor values the collateral less than the firm in equilibrium ($l \leq v$). As shown in Equation (2), the maximum repayment the firm can promise without triggering risk-taking is $r = \bar{r}_1(k)$, which increases with the amount of collateral pledged k. As such, in order to satisfy the creditor's participation constraint under a given liquidation value l, the minimal amount of collateral required to be pledged is $k_1(l)$ which satisfies

$$p_1 \bar{r}_1(k_1(l)) + (1-p_1)k_1(l)l = 1$$

and $k_1(l)$ and $r_1(l) = \bar{r}_1(k_1(l))$ are defined in Equation (7). When the liquidation value l decreases, $k_1(l)$ has to increase in order to preserve incentive and satisfy the creditor's participation constraint.

When the liquidation value is high ($l \geq l_{RT}$), the firm can pledge enough collateral $k_1(l) \leq 1$ to induce prudent investment. When l decreases below l_{RT}, implicitly defined in $k_1(l_{RT}) = 1$, even pledging all the collateral cannot simultaneously satisfy the creditor's participation constraint and induce prudent investment, that is, the debt yield required for the creditor to break-even under the prudent investment is too high, i.e.,

$$r = \frac{(1-p_1)l}{p_1} > \bar{r}_1(1)$$

Consequently, for $l < l_{RT}$, risk-taking p_2 is the only feasible action. In this case the firm promises a higher debt yield $r_2(l)$ but still needs to pledge $k_2(l) < 1$ collateral in order to commit to not privately choosing negative NPV action p_3 after financing.

Since risk-taking entails a smaller NPV and a larger expected fire sales cost due to a higher default risk as compared to the prudent investment, the firm would choose to forgo the investment when l is low enough, which I interpret as credit rationing. To see this, the firm's maximised net payoff from investing is

$$U(l) = \underbrace{p^*(l)X - c(p^*(l)) - 1}_{\text{NPV from investment}} - \underbrace{(1-p^*(l))k(l)(v-l)}_{\text{Expected fire-sale cost}} \tag{8}$$

which is decreasing in l. Hence there exists a l_{CR} such that the surplus generated from the project equals the expected loss from collateral fire sales, i.e. $U(l_{CR}) = 0$. The firm thus optimally forgoes the investment when $l < l_{CR}$. Finally, the firm is indifferent between no investment and risk-taking at l_{CR} and therefore plays a mixed strategy. The probability of taking on the project is denoted by $\lambda \in [0,1]$ which will be pinned down in the general equilibrium.

Let me briefly discuss the role of Assumption 3. The first part regards the expected value of the collateral $v \in (A_1, \bar{v})$ to allow both prudent investment and risk-taking to arise in equilibrium. When v is low enough, there is insufficient collateral to implement prudent investment whereas with a high enough v, the collateral constraint binds *after* risk-taking becomes unprofitable, i.e. $l_{RT} < l_{CR}$, thus ruling out the possibility of risk-taking. Assumption 3 (ii) ensures risk-taking to be not too profitable otherwise credit rationing will not occur even when the expected fire-sale cost is maximal.

To sum up this subsection, Figure 2 graphically summarises the firm's optimal investment decision $p^*(l)$.

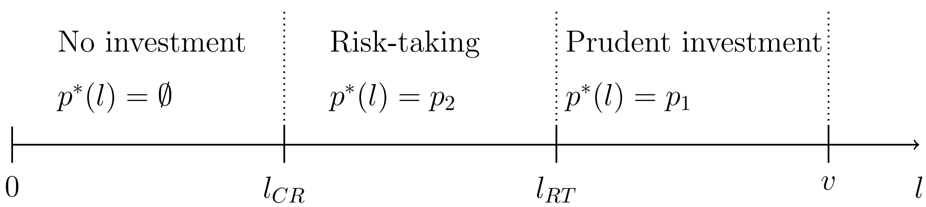


Figure 2: The firm's optimal investment decision at different collateral liquidation value l. Mixed strategies are played at the critical thresholds l_{CR}

2.3 Collateral market: from risk-taking to fire sales

In this section I will describe the supply and demand of the repossessed collateral asset and the determination of its market-clearing price. There is a competitive collateral buyer with capital $\theta \in [0, +\infty)$ to clear the collateral market at $t = 1$. At $t = 0$ he also has an opportunity to invest in a productive technology with decreasing returns to scale that produces gross return $F(\theta)$ at $t = 2$ where $F(0) = 0$, $F''(\theta) < 0$, $\lim_{\theta \to 0^+} F'(\theta) \to +\infty$ and $F'(\hat{\theta}) = 1$ for some $\hat{\theta} > 0$. Augmented with the storage technology which always returns 1, the investment opportunity gives $F''(\theta) = 0$ and $F'(\theta) = 1$ for $\theta \geq \hat{\theta}$. The output of this technology is assumed to be non-verifiable and he therefore cannot compete with the firms to raise capital from the creditors.

These conditions imply that for the buyer to hoard liquidity I for asset purchase at $t = 1$, he has to forgo some productive investment and thus liquidity carries a premium when $\theta - I < \hat{\theta}$. As the buyer behaves competitively, he takes the collateral liquidation value l as given and optimally hoards liquidity I to maximise his net payoff:

$$\Pi(l) \equiv \max_{I \in [0,\theta]} F(\theta - I) + I\frac{v}{l} - \theta \tag{9}$$

and the first order condition is

$$F'(\theta - I^*) \geq \frac{v}{l} \quad \text{with strict equality for } I^* > 0 \tag{10}$$

That is, the marginal return of investing in the productive technology has to equal to that of collateral purchase should the buyer decides to participate in the collateral market. For any given amount of liquidated collateral in the collateral market $\phi \in [0, 1]$ at $t = 1$, the market-clearing condition requires $I^* = \phi l$. Thus for $\phi > 0$, $I^* > 0$, and by substituting

ϕl into the first order condition, one can re-write the liquidation value l as a function of ϕ and θ, that is, $L(\phi;\theta) \in (0, v]$. The following lemma summarises the properties of this market-clearing collateral liquidation value function.

Lemma 1 *(Market-clearing pricing function for collateral $L(\phi;\theta)$) For a given collateral supply $\phi \in (0, 1]$ and the collateral buyer's capital $\theta \in [0, +\infty)$, $L(\phi;\theta)$ satisfies*

(i) $\frac{\partial L}{\partial \phi} \leq 0$

(ii) $\frac{\partial L}{\partial \theta} \geq 0$

(iii) $\lim_{\theta \to 0} L(\phi;\theta) \to 0$ *and for* $\theta \geq \hat{\theta} + v$, $L(\phi;\theta) = v$.

and $L(0;\theta)$ *is any value* $\in [\frac{v}{F'(\theta)}, v]$.

Proof: *direct consequences of total differentiating of the first-order condition Equation 10 and application of the definition of $\hat{\theta}$ where $F'(\hat{\theta}) = 1$ for $\theta \geq \hat{\theta}$.* □

Lemma 1 states that the market-clearing price for the collateral is continuous, decreasing in ϕ and increasing in θ. When the collateral buyer's capital is abundant enough, the collateral is always liquidated in fundamental value v whereas with scarce enough capital, he refuses to buy any collateral at any positive price.

Alternatively one could think of the collateral buyer as a competitive, risk-averse market maker with θ being his degree of risk tolerance. This setup is commonly used in the financial market runs literature such as Morris and Shin (2004) and Bernardo and Welch (2004). To keep the analysis as general as possible, I will only impose properties listed in Lemma 1 on any $L(\phi;\theta)$ and place no restrictions on the second-order derivatives, for example. The interpretation of an outside buyer with a productive investment technology is only used again in the welfare analysis section[14].

