Interest rate spillovers during quantitative easing

Marcello Pericoli*

Bank of Italy

June 30, 2014

Abstract

This paper analyzes the behaviour of interest rates in the United States, the United Kingdom and the euro area, with a focus on the comovements observed during the last five years. We use a standard Gaussian Affine Term Structure Model – augmented with forecasts of interest rates, inflation and rate of growth of GDP – to decompose interest rates into their expected component and the term premium. The results show that the decrease in long-term rates, in particular following the introduction of the second US Quantitative Easing program in November 2010 and the explicit US forward guidance in August 2011, is mainly explained by the drop in term premiums. We show that the large decrease in term premiums observed during the last years is also explained by the increase in domestic government bond holdings of the Federal Reserve and the Bank of England. Furthermore, we show that movements in US long term rates have been mirrored by movements in the corresponding interest rates in the other two economies and that this comovement reflects the link between the respective term premiums, rather that between the expectations on future interest rates. This results has relevant policy implications in view of the exit from the unconventional monetary policy measures, which are being undertaken in the US.

Keywords: term structure model, term premium, unspanned risk factor, quantitative easing

JEL Classifications: C32, E43, G12

*The views expressed are those of the author and do not necessary represent those of the Bank of Italy. I would like to thank Pietro Catte, Jens Christensen, Paolo Del Giovane, Antonio Diez de los Rios, Giuseppe Grande, Pavol Povala, Roberto Piazza, Massimo Sbracia, Stefano Siviero, Marco Taboga, Gabriele Zinna, and participants to the internal Bank of Italy seminar, the 2014 Bank of Canada – Bank of España workshop on 'International Financial Markets' for their comments. Of course, all errors are my own. Email address: marcello.pericoli@bancaditalia.it

1 Introduction

This paper analyzes interest rates developments in the United States, the United Kingdom and the euro area since the introduction of the euro, with a focus on the movements observed during the last five years. First, we decompose interest rates in expected rates and term premiums and, second, we investigate if any cross-country effect has been at work following the introduction of unconventional monetary-policy measures in the United States. The stylized facts indicate that medium- and long-term interest rates show a decreasing trend since 1999, and that movements in the US long-term rates are mirrored by movements in long-term rates in other advanced economies. This suggests that the evolution of US interest rates has spillover effects on monetary conditions in other advanced economies (and on their fiscal stance through the cost of public debt issuance). This conclusion is further supported by the fact that exchange rates vis-à-vis the US dollar do not seem to have played a major role in absorbing differences in financial and economic conditions.

The strong decline in medium- and long-term US interest rates during the financial crisis reflected the introduction of successive waves of unconventional monetary policies in the United States (known as Quantitative Easing, QE1 from September 2008 to March 2010, QE2 from November 2010 to June 2011 and QE3 from September 2012 to date) and of the explicit forward guidance by the Federal Reserve.[1] As clearly stated by the Federal Reserve the aim of such policies was to lower long-term rates by impacting not only their expected component but also their term premium. These policies have resulted in a sharp decrease in medium- and long-term interest rates in other countries as well. In the last five years, the correlation among 10-year interest rates of major advanced economies (with the exception of Japan) has been close to 80% (in line with the value of the previous decade), and has remained high even after the announcement of a possible tapering of the US unconventional measures.[2] All in all, during the crisis these linkages have been particularly beneficial to those countries whose economies were undergoing a severe contraction. However, the implications of those linkages may become unfavourable when business cycles are in different phases; for example,

[1] An explicit forward guidance was announced during the post-Lehman crisis FOMC meetings but particularly stressed at the FOMC meetings of March 2009, August 2011 – when the FOMC announced that it expected to keep the funds rate near zero "at least through mid-2013" –, January 2012 – when the FOMC announced that it expected to keep the funds rate near zero "at least through late 2014" –, 12 September 2012 – when the FOMC announced that it expected to keep the funds rate near zero "at least through mid-2015" –, and December 2012.

[2] Similarly, in January 2009 the Bank of England announced the introduction of a QE program, the Asset Purchase Facility, with the aim of lowering long-term interest rates. In the summer of 2013 the Bank of England and the European Central Bank introduced a forward guidance in their monetary policies.

the global increase in long-term rates which followed the May-June 2013 announcement of a Quantitative Easing (QE) tapering in the United States was detrimental to euro-area economies, whose economic recovery was lagging behind at the time.

Clearly, changes in long-term rates in one currency area do not need to be matched by corresponding changes in other areas; variations in exchange rates, in particular, may compensate for the pressure of interest rate differentials. However, during the crisis period there was limited variations in the exchange rates of the major currencies against the US dollar. All in all, the exchange rate does not seem to have responded to changes either in expectations about future policy rates or in term premiums of fixed-income instruments.

We use a standard Gaussian Affine Term Structure Model (ATSM) augmented with forecasts of interest rates, inflation and rate of growth of GDP to decompose US, euro-area and UK interest rates into their expected component and the term premium. The results show that the decrease in long-term rates in the sample period is mainly explained by the drop in term premiums. We also show that the term premium is linked to the inflation rate in every country; hence the decrease in term premiums is mostly explained by the decreasing trend of inflation rates observed since the early nineties. However, the sharp decline observed during the most recent years is also explained by the increase in domestic government bond holdings of the Federal Reserve and the Bank of England. From September 2008 to 2013 (the "crisis period") the correlation among rates has increased, due to the comovement in term premiums, while that between expected rates has become looser. Finally, a VAR analysis confirms that the US interest rates affect those of the other two areas (both in the pre-crisis and the crisis periods), while they are not systematically affected by developments in the euro and sterling fixed-income markets. In the analysis of the robustness of our results we face the issue of using a standard ATSM with the zero-lower bound.

This paper is organized as follows. A brief literature review is presented in Section 2. Data are presented in Section 3. The decomposition of the interest rates is in Section 4. Section 5 presents the model. Results are in Section 6. Section 7 presents an agnostic vector autoregressive analysis. Some preliminary robustness checks are presented in 8. Section 9 concludes.

2 Literature review

An extensive literature has documented the impact of the QE program on yields of US long-term bonds (Gagnon et al., 2010; Kaminska and Zinna, 2014; Krishnamurthy and Vissing-

Jorgensen, 2011; Li and Wei, 2013). In general, these authors find several channels through which these programs have had an impact on asset prices, such as the decrease in the outstanding amount of long-dated securities, but first and foremost the signalling effect that QE program have had in conveying expectations of lower future federal funds rates. Some papers have analyzed the impact on long-term bond yields of explicit forward guidance before and after the financial crisis (Kuttner, 2000; Gürkaynak et al., 2005; Rudebusch and Williams, 2008; Campbell et al., 2012). These authors argue that some features of QEs resemble forward guidance; for example Krishnamurthy and Vissing-Jorgensen (2011) claim that forward guidance has significantly influenced long-term asset prices in the recent period.[3]

However, very few papers have investigated the impact of such measures on international long-term yields and their components (Bauer and Neely, 2014; Chotibhak et al., 2014). The comovement of medium- and long-term interest rates can be due to the interaction of their components, namely the expected future rate and the term premium. The first component of long-term rates embeds the expectation about the future path of short-term rates; changes in this component can impact the same component in other areas under the assumption that monetary policies are seen by market agents as closely intertwined (signalling channel). The second component of long-term interest rates is given by the term premium, i.e. the premium requested by markets to bear liquidity or interest-rate risks; in case of a decrease in term premiums in a given country investors arbitrage out opportunities increasing their holdings of securities of other countries with similar duration and doing so they decrease the term premiums of these securities (portfolio balance channel).

According to the first channel changes in long-term interest rates propagate to other areas through changes in expected short-term rates, while according to the portfolio channel changes in long-term rates propagate through changes in term premiums. Hence, the signalling channel acknowledges that central bank announcements can affect long-term interest rates by signalling a different path for future policy rates. Conversely, the portfolio balance channel implies that central bank bond purchase affects the term premium embedded in long-term interest rates of other countries as investors arbitrage out price differences of assets with similar features.

It is of relevance to understand which component of long-term rates is more strongly correlated among countries, as those components may be impacted differently by policy action, particularly when exit from extraordinary policy measures is being undertaken in the US. If, for example, the comovement were driven by the expected component, i.e. the

[3] Woodford (2012) presents arguments in favour of the effectiveness of the forward guidance with respect to the QE program.

signaling channel, a commitment to forward guidance for non-US countries could be a policy tool to decouple domestic long-term interest rates from the US ones. Alternatively, if the comovement were explained by the portfolio channel, non-US monetary authorities should intervene on the outstanding amount of domestic long-term bonds available to markets.

This paper relates to the recent stream of literature which analyze the impact of changes in the supply of bonds on term structure models with preferred-habitat investors as it introduces the amount of government securities held by monetary authorities (Bernanke et al., 2004; Hamilton and Wu, 2012; Kaminska and Zinna, 2014; Li and Wei, 2013; Vayanos and Vila, 2009).

It should be emphasized that the decomposition of the long-term interest rates into the expected rate and the term premium is model-dependent. This paper uses a standard Affine Term-Structure model (ATSM) for the interest rates of each country so their components are estimated consistently across markets. Potential mis-specification of the model may then impact the levels of the two components but their comovements can be less influenced by the methodology. However, our estimates of the US term premiums are very close to those of Kim and Wright (2005) that are updated regularly on the Federal Reserve Board web-site. Our model is estimated for each country. So this paper belongs to the recent literature on global bond risk premiums estimated separately by country (Wright, 2011; Hellerstein, 2011; Christensen and Rudebusch, 2012) and differs from the recent multi-country ATSM setup (Bauer and Diez de los Rios, 2012; Chotibhak et al., 2014; Pericoli and Taboga, 2012); this latter class of models assumes the existence of an unique global factor which, by construction, coincides with the global term premium and so it is not appropriate to evaluate potential spillovers across countries. Wright (2011), Bauer and Neely (2012), Hellerstein (2011) and Hibiki and Ueno (2013) present estimates of term premiums across several types of ATSMs for different countries.

3 Data

We use end-of-week zero-coupon interest rates implied in the prices of government coupon bonds from January 1999 until December 2013 (Figure 1). US zero-coupon rates are computed by Gürkaynak et al. (2007) and are available on the Federal Reserve Board web-site. Similarly, UK zero-coupon rates are computed by the Bank of England and are available on its web-site. Data for the euro area are computed using mid-quotes of French (OAT and BTAN) and German (Bund) government coupon bonds by means of a B-spline methodology

(Pericoli, 2014), taken from Thomson Reuters. Exchange rates are also taken from Thomson Reuters.

Forecasts of interest rates, inflation rates and GDP growth rates are released by Consensus Economics once per month. Analysts make one-year ahead forecasts of 3-month and 10-year interest rates. Forecasts of inflation and the rate of growth of GDP are for the current and the following year so we take a monthly weighted average of these forecasts to make a one-year ahead forecast for these two variables. Consensus Economics survey forecasts are available monthly and in real time so they are particularly useful in analysing financial data with weekly frequency. We assume that every monthly forecast is released on the Friday of the week when the surveys are published.

Weekly data for the Federal Reserve holdings of Treasuries and their composition by maturity bucket are taken form the Federal Reserve website; the monthly notional outstanding amount of US Treasuries from the St. Louis Fed database, FRED. Weekly data for the Bank of England Reserve holdings of Gilts are taken form the Bank of England website; the quarterly notional outstanding amount of gilts from the UK Debt Management Office (DMO) database.

