

# Discussion: Efficient estimation and forecasting in dynamic factor models with structural instability

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### Overview of the paper

- Dynamic factor models where all model parameters are considered time-varying
- Mixture between observation driven and parameter driven approaches
- Variances are made TV using observation driven approach
- Loadings and VAR coefficients are made TV using parameter driven approach
- Two algorithms to approximate posterior mode: 1. filtering based, 2. simulation based
- Two applications: 1. macroeconomic forecasting, 2. yield-spread forecasting
- For the discussion I will focus on algorithm 1 and application 1

## Equations

$$\mathbf{x}_t = \Lambda_t \mathbf{f}_t + \epsilon_t \quad \epsilon_t \sim N(0, \mathbf{V}_t)$$

$$\lambda_t = \lambda_{t-1} + \nu_t \quad \nu_t \sim N(0, R_t)$$

$$\mathbf{f}_t = \mathbf{B}_t \mathbf{f}_{t-1} + \eta_t \quad \eta_t \sim N(0, \mathbf{Q}_t)$$

$$\beta_t = \beta_{t-1} + v_t \quad v_t \sim N(0, W_t)$$

$$\mathbf{V}_t = \delta_1 \mathbf{V}_{t-1} + (1 - \delta_1) \text{diag}(\hat{\epsilon}_t \hat{\epsilon}_t')$$

$$R_t = (\mu_1^{-1} - 1) \text{Cov}(\lambda_{t-1} | \mathbf{x}_1, \dots, \mathbf{x}_{t-1})$$

$$\mathbf{Q}_t = \delta_2 \mathbf{Q}_{t-1} + (1 - \delta_2) \hat{\eta}_t \hat{\eta}_t'$$

$$W_t = (\mu_2^{-1} - 1) \text{Cov}(\beta_{t-1} | \mathbf{x}_1, \dots, \mathbf{x}_{t-1})$$

### Algorithm 1

- 1 Compute  $\hat{f}^{\text{PCA}}$
  - 2 Compute  $\hat{\lambda}_t = E(\lambda_t | X; \hat{f}^{\text{PCA}})$ , for  $t = 1, \dots, T$
  - 3 Compute  $\hat{\beta}_t = E(\beta_t | \hat{f}^{\text{PCA}})$ , for  $t = 1, \dots, T$
  - 4 Compute  $\hat{f}_t = E(f_t | X; \hat{\lambda}, \hat{\beta})$ , for  $t = 1, \dots, T$
- Step (2); also gives  $V_t$  and  $R_t$
  - Step (3); also gives  $Q_t$  and  $W_t$

## General comments and questions

- 1 Ambitious paper!!!
- 2 Consistency of step (1) in algorithm 1? Bates, Plagborg-Moller, Stock & Watson (2013) give rates for  $\lambda_{i,t}$ ? In addition  $B_t$  will also require some restrictions.
- 3 Imposing some structure on the loading matrix? Testing for parameter instability?
- 4 If forecasting and computational speed are the goals; why not entirely observation driven?
- 5 Reasoning the particular observation driven structure?

## EWMA vs GAS update

- EWMA update per element

$$V_{i,t} = \delta_1 V_{i,t-1} + (1 - \delta_1) \hat{\epsilon}_{i,t}^2$$

- GAS update per element

$$V_{i,t} = \delta_1 V_{i,t-1} + (1 - \delta_1) S_t \left( -\frac{1}{2} F_{i,t}^{-1} + \frac{1}{2} \hat{\epsilon}_{i,t}^2 F_{i,t}^{-2} \right)$$

- where  $\hat{\epsilon}_{i,t} = x_{i,t} - \hat{\lambda}_{t|t-1} \tilde{f}_t$  and  $\tilde{f}_t$  is the current estimate for  $f_t$
- and  $S_t$  is a scaling term,  $F_{i,t} = \tilde{f}_t' \hat{P}_{i,t} \tilde{f}_t + V_{i,t-1}$
- Main difference is that GAS update also depends on predictive variance loadings
- Possible room for improvement see Blasques, Koopman & Lucas (2014)

## Some further questions?

- 1 Are the variances in step (4) treated as known? if so why? Re-estimating  $V_t$  and  $Q_t$  is possible? When doing forecasting this will make a difference.
- 2 In general: how are the forecasts constructed?
- 3 Is it possible to estimate model parameters ( $\mu$ 's and  $\delta$ 's) in steps (ii) and (iii) using MLE? Similar as in Eickmeier, Lemke & Marcellino (2011). Not much work and saves grid-searches?
- 4 How do you initialize  $V_0$  and  $Q_0$  in general?