Next, I will study how the supply of the collateral asset ϕ is determined. At $t = 0$, the firms and creditors form a conjecture of collateral liquidation value l and all firms adopt their investment strategy as in Proposition 1. Due to the independence of project

[14] In the case of a competitive risk-averse market maker, the collateral buyer always breaks even in any equilibrium and his payoff thus does not play a role in the welfare analysis.

realisation and the mixed strategy probability λ, the measure of firms with failed projects is deterministic and the measure of collateral repossessed by the creditors is

$$\lambda(l)(1-p^*(l))k(l)$$

which is a function of measure of firms undertaking investment, the probability of default of their projects, and the amount of collateral pledged to the creditors. As the hold-to-maturity value of the collateral is worth $\underline{l}$ to the creditors, they prefer liquidating the collateral when the liquidation value l is higher than $\underline{l}$. Denote the probability of selling the collateral by $s(l)$, the measure of collateral supplied in the market ϕ is summarised in the following lemma:

Lemma 2 *(Supply of collateral is affected by expected liquidation value via firms' investment) For a given conjectured liquidation value l, the measure of collateral being liquidated at $t = 1$ is given by*

$$\phi(l) = s(l)\lambda(l)(1-p^*(l))k(l) \tag{11}$$

$$\text{where } \lambda(l) = \begin{cases} 0 & \text{for } l < l_{CR} \\ \text{any } \lambda \in [0,1] & \text{for } l = l_{CR} \\ 1 & \text{for } l > l_{CR} \end{cases} ; s(l) = \begin{cases} 0 & \text{for } l < \underline{l} \\ \text{any } s \in [0,1] & \text{for } l = \underline{l} \\ 1 & \text{for } l > \underline{l} \end{cases} \tag{12}$$

Proof: *See discussion above.*

Figure 3 shows how the supply of collateral depends on the conjectured liquidation value. When the liquidation value is strictly below l_{CR} or $\underline{l}$, there is no collateral liquidated because either no firm undertakes the investment project or creditors prefer to hold the collateral to maturity. At $\max\{l_{CR}, \underline{l}\}$, firms play mixed strategies so that any amount in $[0, (1-p_2)k_2(\max\{l_{CR}, \underline{l}\})]$ of collateral could be supplied. Beyond this critical value, all firms invest and all creditors choose to sell the asset, and thus the supply of collateral is $(1-p^*(l))k(l)$ which is decreasing and convex in l. Finally, there is a discrete jump at l_{RT} as at this level firms invest prudently and fewer defaults reduce the supply of collateral[15].

[15] The existence of the discrete jump, $(1-p_1)k_1(l_{RT}) < (1-p_2)k_2(l_{RT})$, is a consequence of Assumption 1(iii)

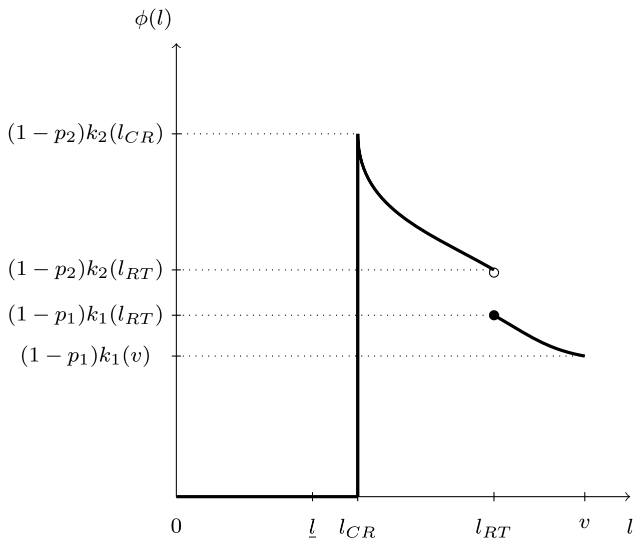


Figure 3: Supply of collateral asset ϕ as a function of conjectured liquidation value l

As the market clearing price of the collateral is decreasing in the amount of collateral supplied and more collateral is supplied when firms engage in risk-taking and pledge more collateral, the second half and reverse direction of the feedback loop in Figure 1 is completed: ex-ante firms' risk-taking incentives deepen the fire sale discount in the collateral market. Due to the interdependence nature of moral hazard risk-taking and the equilibrium liquidation value of the collateral, multiple rational expectation equilibria can arise. In the next section I characterise these equilibria and discuss their implications for financial fragility.

3 General equilibrium: self-fulfilling fire sales

This section is devoted to characterising the equilibria and studying their features and implications for fragility.

Definition 1 *For any given amount of collateral buyer's available capital $\theta \in [0, +\infty)$, a symmetric, competitive rational expectation equilibrium consists of an equilibrium liquidation value $\{l^*\}$ and mixed strategy probabilities $\{s^*, \lambda^*\}$such that*

1. *At $t = 0$, agents conjecture the equilibrium liquidation value to be l^*. Firms maximise their expected payoff by implementing the optimal investment strategy $p^*(l^*)$*

and offering the optimal contract $\{r(l^*), k(l^*)\}$ as in Proposition 1;

2. *At $t = 1$, creditors of insolvent firms seize the collateral and supply $\phi(l^*)$ amount of collateral in the market is $\phi(l^*)$ as in Lemma 2;*

3. *The buyer with available capital θ clears the collateral market at the market clearing price $L(\phi(l^*); \theta)$;*

4. *In equilibrium, agents' expectation of collateral liquidation value is correct. That is, $l^* = L(\phi(l^*); \theta)$.*

I will first prove the existence of equilibrium in the next lemma

Lemma 3 *(Existence of equilibria) For any $\theta \in [0, +\infty)$, there exists at least one equilibrium collateral liquidation value l^* that satisfies the equation:*

$$l^* = L(s(l^*)\lambda(l^*)(1 - p(l^*))k(l^*); \theta) \tag{13}$$

Proof: *See Appendix.*

While Lemma 3 guarantees that equilibrium exists under any amount of the collateral buyer's capital θ, there can be more than one equilibrium collateral liquidation values l^* that satisfy Equation (13)[16]. The next proposition discusses the main result of this paper: how the parameter θ affects the uniqueness and multiplicity of equilibria.

Proposition 2 *(Fragility and collateral buyer's capital θ) With Assumption 1-3 and for collateral with $\underline{l} < l_{RT}$, there exists two distinct values $\underline{\theta}$, $\bar{\theta} \in (0, +\infty)$ such that*

1. *For $\theta \in [\bar{\theta}, +\infty)$, a unique equilibrium in which all firms invest prudently exists and the equilibrium collateral liquidation value is relatively high, $l^*(\theta) \geq l_{RT}$.*

2. *For $\theta \in [0, \underline{\theta}]$, a unique equilibrium in which firms either engage in risk-taking or forgoes investment exists and $l^*(\theta) < l_{RT}$.*

[16]I disregard the potential continuum of equilibria in which the collateral market clears without any supply or demand of the collateral. These equilibria are exactly the same economically except with a different no-trade price.

3. *For* $\theta \in (\underline{\theta}, \bar{\theta})$*, there exist multiple values of* $l^* \in [0, v]$ *that satisfy Equation (13). As such, multiple rational expectation equilibria exist.*

   (a) *When* $l^*(\theta) = l_{CR}$*,* $(1 - \lambda^*(\theta))$ *fraction of the firms are credit rationed where* $\lambda^*(\theta) \in [0, 1]$ *uniquely satisfies*

$$L(\lambda^*(\theta)(1 - p_2)k_2(l_{CR}); \theta) = l_{CR} \tag{14}$$

   and complete credit rationing occurs for θ *such that* $L(0; \theta) \leq l_{CR}$

   (b) *When* $l^*(\theta) = \underline{l}$*, all firms are financed and* $(1 - s^*(\theta))$ *fraction of the creditors in insolvent firms do not sell the collateral in the market and hold it to maturity where* $s^*(\theta) \in [0, 1]$ *uniquely satisfies*