4 Stylized facts

Long-term interest rates in the three areas considered show a very strong comovement during the sample period (Figure 1). US 10-year zero-coupon rates, after averaging between 4 and 5 per cent from 2003 to 2008, start decreasing from September 2008. The 10-year zero-coupon rates in the United Kingdom and in the euro area follow a similar pattern. The crisis period can be divided in two sub-periods, corresponding to the financial and sovereign debt crises, respectively. In the first sub-period, from September 2008 to July 2011, the level of US 10-year rates is lower than that of the preceding five years (shaded gray areas labelled QE1 and QE2). In the second sub-period, from September 2011 until the middle of 2013, the average 10-year rate is almost 150 basis points below the level recorded in the first sub-period (shaded gray area labelled QE3 and various vertical lines corresponding to the Fed's announcements on forward guidance). As mentioned, the dramatic downward shift in long-term rates can be explained by the combined effect of QE programs and of the introduction of explicit forward guidance by the Federal Reserve since August 2011.

The global financial landscape changed in the spring of 2013, when long-term interest rates recorded a sharp increase in all major economies; the trigger was the May 22 testimony of the Chairman of the Federal Reserve, Bernanke, in which he announced the possibility of a gradual QE tapering and the possible conclusion of the QE3 program by the middle of 2014 and (the green shade in Figure 1).[4] Not surprisingly, the tapering announcement led to an increase in the US long-term interest rates. In addition, long-term rates of other areas increased as well, albeit by a smaller amount, notably in the euro area, in the UK and (only initially) in Japan. All in all, between mid-May and mid-September 10-year zero-coupon interest rates increased by 125 basis points in the US, the UK and in Japan, by 70 basis points in the euro area.[5]

4.1 Term premiums, expected interest rates and exchange rates

Interest rates can be decomposed into an expectations component and a term premium, namely an n-period interest rate can be defined as

$$y_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1} E_t y_{t+i}^{(1)} + tp_t^{(n)} \quad , \tag{1}$$

where $y_t^{(n)}$ is the zero-coupon interest rate at time t on an n-period bond, $y_{t+i}^{(1)}$ is the 1-year interest rate at period $t+i$. The first term on the right hand side is the average expected short-term rate over the subsequent n periods, and $tp_t^{(n)}$ is the corresponding term premium. The expectation component captures the marginal investor's expectation of future monetary policy; the term premium captures the additional return that investors require to compensate them for the interest rate risk embedded in long-term bond positions. If investors were risk-neutral, this premium would be equal to zero. Conversely, in normal conditions, investors require a premium to hold a long-term bond that compensates from illiquidity and interest rate risks – this premium is defined term premium. The propagation of shocks of interest rates from one country to another works through these two different components.

[4] On 22 May and 19 June 2013, Bernanke discussed the possibility of "tapering" some of the Federal Reserve's QE policies contingent upon continued positive economic data. On 18 September 2013, the Federal Reserve decided to hold off on scaling back its bond-buying program, which started on December 2013.

[5] In this Section we limit our comments to interest rates and do not comment on the divergent path of the United States cycle and that of the other advanced economies. In other words, we are implicitly assuming that the rise in medium and long-term interest rates seen in Europe in the summer of 2013 is entirely due to the expected changing conditions of the US bond market. However, the different speed of the creeping economic recovery across the two sides of the Atlantic may also have played a role in driving long-term interest rates.

Figure 1: 10-year zero-coupon interest rates

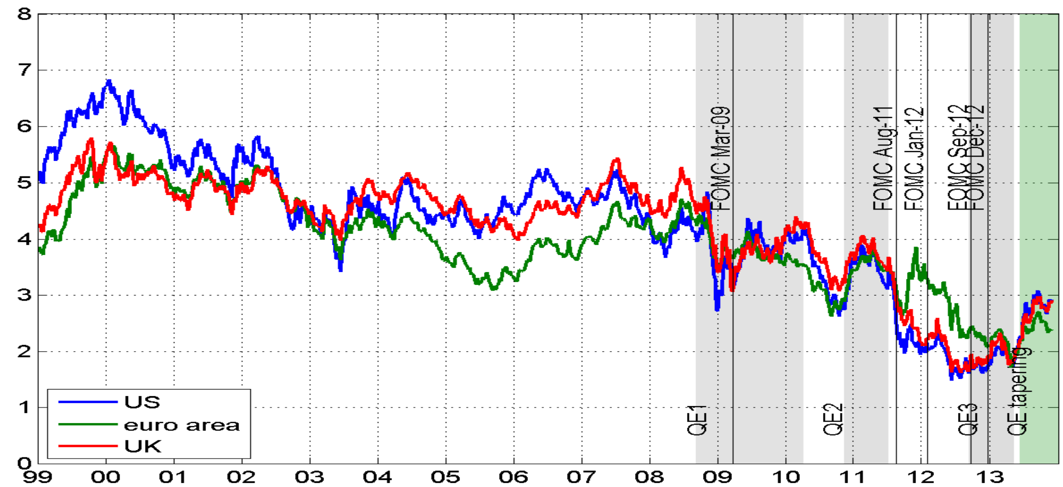
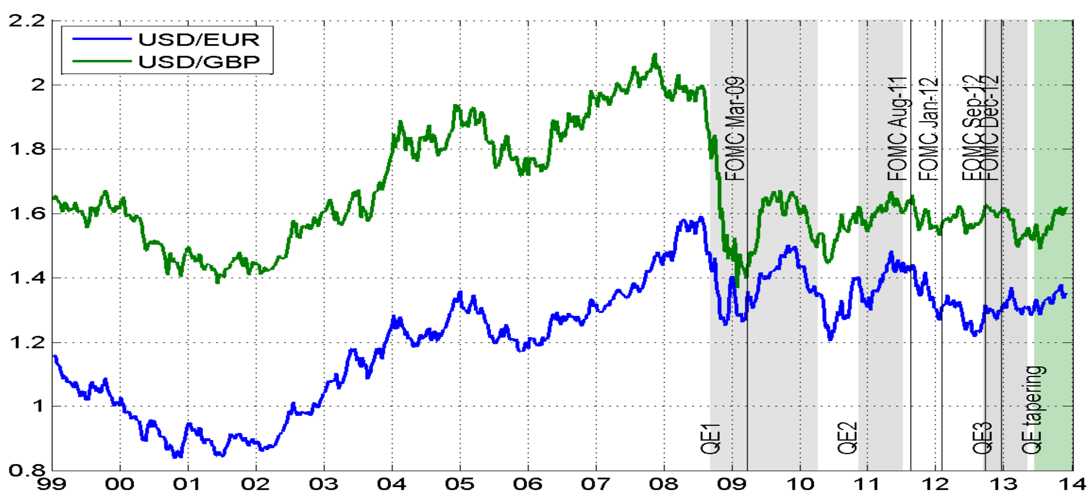


Figure 2: Exchange rates



Note: the shaded areas refer to the periods during which the Federal Reserve carried out its QE1, QE2, and QE3 (gray) and announced the likely QE tapering (green); the vertical lines refer to the FOMC meetings where the forward guidance has been announced. Euro-area zero-coupon rates are computed with French and German government coupon bonds.

Clearly, in an open economy the transmission of changes in interest rates may happen through the two channels mentioned above as well as through changes in the expected exchange rate. However, this last channel does not appear to have been particularly relevant during the crisis period, as supported by the fact that this period has not been characterized by exceptional variations in the two exchange rates against the US dollar (Figure 2). According to the Uncovered Interest Parity (UIP), the differential between two n-period interest rates denominated in different currencies equals the expected change in the exchange rate and a premium component, namely

$$y_t^{(n)} - y_t^{*(n)} = E_t \Delta s_t^{(n)} + fxp_t^{(n)} \quad , \tag{2}$$

where * indicates the foreign country, $E_t \Delta s_t^{(n)}$ indicates the expected variation in the exchange rate of the domestic currency against the foreign currency over n periods, and $fxp_t^{(n)}$ is the foreign-exchange risk premium over the corresponding period. Thus, when we consider the impact of US long-term interest rates on those of the other areas, we should consider also potential changes in the foreign exchange market. We test equation (2) by regressing the change in exchange rate on lagged interest rates, i.e. we run $s_{t+n}/s_t = a + b(y_t^{(n)} - y_t^{*(n)}) + u_t$ under the assumptions that $E_t \Delta s_t^{(n)} = s_{t+n}/s_t$, and find that this ex-post UIP – which implies $a = 0$ and $b = 1$ – does not hold even at the shortest maturity.[6] These results, in line with previous literature, suggest that variations in exchange rates and interest rate differentials are not correlated at any horizon during the sample period (Table 1).[7]

Table 1: ex-post UIP regression

	USD/EUR				USD/GBP			
n	b	t-stat	p-value	R^2	b	t-stat	p-value	R^2
1 month	-0.16	-2.13	0.00	0.006	0.04	0.52	0.00	0.001
3 month	-0.35	-0.80	0.00	0.009	0.19	0.38	0.00	0.002
1-year	-0.74	-0.28	0.00	0.004	-0.28	-0.15	0.00	0.001
2-year	0.55	0.12	0.00	0.001	-2.12	-0.96	0.00	0.034
3-year	1.62	0.35	0.00	0.007	-4.06	-1.96	0.00	0.102
4-year	2.87	0.53	0.00	0.022	-1.79	-0.70	0.00	0.015
5-year	-0.35	-0.05	0.00	0.001	1.02	0.44	0.00	0.004

Notes: Estimates of the regression $s_{t+n}/s_t = a + b(y_t^{(n)} - y_t^{*(n)}) + u_t$ with weekly data from January 1999 to December 2013. As exchange rates are computed on overlapping periods, standard errors are corrected for serial correlation with the Newey-West methodology. Column b report estimates of the b coefficient, column t-stat the corresponding t-statistics, column p-value the F-test probability of a joint test $a = 0, b = 1$, R^2 the adjusted R-squared.

[6] This is a version of the unbiasedness of the forward rate hypothesis where we run the regression $s_{t+n}/s_t = a + b \cdot f_{t+n}/s_t + u_t$ and f_{t+n} is the forward exchange rate. If this hypothesis holds, we have $a = 0$ and $b = 1$. The residual of this regression can proxy the unexpected change in the exchange rate and the foreign exchange premium component.

[7] This interpretation of the results holds if the risk premium is not correlated with the unexpected change in the exchange rate and with the interest rates.

5 The model

We use a standard Affine Term Structure Model set in discrete time

$$\begin{aligned} Y_t &= A + BX_t + R\eta_t \\ X_t &= \mu + \rho X_{t-1} + \Sigma\varepsilon_t \\ \eta &\sim N(0,I), \quad \varepsilon \sim N(0,I), \quad R \perp \Sigma \ , \end{aligned} \tag{3}$$

where Y is a vector which contains weekly interest rates, with maturity between 1 and 10 years, and the monthly forecasts for the 3-month interest rates 12 months ahead and the 10-year interest rate 12 months ahead. X is a vector with three latent factors, the monthly expected inflation and the rate of growth of GDP;[8] see the Appendix for a thorough specification of the model. Based on the state space representation, the factors are filtered according to the Kalman filter; given estimates of the latent factors $\widehat{X}_t$, the parameters can be estimated by maximum likelihood, based on the conditional distribution of $Y_t|Y_{t-1}$ for each observation. The Kalman filter allows to use data with different frequencies, i.e. weekly for the interest rates, and monthly for the expected interest rates, the rate of inflation and the rate of growth of GDP.

The recent literature has argued that there are indeed factors in the term structure of interest rates that are unspanned; an unspanned factor will help to forecast future interest rates, but it will not affect today's term structure and it will not be possible to recover it from observed yields. Macroeconomic variables, such as output growth or inflation, may be unspanned factors, as they are important for forecasting future interest rates, but evidently do not lie in the span of the term structure of interest rates as they are not needed to fit the cross-section of current yields. In a second estimation, as a measure of the QE impact, we also add to the unspanned factors X the ratio of Treasuries (Gilts) holdings in the Federal Reserve (Bank of England) balance sheet with respect to the outstanding total amount of Treasuries (Gilts; Figures A.2 and A.3). The US measure of QE is also plugged in the euro-area and in the UK model to evaluate the impact of the US QE program on the other countries term structures and term premiums .