Table 2: Relative MSE for forecasting German GDP growth

Panel A: EWMA

|    |         | parameter specification |            |            |         |         | Relative MSE at horizon |      |      |      |
|----|---------|-------------------------|------------|------------|---------|---------|-------------------------|------|------|------|
|    |         | $r$                     | $\delta_1$ | $\delta_2$ | $\mu_1$ | $\mu_2$ | 1                       | 2    | 3    | 4    |
| 1  | TVP-DFM | 2                       | 0.83       | 0.83       | 1.00    | 1.00    | 0.77                    | 0.95 | 0.98 | 1.08 |
| 2  | TVP-DFM | 3                       | 0.83       | 0.83       | 1.00    | 1.00    | 0.77                    | 0.95 | 0.98 | 1.09 |
| 3  | TVP-DFM | 2                       | 0.87       | 0.83       | 1.00    | 1.00    | 0.77                    | 0.95 | 0.98 | 1.08 |
| 4  | TVP-DFM | 3                       | 0.87       | 0.83       | 1.00    | 1.00    | 0.77                    | 0.95 | 0.98 | 1.09 |
| 5  | TVP-DFM | 2                       | 0.99       | 0.83       | 1.00    | 1.00    | 0.76                    | 0.96 | 1.00 | 1.10 |
| 6  | TVP-DFM | 3                       | 0.99       | 0.83       | 1.00    | 1.00    | 0.77                    | 0.96 | 1.00 | 1.12 |
| 7  | TVP-DFM | 2                       | 0.83       | 0.99       | 1.00    | 1.00    | 0.82                    | 1.04 | 1.09 | 1.10 |
| 8  | TVP-DFM | 3                       | 0.83       | 0.99       | 1.00    | 1.00    | 0.83                    | 1.06 | 1.10 | 1.20 |
| 9  | TVP-DFM | 2                       | 0.99       | 0.99       | 1.00    | 1.00    | 0.82                    | 1.07 | 1.12 | 1.24 |
| 10 | TVP-DFM | 3                       | 0.99       | 0.99       | 1.00    | 1.00    | 0.83                    | 1.08 | 1.13 | 1.24 |
| 11 | TVP-DFM | 2                       | 0.99       | 0.99       | 0.98    | 0.98    | 0.82                    | 1.07 | 1.12 | 1.24 |
| 12 | TVP-DFM | 3                       | 0.99       | 0.99       | 0.98    | 0.98    | 0.83                    | 1.08 | 1.13 | 1.24 |
| 13 | PC      | 1                       | -          | -          | -       | -       | 0.86                    | 1.02 | 1.02 | 1.08 |
| 14 | PC      | 2                       | -          | -          | -       | -       | 0.87                    | 1.01 | 1.01 | 1.09 |
| 15 | PC      | 3                       | -          | -          | -       | -       | 0.91                    | 1.02 | 1.01 | 1.09 |
| 16 | PC      | 4                       | -          | -          | -       | -       | 0.94                    | 1.06 | 1.08 | 1.15 |
| 17 | AR      | -                       | -          | -          | -       | -       | 1.00                    | 1.00 | 1.00 | 1.00 |



## Comments

- 1 The PCA estimator is based on homoscedastic error-variances  $V_t = I_n \sigma$ . In the macro illustration  $V_t$  is initialized with  $V_0 = I_N$ . When  $\delta_1 < 1$  *two* things change: (1) the variances become heteroskedastic and (2) the variances become time-varying. The improvement in forecasting is entirely attributed to the time-variation in the paper. A comparison with standard MLE would give more insight into the role for heterogeneous variances; see also Bai & Li (2012).
- 2 In the simulation study there is also a comparison w.r.t. two-step estimator of Doz, Giannone & Reichlin (2011), which shows that 4-steps improves two-step when time-variation in the loadings is large. Given that the loadings and factor coefficients are not time-varying in this application a comparison with two-step would be insightful.

## Final remarks

- I enjoyed reading the paper.
- Thank you!