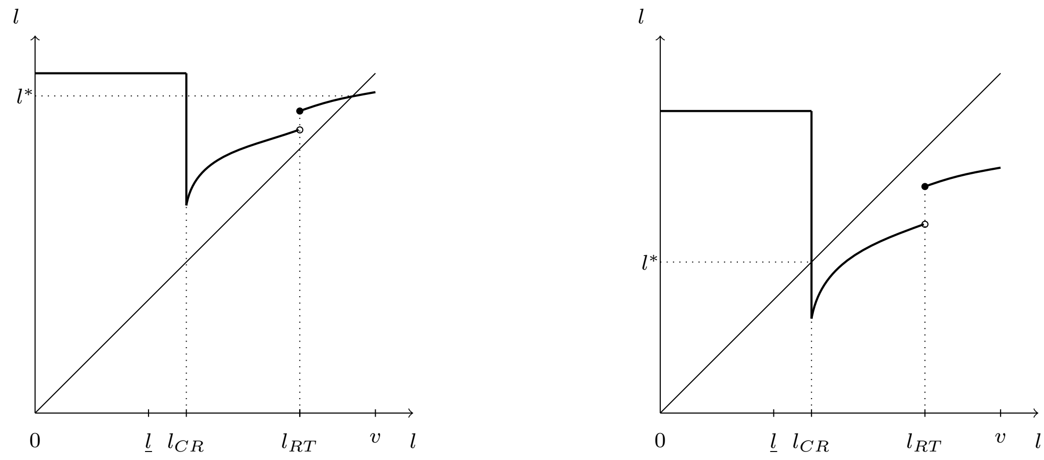
$$L(s^*(\theta)(1 - p_2)k_2(\underline{l}); \theta) = \underline{l} \tag{15}$$

   and no collateral is traded for θ *such that* $L(0; \theta) \leq \underline{l}$

Proof: *See Appendix*

Unique equilibrium under extreme θ Figure 4 plots the indirect collateral liquidation value function $L(\phi(l))$ and the collateral liquidation value l against l itself. An intersection of the two graphs therefore constitutes an equilibrium (a fixed-point l in Equation (13)). Figure 4 shows the two cases of unique equilibrium. Intuitively, when θ is large, the competitive collateral buyer's capital is abundant so that he can clear the market at a relatively high price. Consequently, even when all agents in the market are pessimistic that the collateral is going to be liquidated at a low price, as a result firms take on excessive risk and the amount of collateral liquidated is large, this belief will not be vindicated in equilibrium because the collateral buyer has enough capital to clear the market at a price higher than the anticipated one. The same logic applies to the opposite case with $\theta \leq \underline{\theta}$. As a result, there could only be one equilibrium.

By interpreting the amount of the collateral buyer's capital as a proxy for the aggregate economy, Proposition 2 suggests that the shadow banking system is pro-cyclical, even when the fundamental value of the collateral (v) and firms' investment profitability ($pX - 1 - c$) do not correlate with θ. In a capital-abundant (good) period ($\theta \geq \bar{\theta}$), firms have low default



(a) when $\theta \geq \bar{\theta}$ (b) when $\theta \leq \underline{\theta}$

Figure 4: Cases of unique equilibrium under extreme values of θ

risks, investment returns are high, the amount of credit granted by creditors to firms and by the collateral buyer to the real economy is large, debt yields are low and the collateral liquidation discount is small. In contrast, in a capital-constrained (bad) period firms are stuck in an equilibrium with high default risks, low returns, high borrowing costs, credit being rationed and a large volume of collateral is liquidated at a substantial discount. This pro-cyclicality comes from the fact that the collateral liquidation values are affected by the aggregate capital available for collateral. As a result the moral hazard problem becomes more severe in bad times, creating non-linear amplifications in the system.

Multiple equilibria and fragility When the collateral buyer's capital is between the extreme amount $\underline{\theta}$ and $\bar{\theta}$, the market-clearing price of the collateral becomes more sensitive to the change in the amount of collateral being liquidated. In this case, multiple rational expectation equilibria exist.

The multiple equilibria arises via two channels as shown in Figure 5[17]. The first channel is the *risk-taking channel* which is the case for switching equilibrium liquidation value from l_1^* to l_2^* where

$$l_1^* = L((1-p_1)k_1(l_1^*);\theta) \geq l_{RT} > L((1-p_2)k_2(l_2^*);\theta) = l_2^*$$

[17] I focus the discussion on stable equilibria only

When the anticipated liquidation value changes from l_1^* to l_2^*, there is not enough collateral to maintain incentives at l_2^*, that is, $k_1(l_2^*) > 1$. Firms thus can only engage in risk-taking, resulting in more defaults and a jump in the amount of collateral being liquidated, $(1 - p_2)k_2(l_2^*) < (1 - p_1)k_1(l_1^*)$, thus confirming the anticipated lower liquidation value l_2^*. This fragility phenomenon from risk-taking occurs when θ is in the range that produces l^* which is sufficiently close to the risk-taking threshold l_{RT} and the discrete jump in market-clearing price leads to one equilibrium liquidation value above and the other below l_{RT}. As $L(\phi(l); \theta)$ is continuously increasing in θ from 0 to v for any given l, this range of θ always exists, irrespective of the curvature or the elasticity of the market-clearing price function.

Multiple equilibria can also arise from a *margin channel* as in the case from l_2^* to l_3^*, where both are below l_{RT} and thus the firms' default risks are the same. Note also that in this case there are some credit rationing in the equilibrium with l_3^*, i.e.,

$$l_2^* = L((1 - p_2)k_2(l_2^*); \theta) > L(\lambda^*(\theta)(1 - p_2)k_2(l_3^*); \theta) = l_3^*$$

with some $\lambda^*(\theta) \in (0, 1)$. When the anticipated liquidation decreases from l_2^* to l_3^*, firms have to pledge more collateral $k_2(l_2^*) > k_2(l_3^*)$ to satisfy their incentive and creditors' break-even constraints. As a result more collateral is liquidated and when the market-clearing price function is sensitive enough in the relevant range, the increase in collateral supply pushes the equilibrium liquidation value to l_3^*.

Both types of multiple equilibria discussed above are *self-fulfilling* and feature large variations in collateral asset prices, debt yields, the amount of credit rationed, and firms' profitability. There are also some differences in these two channels. Multiple equilibria caused by risk-taking have significant variations in firms' default risk but the change in margins is ambiguous ($k_1(l_1^*) - k_2(l_2^*)$ cannot be signed). Meanwhile, fragility via margin channel causes large changes in initial margins while firms' default risks remain unchanged. The different effect on margins from the two channels can help to understand the mixed empirical findings on the behaviour of repo haircuts during the Subprime crisis in 2007-2009: Copeland et al. (2011) and Krishnamurthy et al. (2012) found small variations in haircuts in the tri-party repo market while Gorton and Metrick (2012) documented a

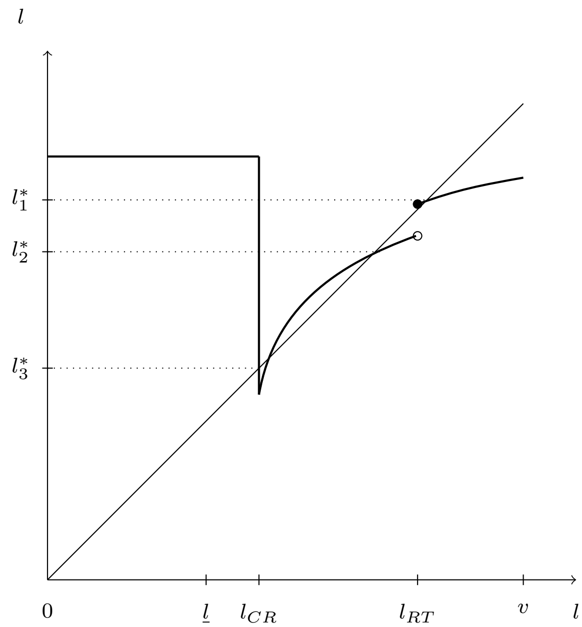


Figure 5: Multiple (stable) equilibria via different channels. *Risk-taking channel:* l_1^* to l_2^*; *Margin channel:* l_2^* to l_3^*

substantial increase of haircut in the bilateral repo market[18].

To conclude this section, Figure 6 summarises how the collateral buyer's capital affects the equilibrium characteristics and fragility in the collateral-based financial system.