[8] The estimation of ATSM with a flexible specification of the market price of risk is beset by a severe small-sample problem arising from the highly persistent nature of interest rates. Bauer et al. (2012) show that, without a correction for the small sample bias, estimates of the expected short-term rate have a very limited variation. Kim and Orphanides (2012) propose using survey forecasts of a short-term interest rate as an additional input to the estimation to overcome the problem. We also add survey short-term forecasts of the ten-year interest rate to anchor the of the long-term premium component.

This model is extremely flexible and is capable to estimate the two components of the zero-coupon interest rates, namely the expected short rate and the term premium (Pericoli, 2012). In particular the term premium can be rewritten as

$$tp_t^{(n)} = \overline{tp}^{(n)} + tp^{(n)}(X_t) + (Jensen\ inequality)\ ,$$

i.e. the sum of a *constant*, $\overline{tp}^{(n)}$, a *function of the state variable* X and a correction for the volatility (Jensen inequality).

6 Results

Term premiums in the three areas decreased between 2000 and 2003 and remained stable until the start of the financial crisis, in September 2008. Since then term premiums recorded a sharp decrease in the three areas. Term premiums in the US and in the UK remained positive in the first crisis period (Sep. 2008 – Aug. 2011) but rapidly dropped in the last quarter of 2011 (Figure 3). The euro-area term premium also decreased and became negative since the last quarter of 2012. The US term premium entered negative territory in mid-2011, after the introduction of QE2 and the announcement of the forward guidance at the August 2011 FOMC meeting. From mid-2011 to mid-2013 the US term premium ranged between 0 and -1.1 percentage points and moved back toward zero only after the QE tapering announcement. This last increase in the term premium seems consistent with the investors' scare of a decrease in the demand of US long-term bonds driven by the announced tapering of the Federal Reserve's bond-buying program.[9] Expected short-term rates over the ten-year horizon are presented in Figure 4.

The decomposition of the long-term interest rates between expected rate and term premium is clearly model-dependent and suffers of the starting assumptions. We tackle the small-sample bias stressed by Bauer et al. (2012) and Bauer et al. (2014) by using the short-term forecasts of the 3-month and of the 10-year interest rates; moreover, in the robustness checks we present the results of a Principal Component Analysis before and during the crisis to show that there has not been a clear change in the relative importance of the factor loadings between the two periods and, then, our model can be reasonably used also when short-term rates are close to the zero-lower bound. All in all, the results show that our

[9] Our estimates of the US 10-year term premium are similar to those computed by Kim and Wright (2005) during the financial crisis; see Figure A.1 in the Appendix.

estimates of the 10-year US term premium are very close to that of Kim and Wright (2005) that are updated regularly on the Federal Reserve Board web-site (Figure A.1). Conversely, 5-year ahead 5-year expected interest rates show a larger variability than those computed by Wright (2011), a smaller one to those computed by Bauer et al. (2014) (see Figure A.7); this suggests that the introduction of the short-term forecast largely mitigates the flatness of this component observed in standard OLS estimates.

During the crisis, term premiums have generally declined across the maturity spectrum (Table **??**). From 1999 to 2008 term premiums showed an increasing term structure with a slope between the 1-year and the 10-year maturity of around 150 basis points, in the US and in the euro area, of 70 basis points in the UK. During the crisis, the term premiums became negative on average and dropped by 100-150 basis points across the maturity spectrum. Conversely, expected interest rates decreased by 1.5 percentage points for the 1-year maturity to 0.30 percentage points for the 10-year maturity. However, expected short-term rates over the ten-year horizon show a steady pattern after the start of the US QE preogram (Figure 4).

Table 2: The term structure of term premiums and expected rates

	Jan. – 1999 / Aug. – 2008			Sep.-2008 / Dec.-2013		
Term premium	US	euro	UK	US	euro	UK
1 year	0.24	0.30	0.91	-1.29	-0.76	-1.05
2 year	0.52	0.50	1.15	-1.37	-0.66	-1.04
3 year	0.76	0.69	1.32	-1.26	-0.51	-0.88
4 year	0.97	0.87	1.44	-1.07	-0.35	-0.68
5 year	1.16	1.04	1.52	-0.83	-0.18	-0.47
6 year	1.33	1.19	1.57	-0.58	-0.01	-0.26
7 year	1.49	1.32	1.60	-0.34	0.16	-0.07
8 year	1.63	1.43	1.62	-0.12	0.30	0.11
9 year	1.76	1.53	1.63	0.08	0.44	0.28
10 year	1.87	1.61	1.63	0.25	0.55	0.42
Expected rate						
1 year	3.33	3.00	3.84	1.64	1.50	1.73
2 year	3.26	2.95	3.64	1.95	1.74	2.00
3 year	3.21	2.91	3.49	2.17	1.93	2.20
4 year	3.18	2.88	3.39	2.32	2.09	2.34
5 year	3.16	2.85	3.32	2.43	2.22	2.44
6 year	3.14	2.82	3.26	2.52	2.33	2.51
7 year	3.12	2.80	3.22	2.59	2.41	2.57
8 year	3.11	2.78	3.19	2.64	2.48	2.62
9 year	3.10	2.77	3.16	2.68	2.54	2.65
10 year	3.10	2.76	3.14	2.71	2.59	2.68

Notes: The Table reports the averages of weekly data for the periods in the legend. Euro-area zero-coupon rates are computed with French and German government coupon bonds.

Figure 3: 10-year term premium

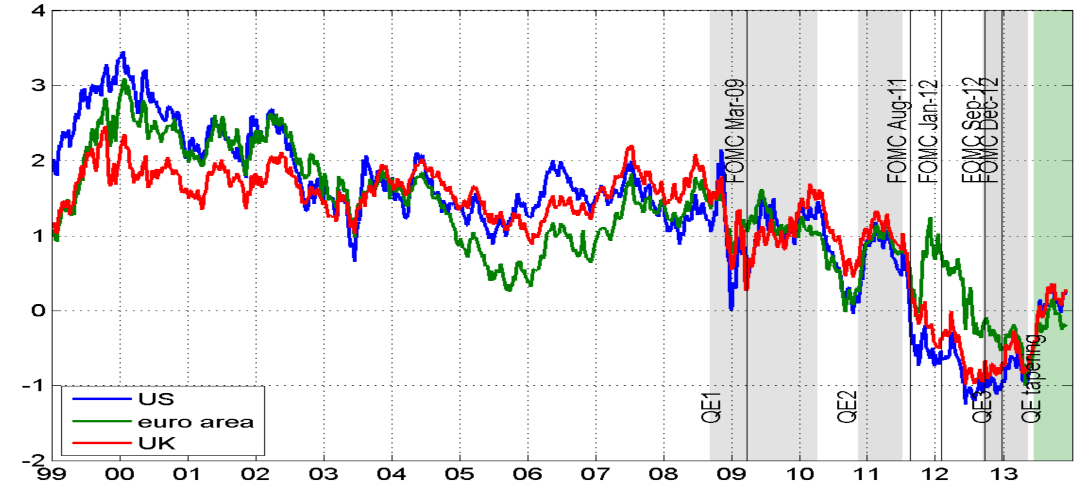
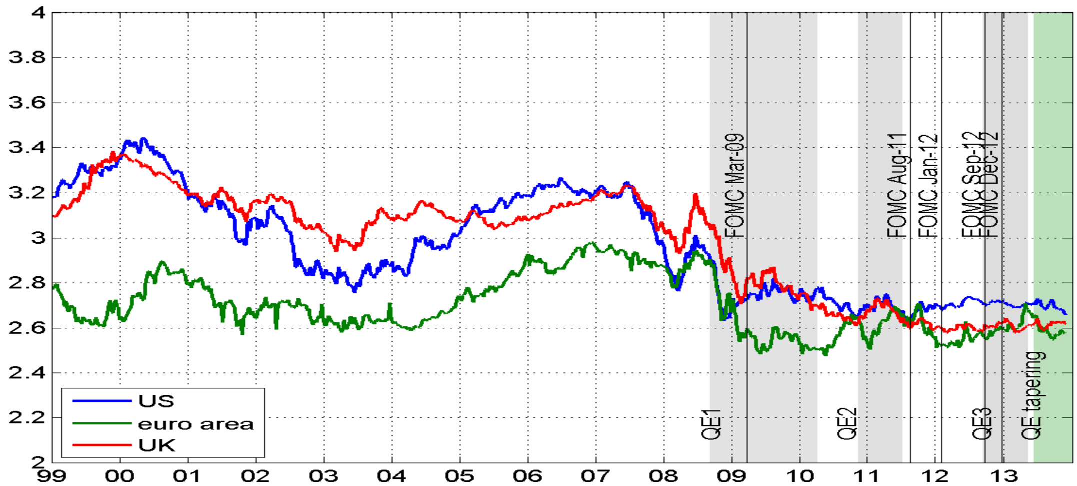


Figure 4: 10-year Expected short-term rates



Note: the shaded areas refer to the periods during which the Federal Reserve carried out its QE1, QE2, and QE3 (gray) and announced the likely QE tapering (green); the vertical lines refer to the FOMC meetings where the forward guidance has been announced. Euro-area zero-coupon rates are computed with French and German government coupon bonds.

The introduction of the US QE program has changed not only the relative contribution of term premiums and expected components to interest rates internally, but also the linkages among areas. An analysis of the correlation among interest rates and their components reveals the different response to the introduction of QE of the channels of spillovers at work before and after the burst of the crisis. The cross-sectional correlation for the couples US/euro-area and US/UK and for rates with corresponding maturities are reported in the Appendix (Figures A.4–A.5). The comparison between the pre-crisis and crisis period highlights sizeable changes in the linkages among interest rates in the three areas. As far as interest rates are concerned, the average correlation of levels (left column) is upward sloping along the maturity spectrum before the crisis for the US/euro-area and the UK/euro-area interest rates while it becomes flat and close to unity in the crisis period. The same holds for the term premium; the correlation between the medium-term premiums is low in the pre-crisis period while it increases in the crisis period. This change is reflected by a downward sloping correlation in the expected rate, which becomes negative in the US/euro-area case. Changes in the comovement thus mostly reflect the portfolio balance channel rather than the signalling channel.

The changes in the correlation between the first difference of interest rate and their components are less pronounced but present similar features (right column).

All in all, the comparison between the two periods highlights a strengthening of the linkages between interest rates and term premiums and a loosening of the linkages between expected interest rates (Figures 3 and 4). Naturally, bivariate analyses such as this one, which focuses on the relation between pairs of financial variables, can be misleading as other factors can be at play. As pointed out by Krishnamurthy and Vissing-Jorgensen (2011), for example, it is likely that the introduction of the US forward guidance has impacted not only the US expected rate but also the US term premium and, more in general, the term premiums of other countries. This suggests us to use a multivariate analysis to detect potential interactions among the components of the interest rates.

6.1 An event study for 2013

The analysis of the recent period is useful to evaluate the impact of the QE tapering announcement and its impact on the other countries' interest-rate components. The increase in long-term interest rates, which started since the May 2013 announcement of a possible QE tapering, has been concomitant in all three areas and can be attributed entirely to the term-premium component (Figures 3 and Table 3); in fact, expected short rates either remained steady, as in the US and in the UK, or moderately decreased, as in the euro area, and in general did not comove across economies (Figure 4). In addition, in the United States the decreasing trend of the expected short-term rate since September 2013 may signal a greater effectiveness of the forward guidance.

As a reaction to this spillover effect, the ECB also introduced a forward guidance policy in July 2013, with the objective of guiding market expectations; this policy measure seems to have been partly effective in decoupling euro-area short-term interest rates from the US ones. By contrast, the forward guidance of the Bank of England, introduced in August 2013, had a more limited effectiveness, in part because the UK economy was strengthening rapidly during that period. Second, the strong comovement of term premiums among areas since last May indicate that, in order to ensure some degree of disconnect between domestic yields and the US ones, the European monetary authorities might envisage policy tools that have a direct impact on term premiums, such as some kinds of unconventional monetary policy measures.

