

Discussion of Mueller and Watson:

“Measuring Uncertainty about Long-Run Predictions”

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(and the Penn Friday Reading Group)

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Really Technical, Really Fun, Really Useful

g_j and g_k are real valued, (6) can be rewritten as $E(\eta_{j,T}\eta_{k,T}) \rightarrow 2 \int_0^\infty S(\omega) w_{jk}(\omega) d\omega$, where $w_{jk}(\omega) = \text{Re}[\left(\int_0^{1+r} g_j(s) e^{-i\omega s} ds\right) \left(\int_0^{1+r} g_k(s) e^{i\omega s} ds\right)]$. With $g_j = \sqrt{2} \cos(\pi j s)$, a calculation shows that $w_{jk}(\omega) = 0$ for $1 \leq j, k \leq q$ and $j+k$ odd, so that $E(X_j X_k) = 0$ for all odd $j+k$,

$$\begin{aligned} E(\eta_{j,T}\eta_{k,T}) &= T^{-2\alpha} \sum_{s,t=1}^{\lfloor(1+r)T\rfloor} \left(\int_{-\pi}^{\pi} e^{-i\lambda(t-s)} R(\lambda) d\lambda \right) \tilde{g}_{j,t} \tilde{g}_{k,s} \\ &= T^{-2\alpha} \int_{-\pi}^{\pi} R(\lambda) \left(\sum_{s=1}^{\lfloor(1+r)T\rfloor} \tilde{g}_{k,s} e^{i\lambda s} \right) \left(\sum_{t=1}^{\lfloor(1+r)T\rfloor} \tilde{g}_{j,t} e^{-i\lambda t} \right) d\lambda \\ &= T^{1-2\alpha} \int_{-T\pi}^{T\pi} R(\omega/T) \left(T^{-1} \sum_{s=1}^{\lfloor(1+r)T\rfloor} \tilde{g}_{k,s} e^{i\omega(s/T)} \right) \left(T^{-1} \sum_{t=1}^{\lfloor(1+r)T\rfloor} \tilde{g}_{j,t} e^{-i\omega(t/T)} \right) d\omega \end{aligned}$$

transform the sample data $\{x_t\}_{t=1}^T$ into the weighted averages $(\bar{x}_{1:T}, X_T)$, with $X_T = (X_T(1), \dots, X_T(T-1))'$, and where $X_T(j)$ is the j th cosine transformation

$$X_T(j) = \int_0^1 \Psi_j(s) x_{\lfloor sT \rfloor + 1} ds = \iota_{jT} T^{-1} \sum_{t=1}^T \Psi_j\left(\frac{t-1/2}{T}\right) x_t \quad (3)$$

with $\Psi_j(s) = \sqrt{2} \cos(j\pi s)$ and $\iota_{jT} = (2T/j\pi) \sin(j\pi/2T) \rightarrow 1$. We make two remarks about this transformation. First, because the Ψ_j weights add to zero, $X_T(j)$ is invariant to location shifts of the sample. Second, the transformation isolates variation in the sample

Goal

Growth rate x_t , $t = 1, \dots, T$

Estimate $\bar{x}_{T+1:T+h} = \frac{1}{h} \sum_{t=1}^h x_{T+t}$ for large h

Find A s.t. $P(\bar{x}_{T+1:T+h} \in A) = 1 - \alpha$

Important MW Advances

- Eliminate effects of high-frequency misspecification

- Incorporate effects of low-frequency parameter-estimation uncertainty

Methods in Search of Applications?

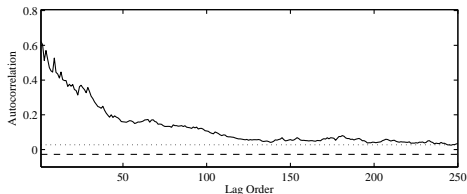
Maybe, but so what?

The methods are great, and there will be plenty of applications.