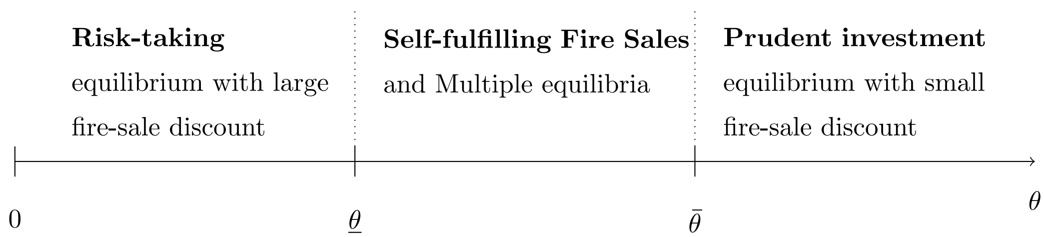


Figure 6: Equilibria characterisation under various exogenous amounts of collateral buyer's capital θ.

[18] Copeland et al. (2011) also finds that lenders in the tri-party repo market are more likely to withdraw funding than to increase haircuts to reduce risk exposure. Credit rationing in this model is analogous to fund withdrawal.

4 Welfare and policy implication: the case for central banks as market-makers of last resort

In this section I will first discuss the welfare implications of the multiple equilibria phenomenon and show that equilibrium with a lower collateral liquidation value is less efficient. Then I argue that this inefficiency creates a role for a social planner, or a central bank in this context, to intervene in and stabilise the collateral market and improve welfare. This role corresponds closely to the idea of *Market-Maker of Last Resort* proposed by various academics and commentators including Willem Buiter and Anne Sibert (see Buiter and Sibert (2007); Buiter (2012)).

I assume the social planner's objective is to maximise the total net utility of all agents. As the creditors always break even in equilibrium, the social welfare function $W(l^*)$ is defined as the sum of the net payoff of the firm $U(l^*)$ and that of the collateral buyer $\Pi(l^*)$, in equilibrium with collateral liquidation value l^*[19].

$$W(l^*) = U(l^*) + \Pi(l^*) \tag{16}$$

where $U(l^*)$ and $\Pi(l^*)$ are defined in Equations (8) and (9). Consider a collateral asset with quality $\underline{l} < l_{RT}$ in a state θ where multiple equilibria exist. The following proposition shows that the equilibria with lower l^* are associated with lower social welfare.

Proposition 3 *(Inefficiency) When multiple equilibria exist, social welfare $W(l^*)$ is larger in the equilibrium with a higher l^*.*
Proof: *See Appendix.*

Let's compare two equilibria with $l_1^* > l_2^*$. There are four potential sources of welfare loss in the equilibrium with l_2^*: (i) the crowding-out effect on the collateral buyer's investment in productive technology, (ii) the inefficiency from the firms' risk-taking decision when $l_1^* \geq l_{RT} > l_2^*$, (iii) the credit rationing of the firms' positive NPV investment when $l_2^* \leq l_{CR}$, and (iv) the creditors' disutility for holding the collateral to maturity when $l_2^* \leq \underline{l}$.

[19]If the collateral buyer is alternatively modelled as a competitive risk-averse market maker, as suggested in Section 2.3, then the equilibria can be Pareto-ranked as both creditors and the collateral buyer always break-even. Only firms have higher payoff in the equilibrium with a higher collateral value.

The self-fulfilling fragility and the inefficiency associated with the lower liquidation value equilibria call for welfare-improving policy intervention. In particular a central bank can coordinate agents into the efficient equilibrium by committing to buy any amount of collateral at a certain price. This kind of asset price guarantee policy can eliminate agents' pessimistic (yet rational) expectation hence ruling out the inefficient equilibria.

Asset Price Guarantee Recall that there are two classes of multiple equilibria that can arise: one involves risk-taking and the other acts through the change in margins. Consider the risk-taking case with two equilibrium liquidation values $l_1^* \geq l_{RT} > l_2^*$. By committing to buy any amount of collateral at a price $l_{PG} \geq l_{RT}$, the equilibrium with risk-taking l_2^* ceases to exist because when agents know the collateral liquidation value would not fall below l_{RT}, firms can pledge enough collateral to induce prudent investment and thus no risk-taking will happen in the first place.

For the case of multiple equilibria through the margin channel, multiple l^* are both below or above l_{RT}. To pick the equilibrium with the highest l^* the central bank just needs to set the price guarantee l_{PG} strictly higher than the second highest l^* and all equilibria but the one with the highest l^* are eliminated.

Interestingly, as long as the price guarantee is strictly below the highest l^*, the price guarantee facility will never be used because in equilibrium the price offered by the outside buyer is higher than that offered by the central bank. Thus the central bank can stabilise the market and improve welfare by simply promising to intervene. This is similar to the result with deposit insurance in Diamond and Dybvig (1983).

Regarding the funding of this asset purchase programme, the central bank can issue bonds worth l_{PG} to finance the purchase or more accurately give a riskless bond worth l_{PG} to creditors in exchange for collateral. These bonds could be backed by future taxes collected from the payoff of firms' projects. Note that firms cannot individually issue claims backed by the project to finance collateral purchase because of adverse selection, as creditors do not observe other firms' solvency.

The credibility of such a commitment could still be an issue in the off-equilibrium since at $t = 1$ the firms and collateral buyer have made their investment decisions and the fire sales of collateral is simply a zero-sum transfer between the creditors and the buyer[20]. The central bank thus has no interest in tax and redistribution unless he puts an increasingly larger weight on the welfare of creditors than that of the buyer ex-post when the collateral liquidation value decreases.

While my model is very stylised and does not deal with the collective moral hazard problem as in Farhi and Tirole (2012) and Acharya (2009), it does provide an economic rationale for the central bank to play an active role in stabilising certain important collateral markets in order to prevent systemic runs. I summarise the discussion of this policy discussion in the following proposition:

Proposition 4 *When multiple equilibria exist, asset price guarantee can eliminate the inefficient equilibria at no cost.*
Proof: *See discussion above.*

5 Collateral quality and fragility

In this section I will show how collateral quality affects fragility. I interpret creditors' hold-to-maturity utility $\underline{l}$ for a particular class of collateral can be interpreted as collateral quality. The analysis below can be considered as a comparison of equilibria supported by two collaterals with different qualities such as U.S. Treasuries and private-label asset-backed securities, or alternatively, the same class of collateral before and after receiving an exogenous shock on its fundamental risk, like mortgage-backed securities around the breakout of the subprime mortgage crisis in 2007.

Lower quality collateral breeds fragility Collateral quality reflects the creditors' eagerness to liquidate. In the same state, collateral with different qualities can have a different number of equilibria. Figure 7 provides an example: for a lower quality collateral $\underline{l}'$, there exist two stable equilibria $l_1^* > l_2^*$ whereas a collateral with higher quality $\underline{l}''$ only supports the equilibrium with the higher liquidation value. This is because creditors' reservation price for the higher quality collateral is higher than the market-clearing price

[20]Except the case with $l^* = \underline{l}$ in which creditors have to inefficiently hold some collateral to maturity.

in the low liquidation value equilibrium $\underline{l}' > l_2^*$. The following proposition generalises this argument that low quality collateral breeds fragility, i.e. if multiple equilibria exist in state θ when the collateral quality is $\underline{l}$, they also exist for a lower quality collateral $\underline{l}' < \underline{l}$ in same state.

Proposition 5 *(Low quality collateral breeds fragility) Denote $\Theta^M(\underline{l})$ as the set of $\theta \in [0, +\infty)$ that permits multiple equlibria to exist when collateral quality is $\underline{l}$. Then the set $\Theta^M(\underline{l})$ is non-expanding in $\underline{l}$.*

Proof: *See Appendix.*

Proposition 5 can explain why the market for high quality collateral like the U.S. Treasuries and agency bonds are rather stable during the crisis while there are substantial variations in repo rates, spreads, and borrowing capacity of lower quality collateral such as private-label ABS and corporate bonds.