Table 3: Changes in 10-year term premiums from April to November 2013

	US	euro area	UK
April-May 2013	52	28	41
May-June 2013	39	34	55
June-July 2013	10	-3	-7
July-August 2013	20	23	33
August-September 2013	-13	-11	-10
September-October 2013	-12	-11	-9
October-November 2013	30	-4	23
April-November 2013	126	55	128

Note: The table reports the changes in 10-term premiums in basis points from the end of two months. The sample runs from the announcement of the US QE tapering to its start after the FOMC meeting of 18 December 2013. The QE variable is computed as the ratio, in percentage points, between the US Treasuries (UK Gilts) in the Federal Reserve (Bank of England) balance sheet and the outstanding amount of US federal (UK) debt.

6.2 The impact of macroeconomic variables on term premiums

The impact of macro variables on interest rates is different in every area but shows a common pattern (Table 4). Inflation turns out the main driver of the term premium with or without the inclusion of the QE variable; namely, its impact is slightly below 1.0 in the euro area, at about 0.6 in the United Kingdom and ranges between 0.7 and 1.1 in the United States. Conversely the impact of the GDP rate of growth is scant and its significance changes with the inclusion of the QE variable. The strong linkages between term premiums and inflation rates in the three areas is also documented by their common trends shown since 1990 (Figure A.6). Finally, we estimate the impact of the US QE measures on the 10-year term premium and see that it is large and significant. If we multiply the average Treasury holdings of the Federal Reserve from September 2008 to December 2013, 6%, by the corresponding estimated coefficient we obtain the relative contribution of the US QE program to the decrease in the US 10-year term premium; for the US, the US QE program has decreased the 10-year term premium by $-0.25 \times 6\% = -1.50$ percentage points (the corresponding impact in the euro area and the UK, respectively, amounts to -1.02 and -1.26 percentage points). For comparison, the impact of the Bank of England's purchase of Gilts on the UK 10-year term premium has been equal to the average Gilt holdings of the Bank of England, 22%, times the corresponding coefficient, namely the impact on the UK 10-year term premium equals $-0.08 \times 22\% = -1.76$ percentage points; the impact is about the same if one consider the US and UK QE programs.[10]

Table 4: Impact of macro variables on 10-year Term premiums

	US	US	euro area	euro area	UK	UK	UK	UK
GDP	-0.11***	-0.04	-0.01	0.47***	-0.02	0.21	-0.01	-0.01
inflation	0.70***	1.13***	0.95***	0.98***	0.55***	0.63***	0.63***	0.59***
QE^{US}	-0.25***	—	-0.17***	—	-0.21***	—	—	-0.13***
QE^{UK}	—	—	—	—	—	—	-0.08***	-0.04***
constant $\overline{tp}^{(n)}$	0.46**	-1.27***	0.16*	-1.13***	-0.93***	-2.11***	-1.36***	-1.03***

Note: Standard errors are robust to specification error as they are obtained by combining the delta method and the Huber Sandwich estimator. Sample: 1999-2013. ***/**/* show that the parameter is significant at the 1%/5%/10% significance level.

Moreover, we use the model to evaluate the impact of a shock to the inflation rate and to the rate of growth of GDP on term premiums with maturity of 3, 5, 7, and 10 years (Figure 7-9). In all three countries the 10-year term premium is impacted by the inflation rate – i.e. an one standard deviation shock produces a change in the 10-year term premium of around $0.6 - 0.8$ percentage points; conversely the impact of a shock to the GDP rate of growth is relatively small and not uniform across the three markets. In general, term premiums across the maturity spectrum respond to shocks to inflation while they react scarcely and differently across countries to shocks to GDP.

The US QE program has also impacted the term premiums across the maturity spectrum in all three areas, as evidenced by Figure 10. In general we have that a one percentage

[10] Recent research also suggests that the two QE programs induced a comparable reduction in long-term government bond yields in each country. For the United States, Gagnon et al. (2010) report a cumulative decline in the ten-year U.S. Treasury yield of 0.91 percentage points. For the United Kingdom, Joyce et al. (2011), report that long-term U.K. government Gilt yields fell a total of about 1 percentage point. Conversely Christensen and Rudebusch (2012) claim that the impact has been large in the UK but negligible in the US where the key effect of the Fed's QE program was to lower policy expectations.

point increase in Fed's holdings of US Treasuries over the outstanding amount decreases the term premium by one basis point. However, the size of the response changes if the shock is maintaned constant for a given period.

Figure 7: Impulse response analysis of US term premiums

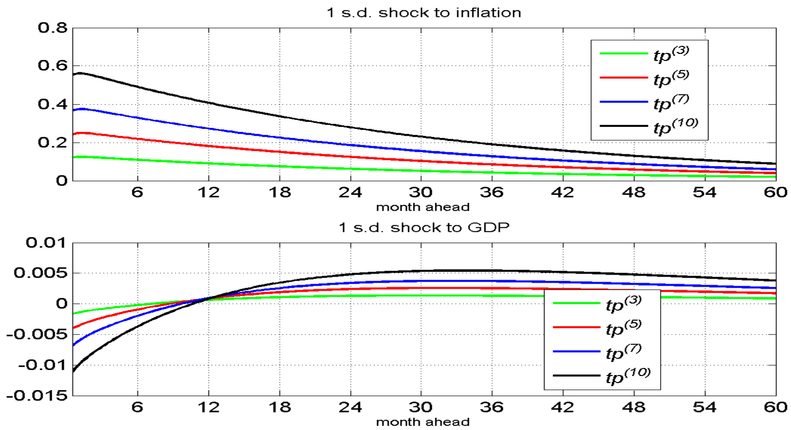


Figure 8: Impulse response analysis of euro-area term premiums

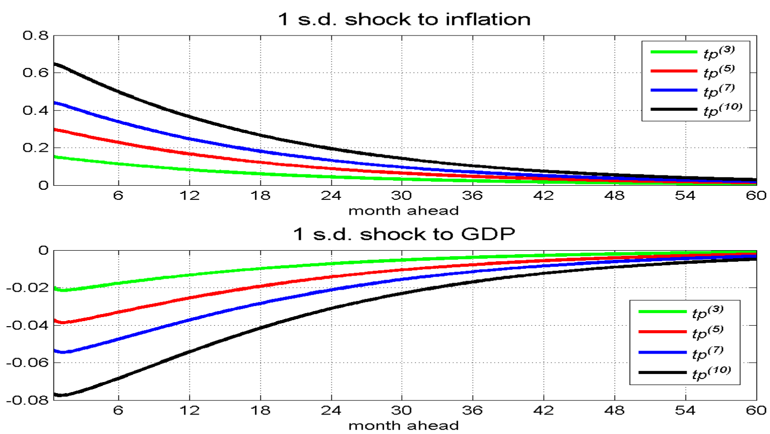


Figure 9: Impulse response analysis of UK term premiums

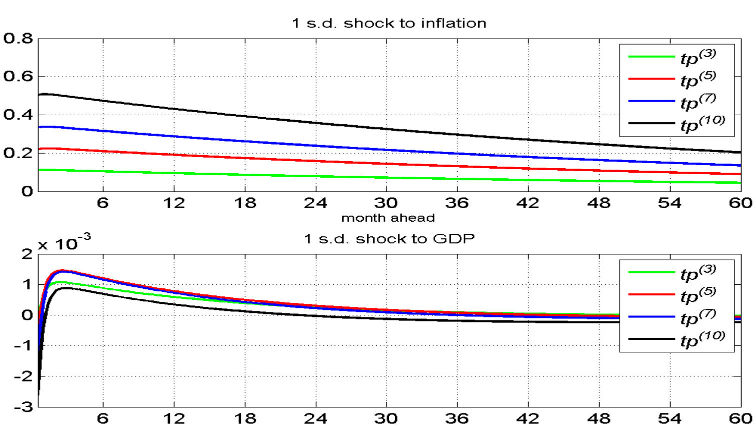
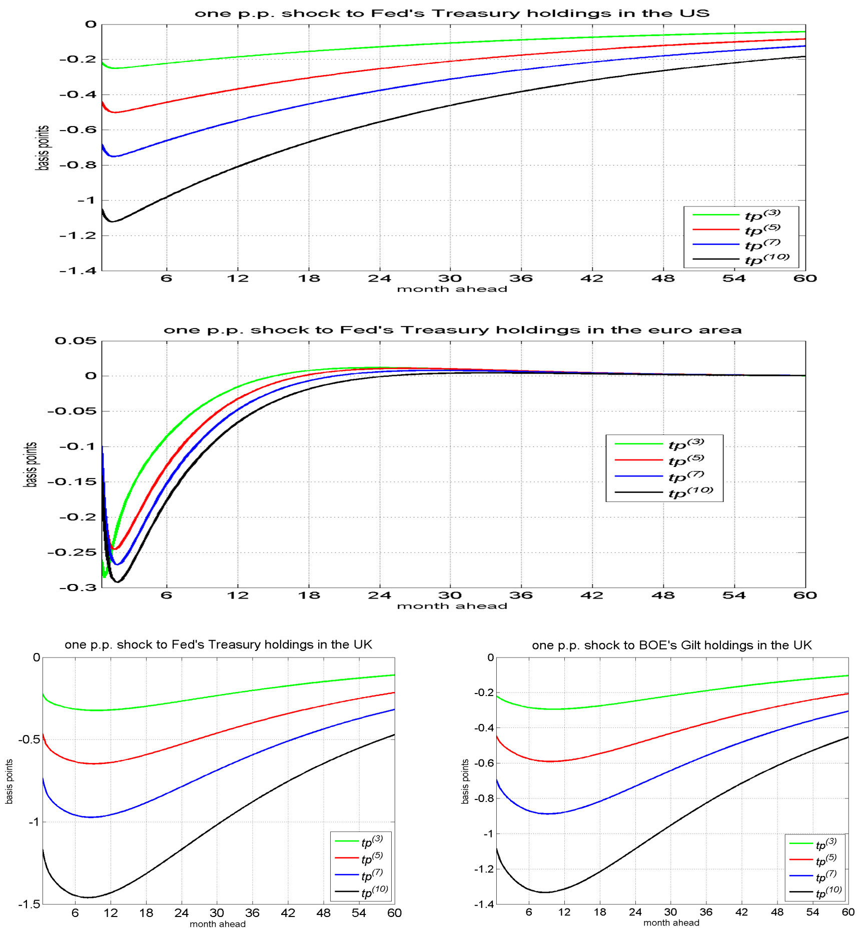


Figure 10: Response of term premiums to QE measures



7 Linkages across long-term rates

The interaction between the components of the long-term interest rates in the three areas is evaluated by means of a Vector Auto-Regressive (VAR) model. We estimate two models, the first with US and euro-area variables, and the second with US and UK variables. For simplicity we limit our multivariate analysis to variables with 10-year maturity. We run the following VAR

$$Z_t = C_0 + \sum_{i=1}^{2} C_i Z_{t-i} + \Omega u_t \tag{4}$$

where the Cs are matrices, and Z is a vector with the following variables: 1) the US 10-year term premium, 2) the US expected rate over the next 10 years, 3) the 10-year euro area (UK) term premium, 4) the euro-area (UK) expected rate over the next 10 years defined as in equation (1), and 5) the logarithm of the exchange rate of the USD against the euro (British pound), Ω is a positive definite matrix, and $u \sim N(0, I)$ is a multivariate normal white noise.[11]

The choice of the ordering reflects the moves in monetary policy during the crisis period. Since September 2008 successive waves of QE programs have been launched in the US, together with the introduction of forward guidance in the US, followed by forward guidance in the euro area and UK only more recently. This suggests that changes in US variables tend to pre-date those in the other two areas. For consistency, the same ordering has been imposed in the pre-crisis period. We put exchange rates last in the ordering but results do not change if they are in the first place.