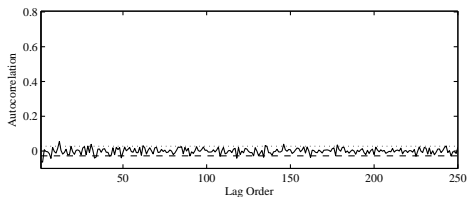
Consider:

- Piketty-type questions
- The equity premium in the 21st century
 - Realized asset return volatility
(High frequency, and undeniable **long memory**)

Long Memory in S&P 500 Realized Volatility



(Hyperbolic) Autocorrelations of Daily Realized Volatility, V_t



(Zero) Autocorrelations of $(1 - L)^4 V_t$

A Trivially Simple Case (to Build Intuition)

DGP: $x_t \sim iid(0, 1)$

Large- h asymptotics: $h \rightarrow \infty$, $T \rightarrow \infty$, $\frac{h}{T} \rightarrow r > 0$

$$\sqrt{T} \left(\underbrace{\frac{1}{h} \sum_{t=1}^h x_{T+t}}_{\text{object of interest}} - \frac{1}{T} \sum_{t=1}^T x_t \right) = \left(\frac{\sqrt{T}}{\sqrt{h}} \frac{1}{\sqrt{h}} \sum_{t=1}^h x_{T+t} - \frac{1}{\sqrt{T}} \sum_{t=1}^T x_t \right)$$

$$\rightarrow_d \left(\frac{1}{\sqrt{r}} z_1 - z_2 \right) \sim N \left(0, \frac{1}{r} + 1 \right)$$

where z_1 and z_2 are independent $N(0, 1)$

Now “simply” generalize from $x_t \sim iid(0, 1)$ to $\Delta x_t \sim I(0)$

Beyond *iid* I: Historical Information

Construct prediction interval conditional on $\tilde{\mathcal{I}}_T = \{X_{T,1:q}\}$

(1) Joint distribution:

$$\begin{pmatrix} \tilde{\mathcal{I}}_T \\ \bar{x}_{T+1:T+h} \end{pmatrix} \rightarrow_d D$$

(2) Then get conditional distribution:

$$p(\bar{x}_{T+1:T+h} | \tilde{\mathcal{I}}_T)$$

- For $x_t \sim I(0)$, $\tilde{\mathcal{I}}_T$ is not useful for forecasting $\bar{x}_{T+1:T+h}$ under large- h asymptotics (short memory)
- For $x_t \sim I(d)$, $d \neq 0$, $\tilde{\mathcal{I}}_T$ is useful for forecasting $\bar{x}_{T+1:T+h}$ even under large- h asymptotics (long memory)

Beyond *iid* II: It's About The Low-Frequency Spectrum

Covariance structure of joint limiting distribution D
depends low-frequency spectrum of x_t

MW parameterize as:

$$S(\omega; b, c, d) \propto \left(\frac{1}{\omega^2 + c^2} \right)^d + b^2$$

b captures local level effects

c captures local-to-unity effects

d captures long-memory effects

Standard Methods Exist for Estimating d Consistently and Robustly (e.g., GPH)

As $\omega \rightarrow 0$,

$$S(\omega) \propto \frac{1}{\omega^{2d}}$$

or:

$$\ln S(\omega) \approx c - 2d \ln \omega$$

So run a regression on low-frequency periodogram ordinates:

$$\ln I(\omega) = \beta_0 + \beta_1 \ln \omega + \varepsilon$$

$$\text{Take } \hat{d} = -\hat{\beta}_1/2$$

Monte Carlo

- One route: more thorough Monte Carlo
 - Another route: *no* Monte Carlo
(do thorough Monte Carlo in a follow-up paper)
- Presently the Monte Carlo appears to be an afterthought, with stochastic volatility curiously stirred into the mix
- Include comparison to forecasts from pure long memory model estimated by GPH, low-frequency Whittle, etc.

Empirics

- For series with long histories it may be possible to do (crude) *checks* of interval calibration.
 - Example: We have about 5000 days of S&P 500 realized volatility. Use 1:100 to forecast 101:200 and see whether the realization is in the interval; use 201:300 to forecast 301:400 and see whether the realization is in the interval; etc. At the end, we have many long-horizon interval forecasts and corresponding realizations. Are approximately 95 percent of the realizations in the (alleged) 95 percent intervals?
- Also explore forecasts from pure long memory model estimated by GPH, low-frequency Whittle, etc.

Miscellaneous

- Relationship to general mean-estimation literature
 - Importance of location-scale invariance
 - Matching CBO is neither here nor there
 - b and c strike me as much less important than d
- Indeed priors in MW empirical work put all mass on $b = c = 0$

The (Massive) Elephant in the Room: **Structural Change**

Imagine standing in 1980 with 30 years of available history, and forecasting average growth over the next 30 years of:

- Cost of a billion floating-point operations
 - Sales of typewriters
 - Number of checks cleared
 - Trading volume in financial derivatives
- Membership in the Soviet Union's Politburo