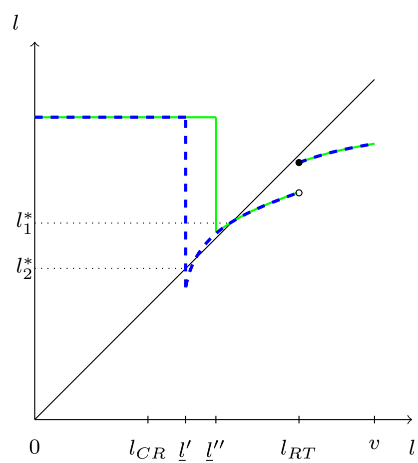
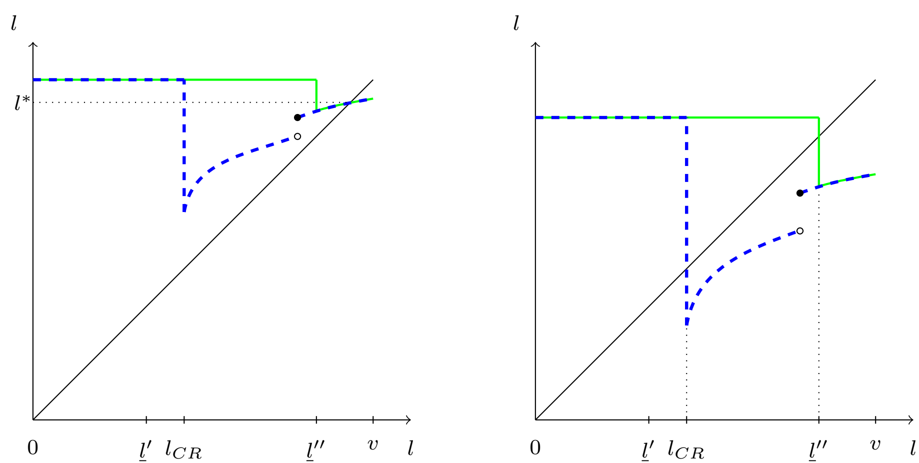


Figure 7: Fragility exists for lower quality collateral (blue, dashed) but not for higher quality collateral (red, dotted) in state θ'.

Counter-cyclical credit spread Another well-documented phenomenon during periods of economic distress is that the credit spreads between safe and relatively risky assets increase significantly. Consider again the two collateral assets above with reservation price $\underline{l}'$ and $\underline{l}''$ but in the extreme states with unique equilibria. To make the comparison starker I will take $\underline{l}'' \geq l_{RT}$. In the good state $\theta \geq \bar{\theta}$, there are minimal differences in terms of spreads and margins between the two collaterals because the competitive collateral buying sector has abundant capital to purchase the collateral; as a result, the difference in creditors' reservation prices for the two collateral assets does not appear in equilibrium.

The difference becomes apparent in a state where the collateral buying sector's capital is scarce. Figure 8 gives such an example. The differences in quality are amplified due to the moral hazard problem: the lower quality collateral triggers risk-taking in the capital-constrained state, further compounding the problem of scarce capital. This result could explain why there are minimal spread and haircut differences for Treasuries and MBSs in capital-abundant periods while the two markets are markedly different during a crisis. One might also regard the Federal Reserve's Large Scale Asset Purchase programme during the recent crisis as injecting liquidity and pushing the market from the right to the left panel in Figure 8. This then suggests a new, moral-hazard based channel to interpret the empirical findings by Krishnamurthy and Vissing-Jorgensen (2013) that the Fed's purchase of MBSs has much a larger reduction in yields than that of Treasuries.



(a) when $\theta \geq \bar{\theta}$ (b) when $\theta \leq \underline{\theta}$

Figure 8: Spreads between the lower quality (blue,dashed) and higher quality (red,dotted) collateral assets in good time (a) and bad time (b) respectively.

6 Repo as optimal contract and cost of automatic stay

In Section 2.2 I restrict attention to collateralised short-term debt contracts with promised repayment r and k measure of collateral being transferred to the lender at $t = 1$, should the firm fail to repay as demanded. In this section I will discuss the optimality and implementability of such a contract. In particular I will show that a repurchase agreement with the exemption of automatic stay can implement the optimal contract. The key friction here is that insolvent firms can threaten to file for bankruptcy protection to delay the transfer of collateral to creditors to $t = 2$, which creates a hold-up problem similar to the one outlined in the incomplete contract literature, as seen in Aghion and Bolton (1992), Hart and Moore (1994),and Diamond and Rajan (2001)[21]. I conclude this section with a discussion of the potentially negative consequences of forbidding the use of stay-exemption.

A general contract consists of a pair $\{r_s, k_s\}$ and $\{r_f, k_f\}$ which specify cash repayment (r) and the amount of collateral transfer (k) in the case of project success or failure respectively. Timing of the payment is irrelevant for now as the information is fully revealed to both parties at $t = 1$. Recall that project cash flow is X and X_f when the project succeeds or fails respectively. The standard moral hazard result shows that $k_s = 0$ and $r_f = X_f$ are optimal. Intuitively, leaving some returns to the firm in the case of failure and giving collateral to the lender in the case of success worsen the incentive problems. Thus the optimal contract will be a debt contract with promised repayment $r = r_s \geq r_f$ and $k = k_f$ measure of collateral given to the lender only if the project fails.

Furthermore, the firm prefers to commit to transfer the collateral to the lender at $t = 1$ because this allows the lender to liquidate the collateral in the market for price l^* which is greater than $\underline{l}$, the lender's valuation of the collateral at maturity $t = 2$. Improving the lender's payoff in the case of failure allows the firm to promise less repayment, relaxes the incentive constraint, and increases the firm's payoff.

Next is the implementation of the optimal contract. First note that as the creditor's signal about her debtor's solvency is non-contractible, the court cannot enforce payment that

[21]The analysis of the optimal contract here is also similar to that in Acharya and Vishwanathan (2011) with a difference that the hold-up problem there is caused by borrower's ex-post asset-substitution problem; assigning control rights to the lender can thus solve the problem.

is contingent on the signal. As such, the collateralised debt contract has to be demandable at $t = 1$. A general secured short-term debt contract, however, will not be enough if the firm can file for bankruptcy protection at $t = 1$ and delay liquidation to $t = 2$. Specifically, I make the following assumption:

Assumption 4 *(Time-consuming bankruptcy and liquidation procedure) If a firm files bankruptcy protection at $t = 1$, the court needs time to verify its bankruptcy, liquidate the assets and can only execute the repayment to creditors at $t = 2$.*

Assumption 4 is broadly in line with the automatic stay provision in the U.S. that inhibits creditors from collecting debt when a firm files for Chapter 11 bankruptcy protection[22]. In practice the bankruptcy and liquidation of complex securities firms can be time-consuming and costly[23]. In the context of this paper, bankruptcy is costly because the collateral is only worth $\underline{l}$ to creditors $t = 2$, due to their aversion to the collateral dividend risk. Hence at $t = 1$ when the firm fails to repay as requested, it can threaten to file for bankruptcy and make a take-it-or-leave-it offer to the lender with an immediate transfer of $k' \leq k$ units of collateral such that $k'l^* = k\underline{l}$. In other words, the firm cannot credibly commit to transfer k units of collateral to the creditor at $t = 1$ when it is insolvent.

As the source of this renegotiation problem is the delay of the liquidation procedure, a short-term repurchase agreement with the exemption of automatic stay avoids this problem by allowing the repo lender to seize the collateral immediately when the borrower defaults[24]. The following proposition summarises this discussion:

[22]For instance on the US Federal Courts website, automatic stay is defined as "an injunction that automatically stops lawsuits, foreclosures, garnishments, and all collection activity against the debtor the moment a bankruptcy petition is filed." See: http://www.uscourts.gov/FederalCourts/Bankruptcy/BankruptcyBasics/Glossary.aspx

[23]For example, Lehman Brothers filed for Chapter 11 in September 2008, exited from it in March 2012, and only made the first payment to creditors in April 2012. See "Lehman Exits Bankruptcy, Sets Distribution to Creditors", *Wall Street Journal*, March 06, 2012.