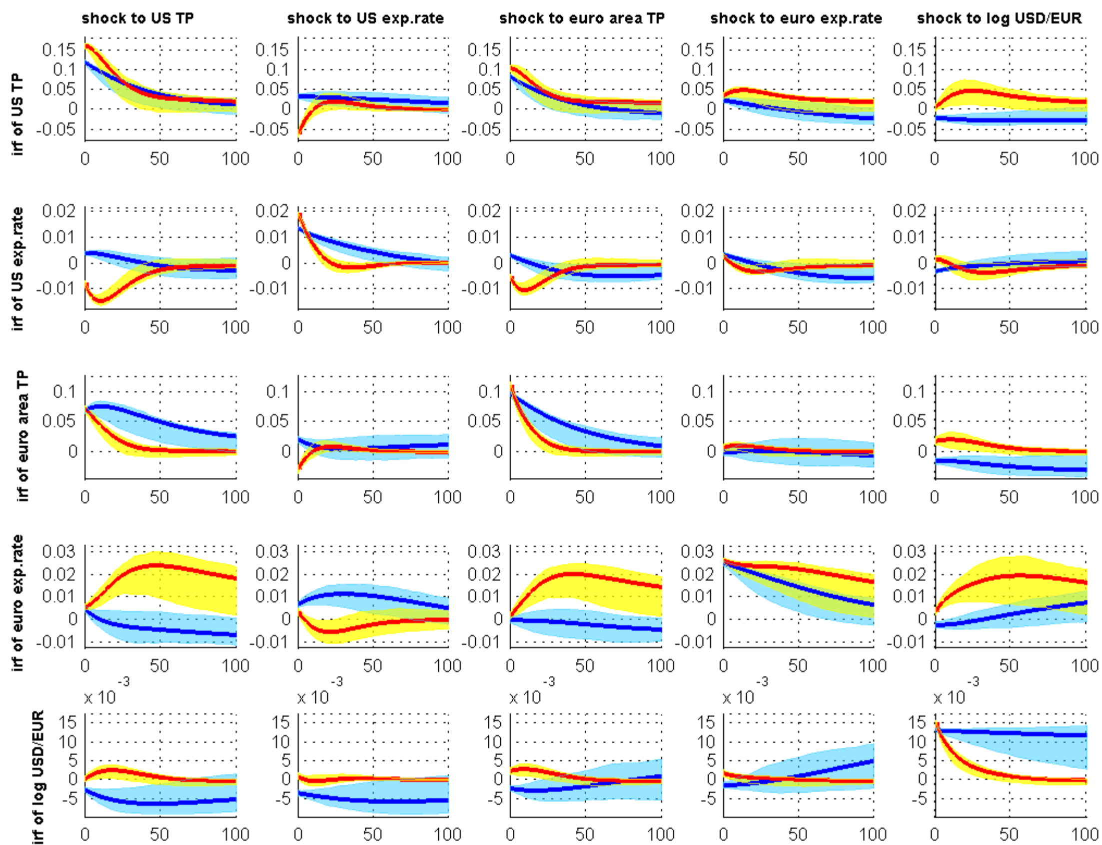
We present the impulse response function of model (4) obtained by a Cholesky decomposition of the covariance matrix for the pairs US/euro-area and US/UK for the pre-crisis (January 1999 – September 2008) and crisis periods (October 2008 – December 2013). An alternative tool can be the analysis of the variance decomposition to evaluate the relative importance of the determinants of expected long-term interest rates and of term premium.

The main results of the VAR analysis (Figures 11-12) are the following.[12] First, in the pre-crisis period the US term premium impacts the euro-area and the UK term premiums; in the crisis period this impact becomes larger. Second, during the crisis period, differently from the previous period, a shock in the expected US rate decreases the US term premium; this result is somewhat puzzling since it would imply a weak effect of the forward guidance. Third, euro-area interest rates have a smaller impact on either US term premiums or US expected short-term rates, while UK interest rates impact on US rates does not change considerably. Finally, exchange rate shocks have no impact on the other variables of the VAR before the crisis while they impact US and euro-area term premiums and the euro-area extected rates during the crisis.

[11] The lag length is selected by means of the Akaike and Schwartz criteria.

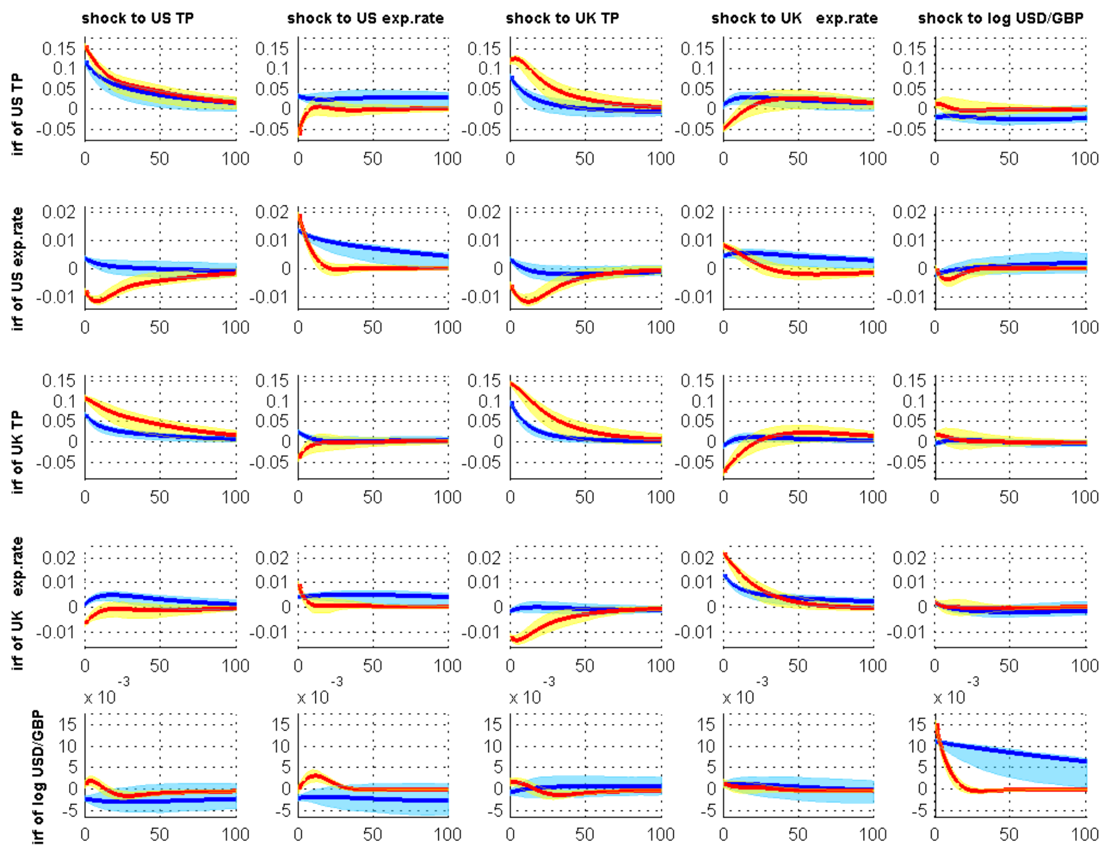
[12] These results are based on the analysis of the 95% confidence bands of the impulse respons functions that we do not show here but are available.

Figure 11: Impulse response analysis US-euro area



Impulse responses to a one standard-deviation shock on the variable shown in column. 90% confidence intervals are shaded. Blue: sample from January 1999 to September 2008. Yellow: sample from October 2008 to December 2013. X-axis: response time in weeks.

Figure 12: Impulse response analysis US-UK



Impulse responses to a one standard-deviation shock on the variable shown in column. 90% confidence intervals are shaded. Blue: sample from January 1999 to September 2008. Yellow: sample from October 2008 to December 2013. X-axis: response time in weeks.

8 Robustness

We have used the ratio of medium and long term bonds held by the Federal Reserve with respect to the total Treasury holdings as a measure of the US QE; this ratio approximates the duration of the Fed Treasuries' portfolio, which may directly affect the term premiums in the global bond markets. Furthermore we have estimated the VAR (4) by changing the ordering of the variables and inserting the exchange rates in the first place. Results, albeit preliminary, are consistent with those presented above.

[With ultra-low interest rates the effectiveness of ATSMs may be severely impaired.[13] In particular, the slope of the term structure, usually positive, may become flat as interest rates shrink across maturities. In order to address this issue we have compared the loadings of the first three principal components of the term structure for the US, euro-area and UK interest rates computed from January 1999 to September 2008 with those computed from October 2008 to December 2013 (Figure A.8). We see that the loadings of the three principal components do not differ substantially and, in particular, the loadings of the slope factor, most likely affected by the zero-lower bound of interest rates, is stable across the three markets, with the exception of the US slope factor that steepens in the second period. This very simple results may suggest that the ATSM can price reasonably well the interest rates even with short-term rates close to zero.]

As far as euro-area interest rates are concerned, my estimates for the euro-area term structure use zero-coupon rates estimated jointly from French and German coupon government bonds, whose spread has widened up to 100 basis points during the most acute phase of the euro-area debt crisis, and so may be thought as not homogenous. The rationale behind the choice of a term structure with French and German bonds is that their combined interest rates were not greatly affected by the large flight-to-liquidity which has pushed German short-term interest rates into negative territory at the peak of the euro-area debt crisis. Then, even if one can assume that French yields may have reflected some credit-risk premium, we are confident that the use of a joint French-German term structure can be seen as a sensible choiche. However, results obtained using only German interest rates with are not different.

9 Conclusion

We use a standard Gaussian Affine Term Structure Model (ATSM) augmented with forecasts of interest rates, inflation and rate of growth of GDP to decompose interest rates into their expected component and the term premium with the aim of detecting the channels at work during the US and UK quantitative easing programs and explaining the spillovers among global interest rates. The results show that the decrease in long-term rates during the crisis is mainly explained by the drop in term premiums. We find that the term premium is linked to the inflation rate in every country. However, the sharp decrease observed during the last years also reflects the quantitative easing policies carried out by the Federal Reserve and

[13] This point has been raised by Jens Christensen and is behind the motivation of building ATSM with a zero-lower bound on interest rates.

the Bank of England through purchases of domestic government bonds. Since the final part of 2008 the correlation among long-term interest rates in the United States, in the United Kingdom and in the euro area has increased; this has reflected a higher degree of comovement between term premiums, while the correlation between the expected rate components has become looser. A VAR analysis also confirms that the US interest rates affect those of the other two economies, while they are not systematically affected by developments in the euro and sterling fixed-income markets.

The results have very intuitive policy implications as they suggests that during the crisis the cross-country transmission of shocks to medium- and long-term rates has been mainly driven by the portfolio balance channel, which mainly operates through the term premium. Central-bank purchases of long-term bonds have contributed, jointly with the decrease in inflation, to lower term premiums, which in turn have spilled over to other markets. However, the introduction of the forward guidance by the ECB limited the variation in euro-area long-term interest rates by anchoring their expected component.

These results are particularly relevant to analyse the rise in long-term rates that took place after the first announcement of a possible US tapering in May-June 2013. The concomitant rise in US, UK and euro-area rates can be attributed entirely to the comovement of term premiums; by contrast, expected short rates either remained steady (in the US and the UK) or decreased moderately (in the euro area), and in general did not comove across economies.

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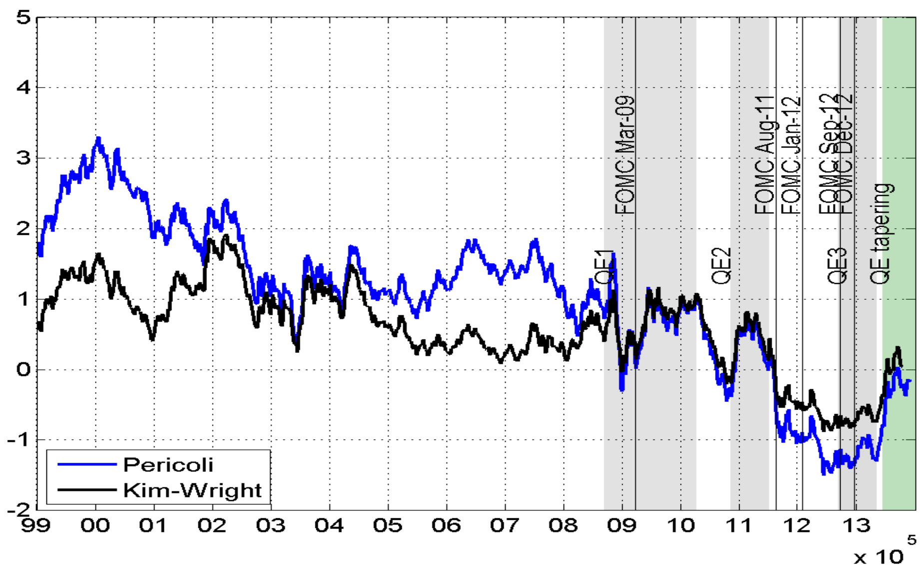
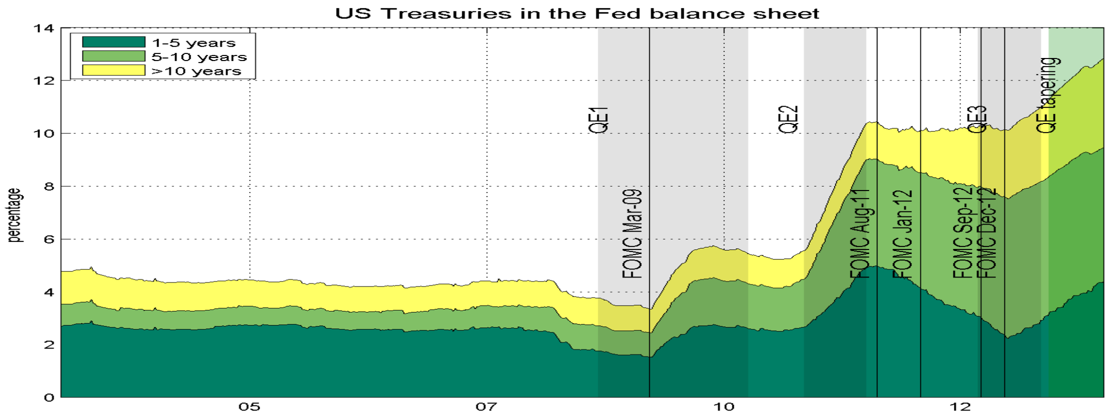
A Appendix

Figure A.1: comparison between 10-year term premiums



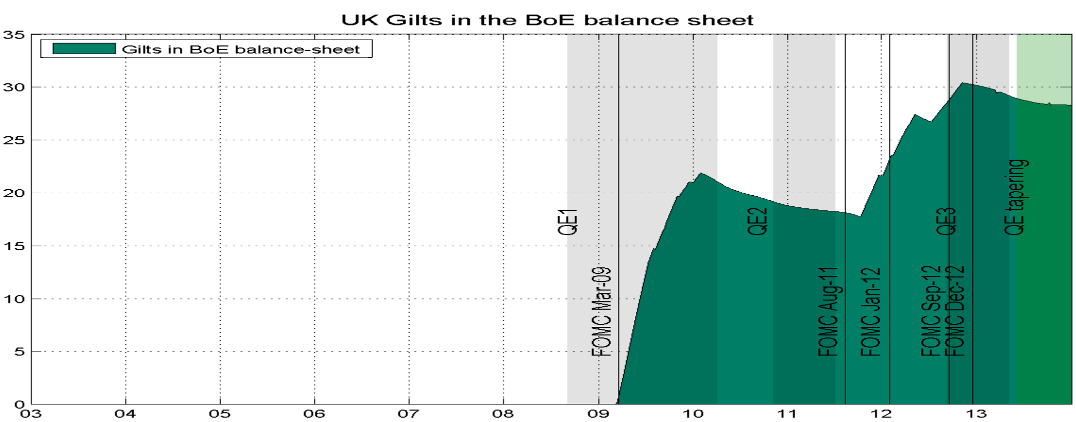
Note: the shaded areas refer to the periods labelled QE1, QE2, and QE3 (gray) and QE tapering (green); the vertical lines refer to the FOMC meetings where the forward guidance has been announced.

Figure A.2: US Treasuries in FED balance sheet



Note: The Figure reports the ratio, in percentage points, between the US Treasuries in the Federal Reserve balance sheet and the outstanding amount of US federal debt. Data are weekly data for the US Treasuries Federal Reserve holdings and converted to weekly from monthly data for the US federal debt. The shaded areas refer to the periods labelled QE1, QE2, and QE3 (gray) and QE tapering (green); the vertical lines refer to the FOMC meetings where the forward guidance has been announced.

Figure A.3: UK Gilts in BoE balance sheet



Note: The Figure reports the ratio, in percentage points, between the UK Gilts in the Bank of England balance sheet and the outstanding amount of public UK debt. The UK QE program started in January 2009, when the Chancellor of the Exchequer authorised the Bank of England to set up an Asset Purchase Facility (APF) to buy high-quality assets financed by the issue of Treasury bills and the DMO's cash management operations. Data are weekly data for the UK Gilts Bank of England holdings and converted to weekly from quarterly data for the UK debt. The shaded areas refer to the periods of the US QE labelled QE1, QE2, and QE3 (gray) and QE tapering (green); the vertical lines refer to the FOMC meetings where the forward guidance has been announced. Sources: Bank of England and UK Debt Management Office.