[24]In principle, an independent sale and repurchase transaction means the collateral rests on the balance sheet of the buyer (repo lender) and thus the automatic stay provision from the default of the seller (repo borrower) should not be applied to the collateral. In practice, nonetheless, repo in the U.S. is treated as *secured loans* and the repo securities are on the balance sheet of the borrower. See Acharya and Öncü (2010) for details and the historical development of the repo market in the U.S.

Proposition 6 *(Repo with stay-exemption as optimal contract) The optimal contract is the collateralised short-term debt contract with promised repayment r at $t = 1$ and immediate transfer of k units of collateral to the creditor at $t = 1$ in the case of default. When Assumption 4 holds such that insolvent firms can renegotiate the debt contract by threatening to file for bankruptcy, a short-term repurchase agreement with the exemption of automatic stay avoids this renegotiation problem and implements the optimal contract.*
Proof: *See discussion above.*

Cost of automatic stay Critics of the special stay-exemption status of repo contracts like Roe (2011) have argued that it could cause the disorderly liquidation of collateral assets when some borrowers default, which in turn drives down the price of the collateral and causes systemic risk. They have proposed reform that makes repo lenders also subject to some degree of automatic stay to prevent the above negative spiral. While my model does have the negative spiral fragility, it also suggests that imposing automatic stay may induce *more* fire sale and thus systemic fragility.

Here the key friction caused by automatic stay is that firms can renegotiate the debt contract ex-post. Firms can reduce the promised k units of collateral to $k' = k\frac{l}{l^*}$ ex-post by threatening to enter into bankruptcy protection, which implies the maximum amount of collateral firms can *credibly* pledge is $\frac{l}{l^*} \leq 1$. Thus the collateral constraint becomes easier to bind and firms are more prone to take excessive risk, resulting in more fire sales and fragility in aggregate. Notice that although under automatic stay the lenders cannot seize the collateral and liquidate it in the market at $t = 1$, it is optimal for firms to fire sale some assets to (partially) repay the lenders. As a result, limiting post-default fire sale worsens firms' risk-taking incentive problem and *increases* pre-default fire sales[25].

7 Concluding remarks

This paper shows a novel form of financial fragility stemming from the feedback effect between the risk-taking incentives of borrowing firms and the illiquidity in the collateral

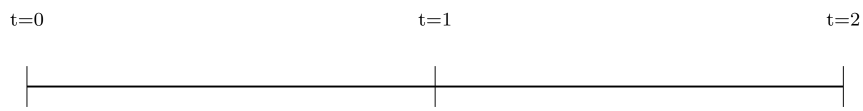
[25] In a policy paper, Begalle et al. (2013) makes a similar distinction between pre-default and post-default fire sales and discuss how they can affect each other.

asset market. This offers a theory of systemic runs in the modern market-based financial system where traditional strategic considerations of depositors within a financial institution may not arise. When firms collateralise their assets to borrow in the form of short-term debt such as repo, I show that a new kind of coordination failure among firms can arise since firms' risk-taking and margin decisions become *strategic complements* due to the interaction between firms' moral hazard and the fire-sale externality in the collateral market. Fire sales can occur in a self-fulfilling manner and aggregate default risk is endogenously chosen by individual firms.

In terms of policy, this paper provides an economic rationale for central banks to intervene in the collateral market. When the market is moderately illiquid, asset price guarantee can eliminate the rational fear of fire-sales of the market participants at no cost and rule out the inefficient crisis equilibrium. In addition, reform aiming to limit post-default fire sales like banning the special bankruptcy stay-exemption status may actually backfire because this could worsen the incentive problems of the borrowing firms.

Appendix

Time-line of events



t=0

- A continuum of firms each needs to borrow \$1 to invest in a project, using an asset-in-place as collateral.
- Each firm offers a collateralised short-term debt to its creditor with promised payment r and k fraction of collateral pledged.
- Each firm privately exerts costly effort to increase the project success probability.
- Collateral buyer with exogenous amount of cash θ optimally hoards cash for collateral purchase at $t = 1$.

t=1

- Projects quality revealed. Each creditor knows whether her firm's project has succeeded or failed.
- Creditors of solvent firms roll over their debt and will receive r at t=2.
- Creditors of insolvent firms seize the k units of collateral and decide whether to sell it in the market.
- Collateral buyer used the hoarded cash to clear the collateral market. The market-clearing price $L(\phi; \theta)$ decreases in the amount collateral sold ϕ and increases in θ.

t=2

- Collateral's dividend realises.
- Succeeded projects' cashflow matures.
- Creditors in solvent firms receive repayment r, and the firms keep the remaining cashflow.

Parametric restrictions in Assumption 3

Parametric restrictions in Assumption 3 are made to ensure $0 \leq l_{CR} < l_{RT} < v$ so that prudent investment, risk-taking and credit rationing can arise in equilibrium. From the implicit definition of l_{RT} and l_{CR}, $k_1(l_{RT}) = 1$ and $U(l_{CR}) = 0$, one can show

$$l_{RT} = \frac{A_1 - p_1 v}{1 - p_1} \text{ and } l_{CR} = v\frac{(1-p_2)A_2 - p_2 NPV_2}{(1-p_2)A_2 + (1-p_2)NPV_2}$$

It is immediate to check that $l_{RT} < v$ and $l_{CR} < l_{RT}$ require $v > A_1$ and $v < \bar{v}$ respectively. To have $U(l_{CR}) = 0$ in equilibrium, one needs $l_{CR} \geq 0$ and $k_2(l_{CR}) \leq 1$ which together give the condition $NPV_2 \leq \min\{v - A_2, \frac{1-p_2}{p_2} A_2\}$. □

Proof

Proof of Proposition 1:

First, both (IC) and (PC) are binding at optimal. If (PC) slacks, the firm can decrease r by a small amount to increase profit while (IC) still holds; If (IC) slacks, the firm can reduce k and increase r by a small amount to keep (PC) binding and (IC) still satisfied while a smaller k increases expected payoff due to lower fire-sale cost. To see this, suppose the contrary that $\{r, k\}$ is optimal but (IC) slacks, that is

$$r < X - \frac{\Delta c_i}{\Delta p_i} + kv$$

Plugging the binding (PC) $r = [1 - (1 - p_i)kl]/p_i$ into the above (IC), one can show $k > \frac{A_i}{p_i v + (1 - p_i)l}$. Consider another contract $\{r', k'\}$ such that $k' = k - \epsilon$ and $r' = r + (1 - p_i)\epsilon l/p_i$, (PC) still binds and for a small $\epsilon > 0$ (IC) also holds. However the firm's expected payoff is strictly higher in the case of $\{r', k'\}$, as $NPV_1 - (1 - k')(v - l) > NPV_1 - (1 - k)(v - l)$, contradicting the optimality of $\{r, k\}$.

By binding (PC) and (IC), the optimal contract $\{r(l), k(l)\}$ is described as in Equation 7. Note that for a given l, it could be both $\{r_1(l), k_1(l)\}$ and $\{r_2(l), k_2(l)\}$ satisfy the remaining (RE) and (PT) constraints, that is, both prudent investment and risk-taking are feasible choices. Since prudent investment is always superior by Assumption 1, the firm optimally chooses $p_1(l)$ and the contract $\{r_1(l), k_1(l)\}$. Hence the firm chooses prudent investment whenever feasible, that is, when $l \geq l_{RT}$. If not, risk-taking is chosen as long as it is profitable, when $l \geq l_{CR}$. □

Proof of Lemma 3:

For a fixed θ, $L(s(l)\lambda(l)(1 - p(l))k(l); \theta)$ is a mapping from $[0, v] \to [0, v]$. Notice that the function $L(l; \theta)$ is upper semi-continuous from the left and closed from the right. The existence of fixed-point follows from the Lemma in Roberts and Sonnenschein (1976). □

Proof of Proposition 2:

There are three steps in this proof: I first show the existence of extreme regions of θ that only exactly one equilibrium exists. Then I show multiple equilibria must exist under some regions of θ and finally, I characterise the bounds of multiple equilibria regions $\underline{\theta}$, $\bar{\theta}$ for different possible shapes of the market-clearing price function $L(\phi(l);\theta)$.