Figure A.4: Correlation, Jan/1999 – Sep/2008

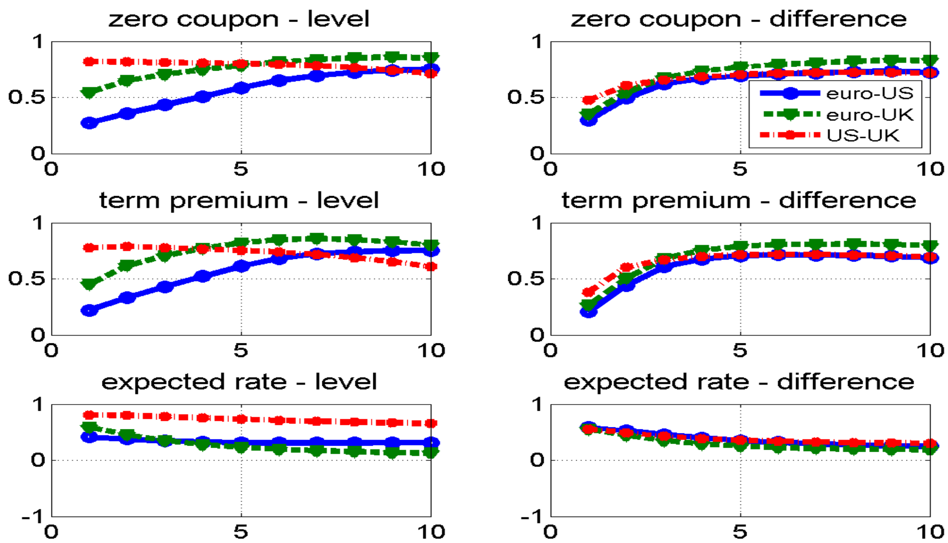


Figure A.5: Correlation, Sep/2008 – Dec/2013

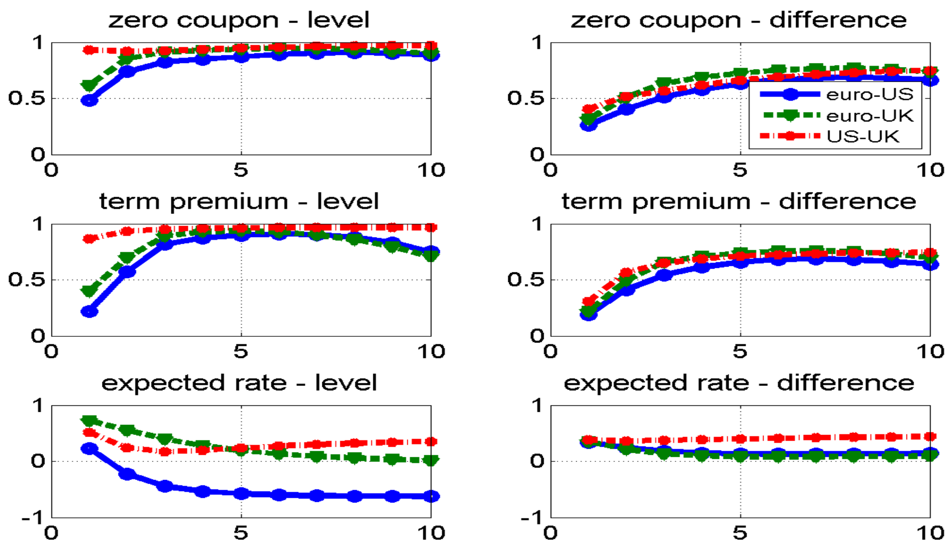


Figure A.6: 10-year Term premiums and macro variables

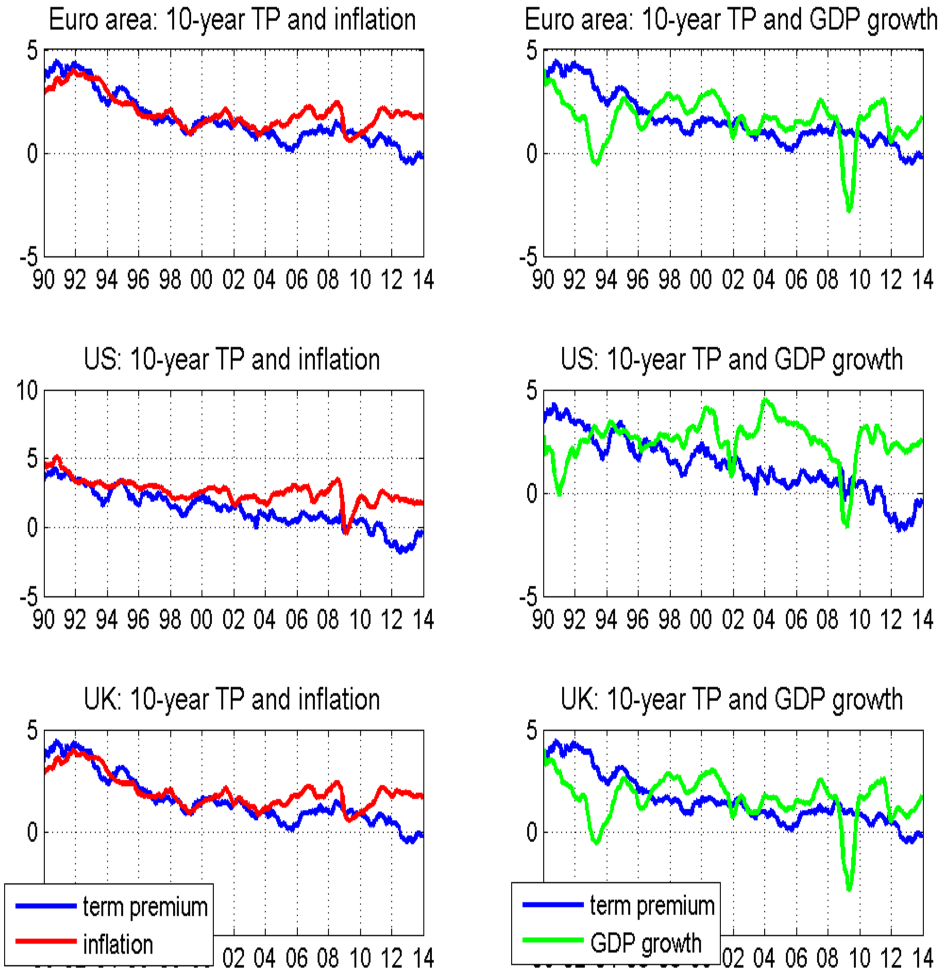
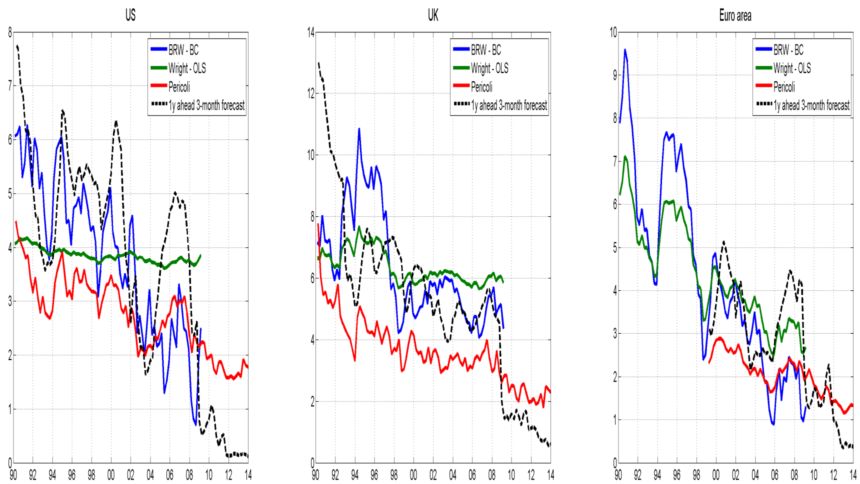
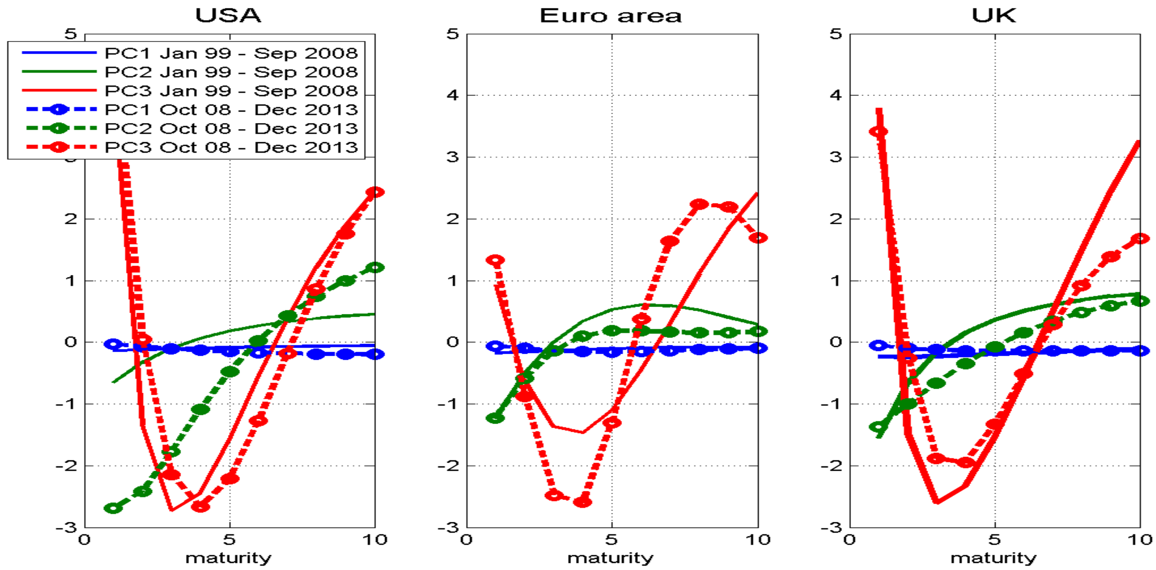


Figure A.7: Expected five-to-ten-year ahead 1-year rates



Note: The Figure reports estimates of expected five-to-ten-year ahead 1-year rates computed with OLS by Wright (2011) and Wright (2014) (Wright-OLS), the bias-corrected estimates computed by Bauer et al. (2014) (BRW- BC), estimates of model 3 (Pericoli), and the 12-month ahead 3-month interest rate forecasts surveyed by Consensus Economics.

Figure A.8: Comparison of PCA factor loadings



Note: The Figure reports the factor loadings of the first three principal components of interest rates from 1 to 10 year maturity for the period from January 1999 to September 2008 and from October 2008 to December 2013. The factors are computed by a principal component analysis (PCA) on the covariance matrix of the deamened interest rates, then the loadings are standardized by the standard deviation of the factors to make the loadings comparable across markets.

B The affine term structure model

Following Dai and Singleton (2000) a standard no-arbitrage standard Gaussian affine term structure model (ATSM), set in discrete time can be written in the state-space form as

$$\begin{aligned} Y_t &= A + BX_t + R\eta_t \\ X_t &= \mu + \rho X_{t-1} + \Sigma\varepsilon_t \\ R &\perp \Sigma \ , \end{aligned} \tag{5}$$

where the first equation is the observation equation and the second is the state equation.

B.1 State equation

The second line of model (3) is the state equation describes the dynamics of the vector of state variables X_t (a k-dimensional vector, $k \in \mathbb{N}$):

$$X_t = \mu + \rho X_{t-1} + \Sigma\varepsilon_t \ , \tag{6}$$

where $\varepsilon_t \sim N(0, I_k)$, μ is a $k \times 1$ vector and ρ and Σ are $k \times k$ matrices. Without loss of generality, it can be assumed that Σ is lower triangular. Furthermore, to ensure stationarity of the process, we assume that all the eigenvalues of ρ strictly lie inside the unit circle. The probability measure associated to the above specification of X_t will be denoted by P. X_t is a matrix containing k latent factors, which can be thought of as $k-1$ real factors and one inflation factor.

The second equation relates the one-period interest rate $y_t^{(1)}$ to the state variables (positing that it is an affine function of the state variables):

$$y_t^{(1)} = -\delta_0 - \delta_1^\intercal X_t \ , \tag{7}$$

where δ_0 is a scalar and δ_1 is a $k \times 1$ vector with the last element equal to zero as the real rate is not affected by the inflation rate.

The third equation is related to bond pricing in an arbitrage-free market. A sufficient condition for the absence of arbitrage on the bond market is that there exists a risk-neutral measure Q, equivalent to P, under which the process X_t follows the dynamics:

$$X_t = \overline{\mu} + \overline{\rho} X_{t-1} + \Sigma\varepsilon_t \ , \tag{8}$$

where $\varepsilon_t \sim N(0, I_k)$ under Q and such that the price at time t of a bond paying a unitary amount of cash at time $t+n$ (denoted by p_t^n) equals:

$$p_t^{(n)} = \mathrm{E}_t^Q \left[\exp\left(-r_t\right) p_{t+1}^{(n-1)} \right] \ , \tag{9}$$

where E_t^Q denotes expectation under the probability measure Q, conditional upon the information available at time t. The vector $\overline{\mu}$ and the matrix $\overline{\rho}$ are in general different from μ and ρ, while equivalence of P and Q guarantees that Σ is left unchanged. The link between the risk-neutral distribution Q and the physical distribution P is given by the (time-varying) price of risk which is affine in the state variables:

$$\lambda_t = \lambda_0 + \lambda_1 X_t \ ,$$

where $\lambda_0 = \Sigma^{-1}(\mu - \overline{\mu})$ and $\lambda_1 = \Sigma^{-1}(\rho - \overline{\rho})$. According to Cameron, Martin and Girsanov's theorem

$$\mathrm{E}_t^P\left[\frac{dQ}{dP}\right] = \prod_{j=1}^{\infty} \exp\left[-\frac{1}{2}\lambda_{t+j-1}^{\top}\lambda_{t+j-1} - \lambda_{t+j-1}^{\top}\varepsilon_{t+j}\right] \ ,$$

so that the real pricing kernel

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t^{\top}\lambda_t - \lambda_t^{\top}\varepsilon_{t+1}\right) \ , \tag{10}$$

can be used to recursively price bonds:

$$p_t^{(n)} = \mathrm{E}_t^P\left[m_{t+1}p_{t+1}^{(n-1)}\right] \ . \tag{11}$$

B.2 Observation equation

The first line of model (3) is the observation equation, which describes, within this Gaussian framework, bond yields as affine functions of the state variables:

$$y_t^{(n)} = -\frac{1}{n}\ln\left(p_t^n\right) = A_n + B_n^{\intercal} X_t \ ,$$

where r_t^n is the yield at time t of a bond maturing in n periods and A_n and B_n are coefficients obeying the following simple system of Riccati equations, derived from (9):

$$\begin{aligned} A_1 &= -\delta_0 \\ B_1 &= -\delta_1 \\ A_{n+1} &= -\delta_0 + A_n + B_n^{\intercal}(\mu - \Sigma\lambda_0) - \frac{1}{2}B_n^{\intercal}\Sigma\Sigma^{\intercal}B_n \\ B_{n+1} &= -\delta_1 + B_n^{\intercal}(\rho - \Sigma\lambda_1) \ . \end{aligned} \tag{12}$$

B.3 Term premium

The term premium is defined as

$$tp_t^{(n)} = y_t^{(n)} - \frac{1}{n}\sum_{i=0}^{n-1} E_t y_{t+i}^{(1)} \ ,$$

and by solving forward the model (3) we obtain

$$\begin{aligned} tp_t^{(n)} &= (A_n - A_1) - \frac{1}{n}B_1^{\intercal}(I + \rho + ...\rho^{n-1})\mu + \\ &\quad + \left[B_n^{\intercal} - \frac{1}{n}B_1^{\intercal}(\rho + ...\rho^n)\right] X_t \\ &= A_{n-1} - \frac{1}{n}B_1^{\intercal}(I + \rho + ...\rho^{n-1})\mu + \\ &\quad \left[B_n^{\intercal} - \frac{1}{n}B_1^{\intercal}(\rho + ...\rho^n)\right] X_t + \\ &\quad B_{n-1}^{\intercal}(\mu - \Sigma\lambda_0) - \frac{1}{2}B_{n-1}^{\intercal}\Sigma\Sigma^{\intercal}B_{n-1} \ . \end{aligned}$$

By rearranging the terms, we have that the term premium is given by the sum of a *constant*, $A_{n-1} - B_1^{\intercal}(I + \rho + ...\rho^{n-1})\mu$, a *function of the variables* X,$[B_n^{\intercal} - B_1^{\intercal}(\rho + ...\rho^n)]$,and a *correction for the volatility* (Jensen inequality), $B_{n-1}^{\intercal}(\mu - \Sigma\lambda_0) - \frac{1}{2}B_{n-1}^{\intercal}\Sigma\Sigma^{\intercal}B_{n-1}$.

B.4 Introduction of the interest rate forecast

Now suppose we have the forecast k periods ahead for the interest rate with maturity n, defined as

$$E_t^{survey}(y_{t+k}^{(n)}) = y_{t+k}^{(n)} + e_{t+k} \ ,$$

that forecasts interest rate $y_{t+k}^{(n)}$ with an error e_{t+k}. So, from the state equation we have

$$\begin{aligned} E_t(X_{t+k}) &= (I + ... + \rho^{k-1})\mu + \rho^k X_t \\ &= (I - \rho)^{-1}(I - \rho^k)\mu + \rho^k X_t \ , \end{aligned}$$

and from the observation equation

$$\begin{aligned} E_t^{survey}(y_{t+k}^{(n)}) &= A_n + B_n^{\intercal}E_t(X_{t+k}) + e_t \\ &= A_n + B_n^{\intercal}(I - \rho)^{-1}(I - \rho^k)\mu + B_n^{\intercal}\rho^k X_t + e_t \ . \end{aligned}$$

In case we use the survey of the forecast of the interest rate between periods T_1 and T_2, its expected value is given by

$$\begin{aligned}
E_t^{survey}\left(y_t^{(T_1,T_2)}\right) &= \frac{T_2}{T_2-T_1}E_t^{survey}(y_t^{(T_2)}) - \frac{T_1}{T_2-T_1}E_t^{survey}(y_t^{(T_1)}) \\
&= \frac{1}{T_2-T_1}\left[T_2A_{T_2} + T_2B_{T_2}^{\intercal}(I-\rho)^{-1}(I-\rho^{T_2})\mu + T_2B_{T_2}^{\intercal}\rho^{T_2}X_t + T_2e_t - \right. \\
&\quad \left. -T_1A_{T_1} - T_1B_{T_1}^{\intercal}(I-\rho)^{-1}(I-\rho^{T_1})\mu - T_1B_{T_1}^{\intercal}\rho^{T_1}X_t + T_1e_t\right] \\
&= \frac{1}{T_2-T_1}\left[T_2A_{T_2} - T_1A_{T_1} + T_2B_{T_2}^{\intercal}(I-\rho)^{-1}(I-\rho^{T_2})\mu - \right. \\
&\quad \left. T_1B_{T_1}^{\intercal}(I-\rho)^{-1}(I-\rho^{T_1})\mu + +(T_2B_{T_2}^{\intercal}\rho^{T_2} - T_1B_{T_1}^{\intercal}\rho^{T_1})X_t\right] + e_t \ .
\end{aligned}$$

Define

$$\begin{aligned}
A_{T_1,T_2} &= \frac{1}{T_2-T_1}\left[T_2A_{T_2} - T_1A_{T_1} + T_2B_{T_2}^{\intercal}(I-\rho)^{-1}(I-\rho^{T_2})\mu - T_1B_{T_1}^{\intercal}(I-\rho)^{-1}(I-\rho^{T_1})\mu\right] \\
B_{T_1,T_2} &= \frac{1}{T_2-T_1}\left[T_2B_{T_2}^{\intercal}\rho^{T_2} - T_1B_{T_1}^{\intercal}\rho^{T_1}\right] \ ,
\end{aligned}$$

so that the expected forward rate can be written as

$$E_t^{survey}\left(y_t^{(T_1,T_2)}\right) = A_{T_1,T_2} + B_{T_1,T_2}\cdot X_t + \varepsilon_{t(T_1,T_2)} \ .$$

B.5 Spanned and unspanned factors

Several papers have considered the possibility some factors in a term structure model could be unspanned. Partition X into $m_1 \times 1$ and $m_2 \times 1$ vectors, and suppose that the last m_2 elements of δ_1 are equal to zero, and the upper-right $m_1 \times m_2$ block of ρ^* is equal to zero but the corresponding block of ρ is nonzero. Then the last m_2 elements of the state vector are required for the physical representation of the state vector (and for forecasting future interest rates), but these factors do not affect the cross-section of bond yields today. Then if $m_1 = 3$ and $m_2 = 3$

$$\rho^* = \begin{bmatrix}
\rho_{11}^* & \rho_{12}^* & \rho_{13}^* & 0 & 0 & 0 \\
\rho_{21}^* & \rho_{22}^* & \rho_{23}^* & 0 & 0 & 0 \\
\rho_{31}^* & \rho_{32}^* & \rho_{33}^* & 0 & 0 & 0 \\
\rho_{41}^* & \rho_{42}^* & \rho_{43}^* & \rho_{44}^* & \rho_{45}^* & \rho_{51}^* \\
\rho_{51}^* & \rho_{52}^* & \rho_{53}^* & \rho_{54}^* & \rho_{55}^* & \rho_{56}^* \\
\rho_{61}^* & \rho_{62}^* & \rho_{63}^* & \rho_{64}^* & \rho_{65}^* & \rho_{66}^*
\end{bmatrix} .$$

In our specification we impose the following representation of the matrices Σ and λ_1

$$\Sigma = \begin{bmatrix} 1 & 0 & 0 & \sigma_{14} & \sigma_{15} & \sigma_{16} \\ 0 & 1 & 0 & \sigma_{24} & \sigma_{25} & \sigma_{26} \\ 0 & 0 & 1 & \sigma_{34} & \sigma_{35} & \sigma_{36} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & 0 & 0 \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & 0 & \sigma_{55} & 0 \\ \sigma_{61} & \sigma_{62} & \sigma_{63} & 0 & 0 & \sigma_{66} \end{bmatrix} , \lambda_1 = \begin{bmatrix} \lambda_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ,$$

so that the physical representation of ρ is given by ($\rho^{*(m_i,m_j)}$ indicates the sub-matrix of ρ^* located in the the $m_i - m_j$ block)

$$\begin{aligned} \rho &= \rho^* + \Sigma\lambda_1 \ , \\ \begin{pmatrix} \rho^{(1,1)} & \rho^{(1,2)} \\ \rho^{(2,1)} & \rho^{(2,2)} \end{pmatrix} &= \begin{pmatrix} \rho^{*(1,1)} & \rho^{*(1,2)} \\ \rho^{*(1,2)} & \rho^{*(2,2)} \end{pmatrix} + \begin{pmatrix} \Sigma^{(1,1)} & \Sigma^{(1,2)} \\ \Sigma^{(2,1)} & \Sigma^{(2,2)} \end{pmatrix} \begin{pmatrix} \lambda_1^{(1,1)} & \lambda_1^{(1,2)} \\ \lambda_1^{(2,1)} & \lambda_1^{(2,2)} \end{pmatrix} \\ &= \begin{pmatrix} \rho^{*(1,1)} & \rho^{*(1,2)} \\ \rho^{*(1,2)} & \rho^{*(2,2)} \end{pmatrix} \\ &+ \begin{pmatrix} \Sigma^{(1,1)}\lambda_1^{(1,1)} + \underbrace{\Sigma^{(1,1)}\lambda_1^{(2,1)}}_{=0} & \underbrace{\Sigma^{(1,2)}\lambda_1^{(1,2)}}_{=0} + \Sigma^{(1,2)}\lambda_1^{(2,2)} \\ \Sigma^{(2,1)}\lambda_1^{(1,1)} + \underbrace{\Sigma^{(2,1)}\lambda_1^{(2,1)}}_{=0} & \underbrace{\Sigma^{(2,2)}\lambda_1^{(1,2)}}_{=0} + \Sigma^{(2,2)}\lambda_1^{(2,2)} \end{pmatrix} , \end{aligned}$$

and

$$\begin{pmatrix} 1 & 0 & 0 & \sigma_{14} & \sigma_{15} & \sigma_{16} \\ 0 & 1 & 0 & \sigma_{24} & \sigma_{25} & \sigma_{26} \\ 0 & 0 & 1 & \sigma_{34} & \sigma_{35} & \sigma_{36} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & 0 & 0 \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & 0 & \sigma_{55} & 0 \\ \sigma_{61} & \sigma_{62} & \sigma_{63} & 0 & 0 & \sigma_{66} \end{pmatrix} \begin{pmatrix} \lambda_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda_{11} & 0 & 0 & \sigma_{14}\lambda_{44} & \sigma_{15}\lambda_{55} & 0 \\ 0 & \lambda_{22} & 0 & \sigma_{24}\lambda_{44} & \sigma_{25}\lambda_{55} & 0 \\ 0 & 0 & \lambda_{33} & \sigma_{34}\lambda_{44} & \sigma_{35}\lambda_{55} & 0 \\ \sigma_{41}\lambda_{11} & \sigma_{42}\lambda_{22} & \sigma_{43}\lambda_{33} & \sigma_{44}\lambda_{44} & 0 & 0 \\ \sigma_{51}\lambda_{22} & \sigma_{52}\lambda_{22} & \sigma_{53}\lambda_{33} & 0 & \sigma_{55}\lambda_{55} & 0 \\ \sigma_{61}\lambda_{11} & \sigma_{62}\lambda_{22} & \sigma_{63}\lambda_{33} & 0 & 0 & 0 \end{pmatrix} .$$

So we have that $\rho^{(1,2)}$, the upper-right $m_1 \times m_2$ block of ρ, should be equal to

$$\rho^{(1,2)} = \begin{pmatrix} \sigma_{14}\lambda_{44} & \sigma_{15}\lambda_{55} & 0 \\ \sigma_{24}\lambda_{44} & \sigma_{25}\lambda_{55} & 0 \\ \sigma_{34}\lambda_{44} & \sigma_{35}\lambda_{55} & 0 \end{pmatrix} \Longrightarrow \rho^{*(1,2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

B.6 The complete setup

We use the Consensus Economics surveys for the 3-month (i.e. 13 weeks) and 10-year (i.e. 520 weeks) interest rates 1 year forward, the vector of observable variables, Y_t, is given by the ten zero coupon rates with maturity $52, 104, ..., 520$ weeks, the expected interest rates between one year and one year and three months, $E_t^{survey}\left(y_t^{(52,65)}\right)$, the expected forward rates between one and eleven years, $E_t^{survey}\left(y_t^{(52,572)}\right)$, the 1-year ahead expected inflation, $E_t^{survey}(\pi_{t+52})$, the 1-year ahead expected growth, $E_t^{survey}(g_{t+52})$, and the central bank holdings of bonds as a percentage of the outstanding amount, QE, namely

$$Y_t = \left[y_t^{(1)}, ..., y_t^{(10)}, E_t^{survey}\left(y_t^{(52,65)}\right), E_t^{survey}\left(y_t^{(52,572)}\right), E_t^{survey}(\pi_{t+52}), E_t^{survey}(g_{t+52}), QE\right]^{\intercal} ,$$

the state vector by

$$X_t = \left[X_t^1, X_t^2, X_t^3, E_t^{survey}(\pi_{t+52}), E_t^{survey}(g_{t+52}), QE\right]^{\intercal} ,$$

where X_t^1, X_t^2, X_t^3 are latent variables; the observation equation, iterated by the short-term interest rate equation $y_t^{(1)} = -\delta_0 - \delta_1^{\intercal} X_t$, is given by

$$Y_t = \begin{bmatrix} A_{52} \\ \vdots \\ A_{520} \\ A_{52,65} \\ A_{52,572} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} B_{52} \\ \vdots \\ B_{520} \\ B_{52,65} \\ B_{52,572} \\ 1 \\ 1 \\ 1 \end{bmatrix} X_t + \begin{bmatrix} \underset{(10\times10)}{R} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\tau_{52,65}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\tau_{52,572}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{52} \\ \vdots \\ \varepsilon_{520} \\ \varepsilon_{\tau_{52,65}} \\ \varepsilon_{\tau_{52,572}} \\ 0 \\ 0 \\ 0 \end{bmatrix} ,$$

where the As and Bs are computed in (12), R is a diagonal matrix with a unique parameter σ_y, i.e. $R = \sigma_y \cdot I(10)$; the state equation is given by

$$X_t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mu_\pi \\ \mu_g \\ \mu_{QE} \end{bmatrix} + \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} & \rho_{15} & 0 \\ \rho_{21} & \rho_{22}^{*} & \rho_{23} & \rho_{24} & \rho_{25} & 0 \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} & \rho_{35} & 0 \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} & 0 & 0 \\ \rho_{51} & \rho_{52} & \rho_{53} & 0 & \rho_{55} & 0 \\ \rho_{61} & 0 & 0 & 0 & 0 & \rho_{66} \end{bmatrix} X_{t-1} + \begin{bmatrix} 1 & 0 & 0 & \sigma_{14} & \sigma_{15} & \sigma_{16} \\ 0 & 1 & 0 & \sigma_{24} & \sigma_{25} & \sigma_{26} \\ 0 & 0 & 1 & \sigma_{34} & \sigma_{35} & \sigma_{36} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & 0 & 0 \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & 0 & \sigma_{55} & 0 \\ \sigma_{61} & \sigma_{62} & \sigma_{63} & 0 & 0 & \sigma_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_1 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} ,$$

where we assume that the lagged QE program only impacts the first latent factor and itself; the market-risk equation is given by

$$\lambda_t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \lambda_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} X_t$$

The identification of the model is warranted by the restrictions $\delta_1 > 0$, $\rho^{*(1,2)} = \mathbf{0}$, and the zeros in the covariance matrix of the state equation. The parameters to be estimated are 52.