Step 1: non-empty regions of θ with unique equilibrium

For $\theta \in [\hat{\theta}+v, +\infty)$, $L(\phi(l);\theta) = v$ for all $l \in [0, v]$ according to Lemma 1. Thus there is only prudent investment equilibrium in this region as $l_{RT} < v$. On the other hand, for $l < \max\{\underline{l}, l_{CR}\}$, $\phi(l) = 0$ while the maximum price the collateral buyer willing to pay for the first unit is $\frac{v}{F'(\theta)}$. As $\lim_{\theta \to 0+} F'(\theta) \to +\infty$ and $F''(\theta) < 0$, there exists a $\theta' > 0$ such that $\frac{v}{F'(\theta')} = \max\{\underline{l}, l_{CR}\}$. Then for $\theta \in [0, \theta')$, there is unique equilibrium with complete credit rationing (when $\underline{l} < l_{CR}$) or risk-taking and no collateral traded (when $\underline{l} > l_{CR}$).

Step 2: non-empty set of θ with multiple equilibria

The key of this step is the upward jump of $L(\phi(l);\theta)$ from $l \to l_{RT}$. At $l = l_{RT}$, $\phi(l_{RT}) = (1-p_1)k_1(l_{RT}) = (1-p_1) > (1-p_2)k_2(l_{RT})$, where the strict inequality is implied by Assumption 1(iii). By continuity of $L(;\theta)$, there exists a θ'' such that $L(\phi(l_{RT});\theta'') = l_{RT}$ hence $l^* = l_{RT}$ is an equilibrium with prudent investment at θ''. I am going to show that there also exists at least another equilibrium in the region $l \in [\max\{\underline{l}, l_{CR}\}, l_{RT})$ at this θ''. Due to the discontinuity of $\phi(l)$ at l_{RT}, $L((1-p_2)k_2(l_{RT});\theta'')$ is strictly below l_{RT} and then $L(\phi(l);\theta'')$ must cross the 45-degree line at some $l^* \in [\max\{\underline{l}, l_{CR}\}, l_{RT})$. To reduce notation, I will discuss the case with $l_{CR} > \underline{l}$. If $L((1-p_2)k_2(l_{CR});\theta'') \geq l_{CR}$, then by Intermediate Value Theorem, there exists a $l^* \in [l_{CR}, l_{RT})$ such that $L((1-p_2)k_2(l^*);\theta'') = l^*$ because $L(\phi(l);\theta'')$ is continuous in l and $L((1-p_2)k_2(l_{RT})) < l_{RT}$; If $L((1-p_2)k_2(l_{CR});\theta'') < l_{CR} < L(0;\theta'')$, then there exist a $\lambda^* \in (0,1)$ such that $L(\lambda^*(1-p_2)k_2(l_{CR});\theta'') = l_{CR}$ as at l_{CR}, $L(;\theta'')$ can take any value between $L((1-p_2)k_2(l_{CR});\theta'')$ and $L(0;\theta'')$ due to Lemma 2. In conclusion, there exists multiple equilibria at θ''.

Step 3: Characterise the bounds of $\underline{\theta}$ and $\bar{\theta}$

Let's start with the upper bound $\bar{\theta}$. For $\theta > \theta''$, multiple equilibria can exist because Equation 13 has multiple solutions in the region $[l_{RT}, v]$ or at least one solution in $[\max\{\underline{l}, l_{CR}\}, l_{RT})$ or both. Denote θ_1 and θ_2 as the smallest $\theta > \theta''$ that $L((1-p_1)k_1(l);\theta_1) = l$ has exactly one solution in $[l_{RT}, v]$ and $L((1-p_2)k_2(l);\theta_2) = l$ has no solution in $[\max\{\underline{l}, l_{CR}\}, l_{RT})$ respectively. Both θ_1 and θ_2 exist as members in the non-empty set of θ with unique prudent investment equilibrium satisfy these properties. Define $\bar{\theta} = \max\{\theta_1, \theta_2\}$ and as $L(;\theta)$ increases in θ, there is a unique equilibrium with prudent investment for any $\theta \in [\bar{\theta}, +\infty)$. Note that by construction $\theta'' < \bar{\theta}$.

Similarly for $\underline{\theta}$. Denote θ_3 and θ_4 as the largest $\theta < \theta''$ that $L((1-p_2)k_2(l);\theta_3) = l$ has exactly one solution in $[\max\{\underline{l}, l_{CR}\}, l_{RT})$ and $L((1-p_1)k_1(l);\theta_4) = l$ has no solution in $[l_{RT}, v]$ respectively. Define $\underline{\theta} = \min\{\theta_3, \theta_4\}$ and as $L(;\theta)$ increases in θ, there is unique equilibrium with risk-taking (and credit rationing) for any $\theta \in [0, \underline{\theta}]$. Note that by construction, $\underline{\theta} < \theta''$. Finally by the fact that $L(\phi(l);\theta)$ is continuous and strictly increases in θ for $\phi(l) > 0$, any $\theta \in (\underline{\theta}, \bar{\theta})$ contains multiple equilibria and this region is non-empty as $\theta'' \in (\underline{\theta}, \bar{\theta})$. □

Proof of Proposition 3:

By the definition of $U(l^*)$ and $\Pi(l^*)$ in Equation (8) and (9) and the market-clearing condition $F'(\theta - \phi(l^*)l^*) = v/l^*$, the social welfare function $W(l^*)$ can be expressed as

1. When $l^* > \max\{l_{CR}, \underline{l}\}$, $\phi(l^*) = (1-p(l^*))k(l^*)$

$$W(l^*) = NPV(l^*) + \int_0^{\theta - \phi(l^*)l^*} [F'(x) - 1]dx \tag{17}$$

2. When $l^* = l_{CR}$, $\phi(l_{CR}) = \lambda(1-p(l_{CR}))k(l_{CR})$

$$W(l_{CR}) = \lambda NPV(l_{CR}) + \int_0^{\theta - \phi(l_{CR})l_{CR}} [F'(x) - 1]dx \tag{18}$$

3. When $l^* = \underline{l}$, $\phi(\underline{l}) = s(1-p(\underline{l}))k(\underline{l})$

$$W(\underline{l}) = NPV(\underline{l}) + \int_0^{\theta - \phi(\underline{l})\underline{l}} [F'(x) - 1]dx - (1-s)(1-p(\underline{l}))k(\underline{l})(v - \underline{l}) \tag{19}$$

It is then immediate to see that a higher l^* will increase $W(l^*)$ in all cases. $NPV(l^*) = p(l^*)X - 1 - c(p(l^*))$ increases in l^*; $\int_0^{\theta-\phi(l^*)l^*}[F'(x) - 1]dx$ is the net return from collateral buyer's productive investment and increases in l^* as $\phi(l^*)l^*$ decreases in l by market-clearing condition. In case 2, $(1-\lambda)$ of firms do not invest and in case 3, $(1-s)$ of creditors could not sell the collateral to the buyer and both entail welfare loss. Therefore equilibria with lower l^* has a lower $W(l^*)$. $\square$

Proof of Proposition 5:

Suppose $\theta \in \Theta^M(\underline{l})$, $L(\phi(l;\underline{l});\theta) = l$ has multiple solutions $\{l^*\}$. What I need to show is that when $\underline{l}$ decreases to any $\underline{l}' < \underline{l}$, there are as least as many solutions. First note that as ϕ is only affected by $\max\{l_{CR}, \underline{l}\}$, changes $\underline{l}$ below l_{CR} will not have any effect in equilibrium. Every member in the set $\{l^*\}$ is at least as large as $\underline{l}$ and when they are strictly larger than $\underline{l}$, they will still be part of the solution for any $\underline{l}' < \underline{l}$. When $\underline{l}$ is one of the solutions and is changed to $\underline{l}'$, there are two cases: If $L((1-p(\underline{l}'))k(\underline{l}');\theta) < \underline{l}'$, the original solution $\underline{l}$ changes to $\underline{l}'$ with $L(s^*(1-p(\underline{l}'))k(\underline{l}');\theta) = \underline{l}'$ for some $s^* \in [0,1]$. This is because at $\underline{l}'$, $L()$ is a correspondence taking any value from $L((1-p(\underline{l}')k(\underline{l}');\theta)$ to $L(0;\theta) \geq \underline{l} > \underline{l}'$. On the other hand, if $L((1-p(\underline{l}'))(\underline{l}');\theta) > \underline{l}'$, the original solution $\underline{l}$ changes to some $\underline{l}'' \in [\underline{l}', \underline{l}]$ where $L((1-p(\underline{l}''))k(\underline{l}'');\theta) = \underline{l}''$. This follows from Intermediate Value Theorem as $L((1-p(l))k(l);\theta)$ is a continuous function in l and $L((1-p(\underline{l}))k(\underline{l});\theta) \leq \underline{l}$. Therefore, decreasing $\underline{l}$ to $\underline{l}'$ does not reduce the number of solutions $\{l^*\}$. $\square$

References

Acharya, V. V. (2009). A theory of systemic risk and design of prudential bank regulation. *Journal of Financial Stability*, 5(3):224–255.

Acharya, V. V., Gale, D., and Yorulmazer, T. (2011). Rollover risk and market freezes. *Journal of Finance*, 66(4):1177–1209.

Acharya, V. V. and Öncü, S. (2010). The repurchase agreement (repo) market. *Regulating Wall Street: The Dodd-Frank Act and the New Architecture of Global Finance*, pages 319–350.

Acharya, V. V. and Vishwanathan, S. (2011). Leverage, moral hazard, and liquidity. *The Journal of Finance*, 66(1):99–138.

Adrian, T. and Shin, H. (2011). Financial intermediary balance sheet management. *FRB of New York Staff Report*, (532).

Aghion, P. and Bolton, P. (1992). An incomplete contracts approach to financial contracting. *The review of economic Studies*, 59(3):473494.

Becker, B. and Ivashina, V. (2013). Reaching for yield in the bond market. Working Paper 18909, National Bureau of Economic Research.

Begalle, B., Martin, A., McAndrews, J., and McLaughlin, S. (2013). The risk of fire sales in the tri-party repo market. *working paper*.

Bernardo, A. E. and Welch, I. (2004). Liquidity and financial market runs. *The Quarterly Journal of Economics*, 119(1):135–158.

Besanko, D. and Thakor, A. V. (1987). Collateral and rationing: sorting equilibria in monopolistic and competitive credit markets. *International Economic Review*, 28(3):671–689.

Bolton, P., Santos, T., and Scheinkman, J. A. (2011). Outside and inside liquidity. *The Quarterly Journal of Economics*, 126(1):259–321.

Brunnermeier, M. K. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic Perspectives*, 23(1):77–100.

Brunnermeier, M. K. and Pedersen, L. H. (2009). Market liquidity and funding liquidity. *Review of Financial Studies*, 22(6):2201–2238.

Buiter, W. (2012). The role of central banks in financial stability: How has it changed? *CEPR Discussion Paper no. 8780.*

Buiter, W. and Sibert, A. (2007). The central bank as the market maker of last resort: From lender of last resort to market maker of last resort. *Voxeu.org*.

Chan, Y.-S. and Thakor, A. V. (1987). Collateral and competitive equilibria with moral hazard and private information. *The Journal of Finance*, 42(2):345–363.

Coco, G. (2000). On the use of collateral. *Journal of Economic Surveys*, 14(2):191–214.

Copeland, A., Martin, A., and Walker, M. (2011). Repo runs: Evidence from the tri-party repo market. *New York Federal Reserve Bank Staff Report*, 506.

Covitz, D., Liang, N., and Suarez, G. A. (2013). The evolution of a financial crisis: Collapse of the asset-backed commercial paper market. *The Journal of Finance*, 68(3):815–848.

Danielsson, J., Shin, H. S., and Zigrand, J.-P. (2011). Balance sheet capacity and endogenous risk. FMG Discussion Papers dp665, Financial Markets Group.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3):401–19.

Diamond, D. W. and Rajan, R. G. (2001). Liquidity risk, liquidity creation, and financial fragility: A theory of banking. *Journal of Political Economy*, 109(2):287–327.

Diamond, D. W. and Rajan, R. G. (2011). Fear of fire sales, illiquidity seeking, and credit freezes. *The Quarterly Journal of Economics*, 126(2):557–591.

Eisenbach, T. M. (2011). Rollover risk: Optimal but inefficient. *working paper.*

Farhi, E. and Tirole, J. (2012). Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review*, 102(1).

Goldstein, I. and Pauzner, A. (2005). Demand–deposit contracts and the probability of bank runs. *Journal of Finance*, 60(3):1293–1327.

Gorton, G. and Metrick, A. (2012). Securitized banking and the run on repo. *Journal of Financial Economics*, 104(3):425–451.

Gorton, G. B. (2012). Some reflections on the recent financial crisis. Working Paper 18397, National Bureau of Economic Research.

Gromb, D. and Vayanos, D. (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics*, 66(2-3):361–407.

Hart, O. and Moore, J. (1994). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics*, 109(4):841879.

He, Z., Khang, I. G., and Krishnamurthy, A. (2010). Balance sheet adjustments during the 2008 crisis. *IMF Economic Review*, 58(1):118–156.

He, Z. and Xiong, W. (2012). Dynamic debt runs. *Review of Financial Studies*, 25(6):1799–1843.

Holmström, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *the Quarterly Journal of economics*, 112(3):663–691.

Hombert, J. (2009). Optimal financial structure and asset prices. *Working Paper*.

Jensen, M. C. and Meckling, W. H. (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, 3(4):305–360.

Kacperczyk, M. and Schnabl, P. (2013). How safe are money market funds? *The Quarterly Journal of Economics*.

Krishnamurthy, A. (2010a). Amplification mechanisms in liquidity crises. *American Economic Journal: Macroeconomics*, 2(3).

Krishnamurthy, A. (2010b). How debt markets have malfunctioned in the crisis. *Journal of Economic Perspectives*, 24(1).

Krishnamurthy, A., Nagel, S., and Orlov, D. (2012). Sizing up repo. NBER Working Papers 17768, National Bureau of Economic Research, Inc.

Krishnamurthy, A. and Vissing-Jorgensen, A. (2013). The ins and outs of lsaps. In *Federal Reserve Bank of Kansas Citys Jackson Hole Symposium on the Global Dimensions of Unconventional Monetary Policy, Jackson Hole, Wyoming, August*.

Malherbe, F. (2014). Self-fulfilling liquidity dry-ups. *The Journal of Finance*, 69(2):947–970.

Martin, A., Skeie, D., and von Thadden, E.-L. (2014). The fragility of short-term secured funding markets. *Journal of Economic Theory*, 149:15–42.

Morris, S. and Shin, H. S. (2004). Liquidity black holes. *Review of Finance*, 8(1):1–18.

Roberts, J. and Sonnenschein, H. (1976). On the existence of cournot equilbrium without concave profit functions. *Journal of Economic Theory*, 13(1):112–117.

Rochet, J.-C. and Vives, X. (2004). Coordination failures and the lender of last resort: Was bagehot right after all? *Journal of the European Economic Association*, 2(6):1116–1147.

Roe, M. J. (2011). The derivatives markets payment priorities as financial crisis accelerator. *Stanford Law Review*, 63:539.

Shleifer, A. and Vishny, R. (2011). Fire sales in finance and macroeconomics. *Journal of Economic Perspectives*, 25(1):29–48.

Stein, J. C. (2012). Monetary policy as financial stability regulation. *The Quarterly Journal of Economics*, 127(1):57–95.

Stiglitz, J. E. and Weiss, A. (1981). Credit rationing in markets with imperfect information. *The American economic review*, 71(3):393–410.

Tirole, J. (2006). *The Theory of Corporate Finance*. Princeton University Press.

Vayanos, D. (2004). Flight to quality, flight to liquidity, and the pricing of risk. *working paper, LSE.*